PHYSICS

A Textbook for Grade 12

P12TB

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Foreword

Liberia, having gone through a period of utmost turmoil till 2003, due to the civil wars, is still reeling under its effect and the added trauma of Ebola in 2014 and effects of the COVID-19 outbreak in 2020. The Liberian government, in the past decade, has made valiant efforts to bring order to the lives of its people. In one such effort, the Ministry of Education (MoE) brought changes to the National Curriculum Framework which are relevant to the present generation, and which would prepare them to meet the challenges of the changing trends of the world. The National Curriculum Framework (NCF) 2018 recommends a change in basic assumptions in the teaching learning process from behaviorist to constructivist approach — moving from hardcore print material to the digital world. Keeping in consideration the sociocultural context and varied experiences of learners as laid down in the Framework, our Teaching Learning Materials are expected to be competent to use multiple methods and techniques like e-learning resources, energized textbooks, and readily available reference material to engage the learners.

As a first initiative, the MoE, through its World Bank-funded Improving Results in Secondary Education (IRISE) project, has adapted textbooks for Grades 10 to 12 in five subjects — English Language and Literature, Mathematics, Biology, Physics and Chemistry.

The National Curriculum Framework, 2018, recommends that children's learning at school is a reflection of their life outside the school and shows them the path to become a responsible citizen who makes knowledge-based choices.This principle marks a departure from the legacy of teacher centered learning to student centered learning. The syllabi and textbooks developed on the basis of the NCF indicate a serious attempt to implement the idea of Activity Base Learning (ABL). We hope these measures will take us ahead in the direction of building a system of education as outlined in the NCF.

Combined with the efforts by the school principals and teachers this will encourage children to reflect on their own learning and to pursue imaginative activities and questions.With this in mind, perhaps for the first time in our country, we are able to provide separate subject specific textbooks accompanied with guides for teachers for 10–12 grades. Not only have these been developed, adapted and modified to the Liberian context, each of the eight Minimum Learning Competencies (MLCs) have been included in each textbook. So as to reach every high school student, for the first time in the country's history we have included the digitized form of the textbook accessible by a Quick Response (QR) code given in each book. Not only does it have the digitized textbook, but it provides additional learning materials for use by students, teachers and interested persons. The links to these e-resources and digitized material is being made available on the MoE's website.

The Textbooks and Teacher Guides have reached the hands of the students after a rigorous quality evaluation by carefully handpicked subject specialists by the MoE, to whom the Ministry expresses gratitude. For the success of this project, I acknowledge the contributions of the IRISE Project Team in the World Bank, and in particular, the Task-Team Leaders; the Project Implementation Team in Liberia headed by its Coordinator Abraham A. Kiazolu II, supported by the Executive Director of the Center of Excellence for Curriculum Development and Textbooks Research, Mrs. Julia K. Sandiman-Gbeyai and her technical working group (TWG), and the International Textbook Consultant and Advisor, Dr Shveta Uppal engaged by the MoE. These notwithstanding would not have been possible without the guidance of the Senior Management Team (SMT) of the Ministry of Education, and in particular, the Deputy Ministers for Instructions, Administration, and Planning, Research and Development, respectively.

Professor Dao Ansu Sonii, Sr.
Minister of Education
Republic of Liberia

Monrovia, Republic of Liberia
January 24, 2023

Acknowledgments

The development of textbooks contributes to the quality of teaching and learning that go on in the classroom.

The Ministry of Education (MoE) has aligned its Curriculum for Grades 10–12 to the National Curriculum Framework (NCF) of 2018. To ensure the provision of Teaching Learning Materials (TLMs) that support the revised curriculum, the Ministry has sought, reviewed and adapted a new set of textbooks and teacher guides along with digitized contents and e-learning resources for the five core subjects taught at the Senior Secondary education level, namely English Language and Literature, Mathematics, Biology, Chemistry and Physics, through an internationally competitive bidding process from the market supported by the World Bank funded Improving Results in Secondary Education (IRISE) Project.

With profound gratitude and honor, we recognize the Senior Management Team of the Ministry, headed by the Coach, Professor D. Ansu Sonii, Sr., for the strategic decision to make teaching learning materials available and accessible to all in the Liberian Senior Secondary School System, and for providing directions through the process of securing these textbooks and other teaching learning materials for our students and teachers. Our special thanks and appreciation to the World Bank for the financial support towards this policy intervention, and its education task-team including Alonso Sanchez, Oni Lusk-Stover and Binta B. Massaquoi for all their technical inputs offered throughout the process to ensure the kind of quality TLMs the Liberian students deserve are made available for improved learning outcomes.

We would like to specifically recognize the invaluable contributions of the 15 subject experts selected by the MoE from across the various education systems and the West African Examinations Council (WAEC) to evaluate, review and sign off on these teaching learning materials. They didn't just deliver according to our expectations, but also ensured the contextual relevance of the materials to the Liberian Secondary Education Curriculum and its minimum learning competencies (MLCs). These subject experts include Professor Isaac Saye-Lakpoh Zawolo – *Superintendent* of the Monrovia Consolidated School System (MCSS), Mr. Matthew V.Z. Darblo, Sr. – *Mathematics Instructor* at the University of Liberia (UL), Mr. Charles Tieh Bropleh – *Mathematics Specialist* (MoE), Mrs. Linda Y. Dean – *English Specialist*, Mr. Hassan M. Bangura – *English Language and Literature Expert*, Mr. J. Emmanuel Milton – *English Specialist* (MoE), Mr. Moses K.M. Togbah – *Physics Specialist*, Mr. Prince A. Dossen – *Physics Specialist*, Mr. Benjamin Koryah – *Physics Instructor* at the University of Liberia (UL), Mr. Dominic Dugbe Doe – *Chemistry Specialist*, Mr. Patrick A. Anderson, Sr. – *Director o*f the Division of Technical and Vocational Education (MoE), Mr. Kandakai Massaquoi – *Chemistry Specialist*, Ms. Patricia N. Doe – *Head* of Biology Department, African Methodist Episcopal University (AMEU), Mr. Job Carpenter – *Biology Specialist* and Mr. Prince Philip K.A. Aderibigbe – *Biology Specialist*.

The MoE is sincerely grateful to Dr Shveta Uppal, the *International Textbook Consultant* engaged by the IRISE Project to provide technical guidance and quality assurance support to the revising of the Textbooks Management Guidelines (TMG) and the procurement process leading to the provision of textbooks, teacher guides, digital contents and e-learning resources for the Senior Secondary School System in Liberia in accordance with the revised TMG. Heartfelt thanks and appreciations also to the *Executive Director* for the Center of Excellence for Curriculum Development and Textbooks Research, Mrs. Julia K. Sandiman-Gbeyai, and members of her Technical Working Group (TWG) for taking up the responsibility to lead the process of making textbooks and other TLMs available to Liberian students and teachers.

Lastly, we acknowledge the IRISE Project Delivery Team led by Mr. Abraham A. Kiazolu, II – *Project Coordinator*, Mr. Fuseini A. Abu – *International Procurement Specialist* and Mr. Lawrence S. Taylor – *Project Control Specialist* who coordinated the entire process.

We remain grateful to you all!

Hon. Alexander N. Duopu, Sr.,
Deputy Minister for Instruction
Ministry of Education, Republic of Liberia
#The Teacher

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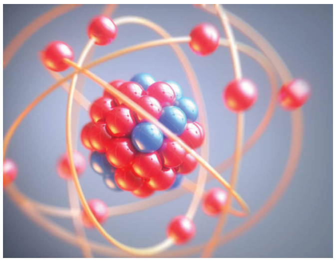
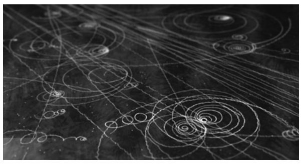




P12CH01

CHAPTER 1

REFRACTION AND DISPERSION OF LIGHT

Chapter Contents

Chapter Outcome

Learners will able to:

- recognize the sources of colors in respect to producing others colors and identify the lenses and their uses in the correction of eye defects.

Chapter Objectives

After completing this chapter, you will be able to:

- recognize situations in which refraction will occur;
- identify which direction light will bend when it passes from one medium to another;
- solve problems using Snell's law;
- describe critical angle in relation to total internal reflection;
- distinguish between primary colors and primary pigments.

Introduction

In this chapter, you will study optical phenomena associated with the refraction of light as it passes from one transparent material to another. The transparent material through which light travels is called the medium (plural media). Glass, water, diamond, and quartz are all examples of transparent ***media*** through which light can pass.

Optical devices, such as cameras, microscopes, and telescopes, use the principles of reflection and refraction to create images that we can use for many scientific applications. An understanding of how lenses function is also essential to the study of optometry.

Key Terms

- Refraction
- Medium
- Speed of light

If you look at the surface of a lake, you can see sunlight reflecting off the water. But you also see rocks and plants under the surface. Because you see them, light must have entered the water, reflected off the rocks and plants, returned to the water surface, and then traveled from the surface to your eyes. When you look carefully at objects in the water, however, you notice that they look closer than normal. This phenomenon occurs because light is bent at the boundary between the water and the air around it. The bending of light as it travels from one medium to another is called refraction. In this section you will learn how and when refraction occurs including the laws of refraction.

Refraction

When light crosses the boundary between two transparent materials, some of it is reflected and some is transmitted. For example, when a light ray traveling in air is incident on a transparent material such as glass, it is partially reflected and partially transmitted (Figure 1). But the direction of the transmitted light is different from the direction of the incident light, so the light is said to have been ***refracted***; in other words, it has changed direction.

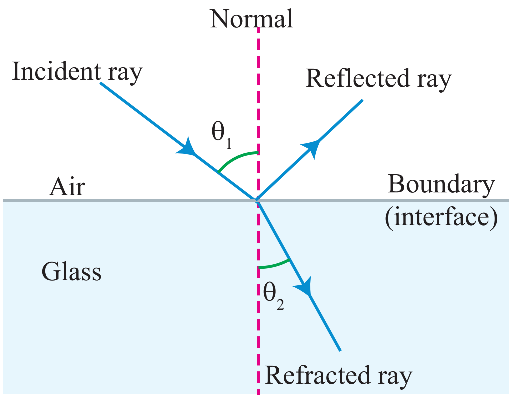


Figure 1.

- ***Refraction*** *refers to the change in direction of a light ray at a boundary where it passes from one medium to another.*

As in the case of reflection, the angles of the *incident* and *refracted rays* are measured with respect to the normal. For studying refraction, the normal line is extended into the refracting medium, as shown in Figure 1. The angle between the refracted ray and the normal is called the angle of refraction, θ_2, and the angle of incidence is designated as θ_1.

Discuss 1: In Figure 1, what is the angle between the normal line and the boundary between air and glass?

Refraction occurs when the velocity of light changes

As you know, the speed of light in vacuum is $c = 3.00 \times 10^8$ m/s. Vacuum is an empty space through which light travels with the fastest speed. However, when light travels through a dense material like water, its speed is reduced.

Discuss 2: Verify that the path of a light ray that crosses a boundary between two different media is reversible. Support your answer using a light ray crossing a boundary between the two transparent materials air and glass in Figure 1.

Exercises

1. How does the angle of incidence compare with the angle of refraction when a light ray passes from air into glass at a non-zero angle of incidence?
2. How does the angle of incidence compare with the angle of refraction when a light ray leaves glass and enters air at a non-zero angle of incidence?

Refractive Index

Key Terms

- index of refraction
- Snell's law
- apparent depth

What happens when you shine a narrow beam of light at the surface of a piece of glass? It bends as it crosses the boundary from air to glass. The bending of light, called refraction, was first studied by René Descartes and Willebrord Snell around the time of Kepler and Galileo. In this section you learn how to determine the amount of refraction using Snell's law.

Experiments show that the speed of light in water, v, is smaller than the speed of light in a vacuum, c, by a factor of 1.33. That is,

$$v = \frac{c}{1.33}$$

The index of refraction of a material is the factor by which the speed of light in the material is reduced. Therefore, the index of refraction of water is 1.33.

- *In general, the index of refraction* n *for a material is the ratio of the speed of light in a vacuum* c *to the speed* v *of light in that material.*

$$n = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in a material}}$$

$$n = \frac{c}{v}$$

Notice that the larger the index of refraction, the smaller the speed of light. Values of the index of refraction for a variety of materials are given in **Table 1**.

Table 1 Index of Refraction for Common Materials

Material	Index of refraction
Diamond	2.42
Crown glass	1.52
Flint glass	1.66
Quartz (glass)	1.46
Ice	1.31
Benzene	1.50
Water	1.33
Air	1.000293

Exercises

Light rays cross interfaces from medium 1 into medium 2 and then into medium 3 (Figure 2). What can we say about the relative sizes of the indices of refraction of these media?

(a) $n_1 < n_2 < n_3$

(b) $n_2 < n_1 < n_3$

(c) $n_2 < n_3 < n_1$

(d) $n_3 < n_1 < n_2$

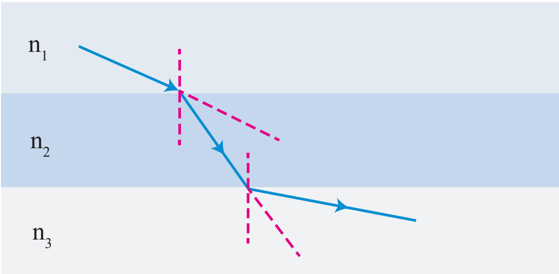


Figure 2.

Explanation: The rays are bent toward the normal when crossing into medium 2, so $n_2 > n_1$. But rays are bent away from the normal when going into medium 3, so $n_3 < n_2$. To find the relationship between 1 and 3, ignore medium 2. So, the rays are bent away from the normal if they would pass from 1 directly into 3. Thus, we have: $n_3 < n_1 < n_2$.

The correct answer is D.

Exercises

Parallel light rays cross interfaces from air into three different media, 1, 2 and 3, as shown in Figure 3. In which of the media is the light traveling fastest?

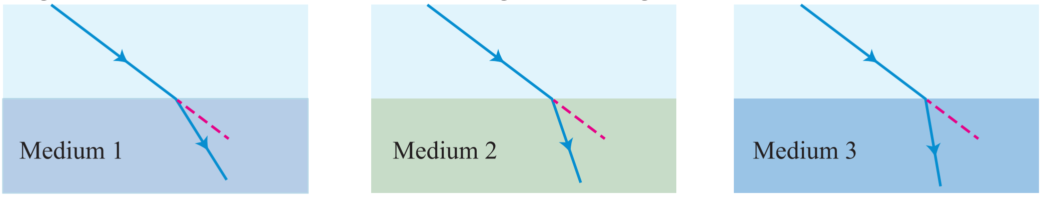


Figure 3.

(a) Medium 1

(b) Medium 2

(c) Medium 3

(d) All the same

Explanation: Comparing the bending of light in the three media, we see that the lesser the bending the greater is the speed.

The correct answer is A.

Example

Light travels at a speed of 2.29×10^8 m/s in a certain material. What material could this be? (Use the speed of light in a vacuum to be 3×10^8 m/s)

Solution

The speed of light in the given material is: $v = 2.29 \times 10^8$ m/s

Solving the equation $v = c/n$ for n, we get

$$n = \frac{c}{v} = \frac{3 \times 10^8 \text{m/s}}{2.29 \times 10^8 \text{m/s}} = 1.31$$

Referring to Table 1.1 for the indices of refraction of materials, the given material is ice.

Example

The speed of light in vacuum is 3×10^8 m/s. What is the speed of light in glass ($n_g = 1.5$)?

Solution

$c = 3 \times 10^8$ m/s

$n_a = 1.0$ (index of refraction of air)

$n_g = 1.5$ (index of refraction of glass)

The speed of light in glass is

$$v = \frac{c}{n} = \frac{3 \times 10^8 m/s}{1.5} = 2.0 \times 10^8 m/s$$

Note: The index of refraction is a dimensionless quantity (i.e., it has no unit).

Exercises

For all transparent materials, the index of refraction is

(a) less than 1 (b) greater than 1 (c) equal to 1.

Verify your answer using Table 1.1 and the equation $v = c/n$

Discuss 3: What is wrong with the statement "The index of refraction of a certain medium is less than one"?

Snell's Law

The angle of refraction θ_2 depends on the angle of incidence θ_1 and on the indices of refraction, n_2 and n_1, of the two media. The relation between these quantities is known as **Snell's law of refraction**, after the Dutch mathematician Willebrord Snell (1591–1626), who discovered it experimentally.

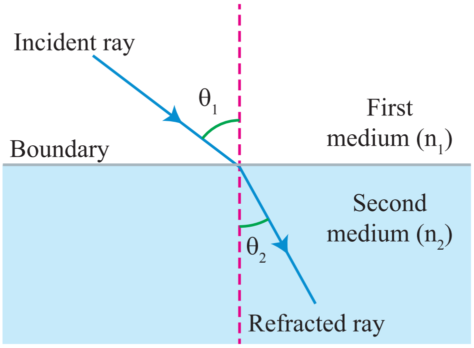


Figure 4.

The law of refraction can be stated as follows:

1. When light travels from a material with refractive index n_1 into a material with refractive index n_2, the refracted ray, the incident ray, and the normal to the interface between the materials all lie in the same plane.
2. The angle of refraction θ_2 is related to the angle of incidence θ_1 by

$n_1 \sin \theta_1 = n_2 \sin \theta_2$ (Snell's Law)

Figure 4 illustrates this situation: If $n_2 > n_1$, then the ray bends towards the normal and $\theta_2 < \theta_1$.

Exercises

What happens when a light ray passes from water to air?

Explanation: Water is denser than air. So, the ray will be refracted away from the normal. In this case the angle of refraction θ_2 is larger than the angle of incidence θ_1 (Figure 5(a)).

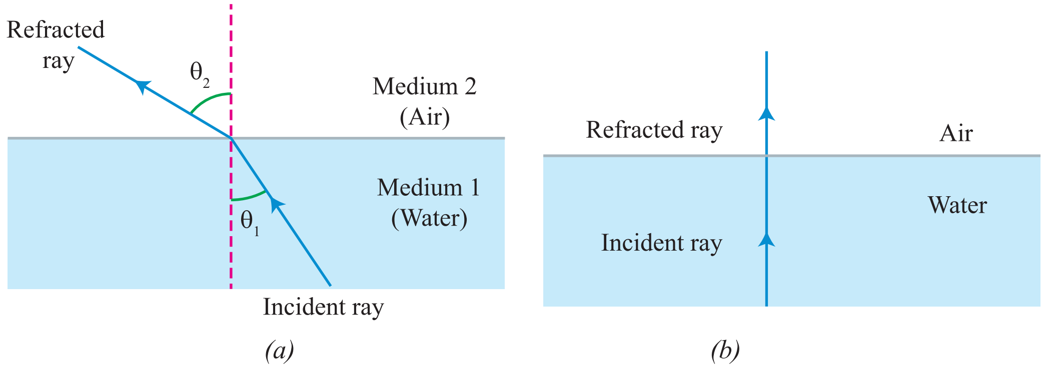


Figure 5.

Note: If the light ray strikes a surface perpendicularly (i.e. along the normal), the angle of incidence is zero, and the angle of refraction will also be zero (Figure 5 (b)). The refracted ray leaves perpendicular to the surface and does not change direction.

Example

Light traveling in water is incident on a piece of crown glass at an angle of incidence of 53°. (a) In which direction will the transmitted ray be bent? (b) What is the angle of refraction?

Solution

(a) The index of refraction of crown glass ($n_2 = 1.52$) is greater than that of water ($n_1 = 1.33$). In other words, glass is denser than water; so, the ray will bend towards the normal.

(b) Since $n_2 > n_1$, according to the law of refraction, $n_1 \sin \theta_1 = n_2 \sin \theta_2$, we can predict that $\theta_2 < \theta_1$. Mathematically,

$n_1 = 1.33$ $\quad \theta_1 = 53^o$

$n_2 = 1.52$ $\quad \theta_2 = ?$

Applying Snell's Law, we have

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\sin \theta_2 = \frac{n_1 \sin \theta_1}{n_2} = \frac{(1.33) \sin 53^o}{1.52} = \frac{(1.33)(0.8)}{1.52} = 0.7$$

$$\theta_2 = \sin^{-1}(0.7) = 44.4^o$$

Displaced Light When a ray of light passes through a glass slab, it refracts first toward the normal, then away from the normal as shown in Figure 6. The net result is that the ray continues in its original direction but is displaced sideways.

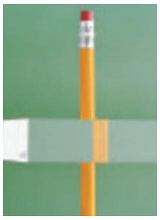
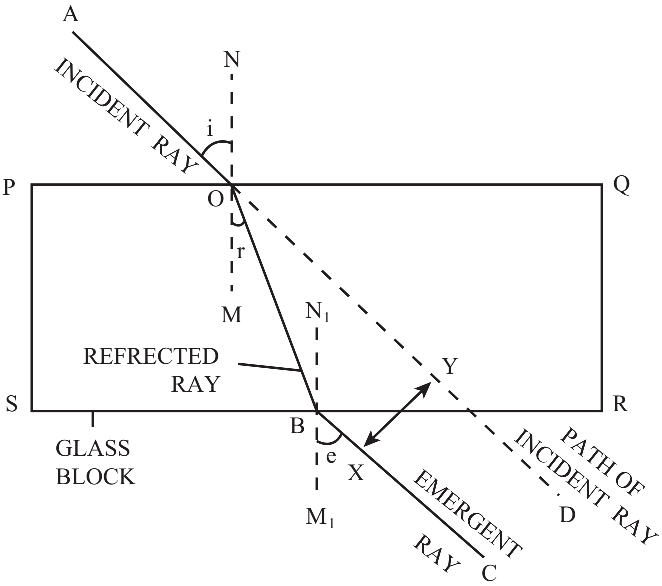


Figure 6.

Refraction through rectangular prism

A three dimensional solid shape having six faces, two at the top and bottom and rest four are lateral faces. Due to its shape, a rectangular prism is also called a rectangular glass slab and a cuboid.



The ray of incident light enters rectangular prism from air (rarer medium) to glass (denser medium) causing the light to bend towards normal due to difference in the refractive index of two mediums. After travelling in the rectangular prism, the ray of light emerges from glass (denser medium) back to air (rarer medium) bending away from normal.

The path of light passing through the rectangular prism PQRS is shown in the above figure. The ray of light incidents on the surface PQ with incidence angle i with normal NM and refracted ray making an angle r. The ray of light emerges from the surface SR with an angle e as shown in the above figure.

Using Snell's law, $\sin(i)/\sin(r) = n_2/n_1$ …….. (1)

Since, NM is parallel to N1M1, refracted ray as the transversal, r is the incident angle at surface SR.

Thus, $\sin(r)/\sin(e) = n_1/n_2$ …….. (2)

multiplying (1) and (2), we get

$[\sin(i)/\sin(r)][\sin(r)/\sin(e)] = [n_2/n_1][n_1/n_2]$

$\sin(i)/\sin(e) = 1$

$\sin(i) = \sin(e)$; hence, $i= e$

This means that incident ray and emergent ray are parallel to each other and does not lie on the same line and emergent ray is laterally shifted by some amount from the incident ray.

Example

A beam of light traveling in air strikes a glass slab at an angle of incidence of 45° as shown in Figure 7. (a) If the index of refraction of the glass is 1.5, what is the angle of refraction for the light transmitted into the glass? (b) Prove that the beam emerging from the other side of the glass is parallel to the incident beam—that is, $\theta_4 = \theta_1$.

Solution

$n_1 = 1.0$ (air) $\theta_1 = 45^o$

$n_2 = 1.5$ (glass) $\theta_2 = ?$

Apply Snell's law,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{(1.0)\ \sin\ 45^o}{1.5}$$

$$\sin\theta_2 = \frac{(1.0)(0.707)}{1.5} = 0.47$$

$$\theta_2 = \sin^{-1}(0.47) = 28^o$$

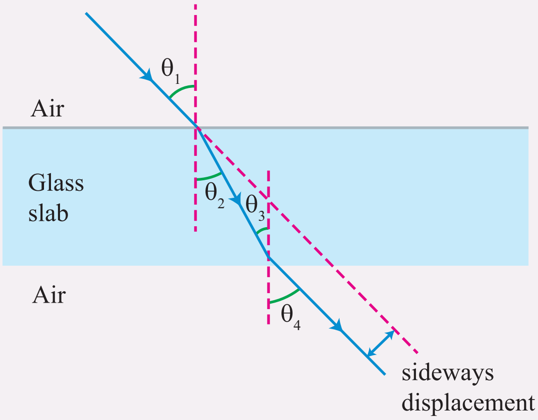


Figure 7.

(a) Here refraction occurs twice. First, in going from air to glass, the ray is bent toward the normal. Then, in going from glass to air, the ray is bent away from the normal. If $\theta_1 = \theta_4$, then the emergent ray is parallel to the incident ray. Applying the law of refraction at both surfaces,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

From the figure, $\theta_2 = \theta_3$,

$$n_2 \sin\theta_3 = n_1 \sin\theta_4$$

Therefore,

$$n_1 \sin\theta_1 = n_1 \sin\theta_4 \text{ or } \theta_1 = \theta_4$$

This means that the ray emerges from the glass at the same angle that it entered, for any angle of incidence θ_1. The slab of glass has not changed the direction of the light. The ray continues in its original direction, but has been displaced slightly by an amount proportional to the thickness of the slab. If the two surfaces of the glass were not parallel (i.e., if it is not a rectangular slab), the direction of the light would be changed, as seen in the following conceptual exercise of a prism.

Note: If a pencil is placed behind such a slab of glass as shown in Figure 6, it appears to be disjointed, because light is refracted as it passes through the slab.

Activity 1

To observe a light that is displaced sideways by a transparent material you need the following materials: a light source, a glass slab, a rectangular glass container filled with water, two sheets of white paper, a protractor, and a ruler.

Place the glass slab on the paper and draw around it. Turn on the light source and send a beam of light onto the glass as shown in Figure 8 (a). Observe how the light ray changes its direction as it enters the glass medium. Mark the path on the paper showing where it is incident and where it is emerged. Remove the glass and join the marks with a ruler. Label the normal (N), the incident light, the refracted light, and the emergent light on the paper. Measure the angles.

Now, repeat the above procedure using the rectangular glass shown in Figure 8 (b) that is filled with water. Do you observe the light bending by the same amount as it passes from air into water?

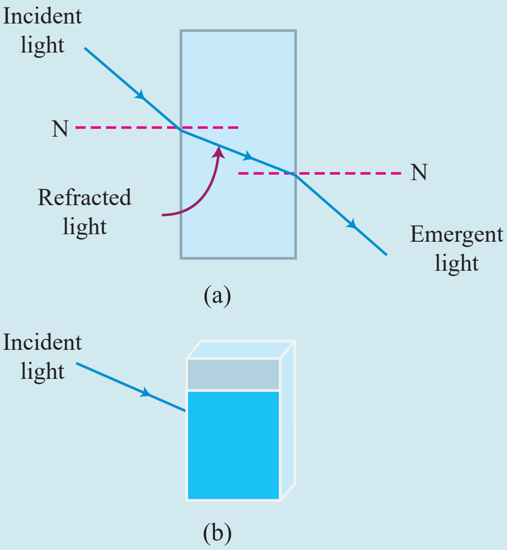
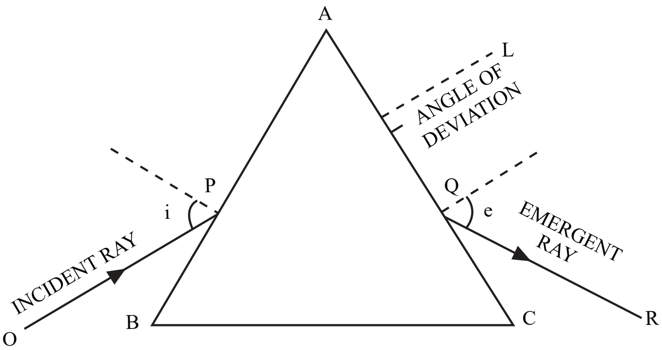


Figure 8.

Refraction through glass prism

A glass prism has two triangular bases with three rectangular faces. The angle between each surface is known as the angle of prism.



ABC is the principle section of a triangular glass prism as shown in the figure. OP is the ray of light incident on the face AB of the prism. This ray enters a denser medium and therefore bends towards the normal and travels along PQ in the prism.

At face AC, the ray PQ suffers another refraction bending away from the normal as it emerges along QR. The angle of deviation δ is the angle between the incident and the emergent rays as shown in the above figure.

From quadrilateral APNQ, $A + N = 180^{o}$

From triangle PNQ, $r_1 + r_2 + N = 180^{o}$

Therefore, $A = r_1 + r_2$

From the triangle MPQ, we have $\delta = \delta_1 + \delta_2$

$\delta_1 = i - r_1$, and $\delta_2 = e - r_2$

$\delta = (i - r_1) + (e - r_2)$

$\delta = (i + e) + A$

When deviation δ is minimum;

$\delta = (\delta_m)$, $i = e$ and $r_1 = r_2 = r$

$\delta_m = 2i - A$

$$i = \frac{(A + \delta_m)}{2}, \text{ and } r = \frac{A}{2}$$

From Snell's law, the refractive index of the material of the prism is:

$$n = \left[\frac{\sin(i)}{\sin(r)}\right] = \frac{\sin\left[\frac{(A + \delta_m)}{2}\right]}{\left[\sin\left(\frac{A}{2}\right)\right]}$$

Exercises

A ray of light is incident on a glass prism as in Figure 9 (a). Indicate the path followed by the ray after passing through the prism.

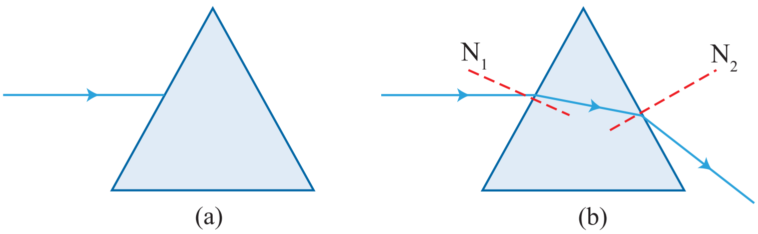


Figure 9.

Explanation When the ray enters the prism it is bent toward the normal (N_1) which deflects it downward, as shown in Figure 9 (b). When it leaves through the opposite side of the prism, it is bent away from the normal (second normal, N_2). Because the sides of a prism are angled in opposite directions, bending away from the normal in the second refraction also causes a downward deflection.

Example

It is found experimentally that a light ray entering a liquid from air at an angle of incidence of 37° exhibits an angle of refraction of 30° in the liquid. What is the speed of light in the liquid?

Solution

$n_1 = 1$ (air) $\quad c = 3 \times 10^8$ m/s

$\theta_1 = 37^o$ $\quad v = ?$

$\theta_2 = 30^o$

First, apply Snell's law to find the index of refraction of the liquid n_2,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1.0)\sin 37^o}{\sin 30^o} = \frac{1.0 \times 0.6}{0.5} = 1.2$$

Now the speed of light in the liquid is,

$$v = \frac{c}{n} = \frac{3 \times 10^8 \text{ m/s}}{1.2} = 2.5 \times 10^8 \text{ m/s}$$

Example

A ray of light traveling in air enters water at a certain angle. How long does it take for light to travel 6.00 m in the water?

Solution

$s = 6.00$ m

$n = 1.33$

The speed of light in water is c/n. Therefore, the time required to cover 6.00 m is

$$t = \frac{s}{v} \text{ but } v = \frac{c}{n}$$

$$\text{Then, } t = \frac{s}{v} = \frac{s}{\frac{c}{n}} = \frac{3.00 \text{ m}}{\frac{3.00 \times 10^8 \text{ m/s}}{1.33}} = 1.33 \times 10^{-8}\text{s}$$

Example

In a certain time, light travels 3.50 km in air. During the same time, light travels only 2.50 km in a liquid. What is the refractive index of the liquid?

Solution

$s_1 = 3.50$ km = 3500 m (distance traveled in air)

$s_2 = 2.50$ km = 2500 m (distance traveled in liquid)

$c = 3 \times 10^8$ m/s

$t_1 = t_2 = t$ (Equal time interval)

First, the speed of light in the liquid must be obtained. To do this we calculate the time taken for light to travel 3500 m in air and use this time to determine the speed of light in the liquid.

$$t = \frac{s}{c} = \frac{3500 \text{ m}}{3 \times 10^8 \text{ m/s}} = \frac{35}{3} \times 10^{-6} \text{s}$$

Then, $$v = \frac{s}{t} = \frac{2500 \text{ m}}{\frac{35}{3} \times 10^{-6} \text{ s}} = 2.14 \times 10^8 \text{ m/s}$$

Index of refraction and speed of light are related by the equation,

$$n = \frac{c}{v} = \frac{3 \times 10^8 \text{ m/s}}{2.14 \times 10^8 \text{ m/s}} = 1.4$$

Note that in this equation the first medium is air where $n_1 = 1$. For any two media having indices of refraction of n_1 and n_2, the following expression also holds.

$$\frac{n_2}{n_1} = \frac{c}{v} \text{ or } \frac{n_2}{n_1} = \frac{v_1}{v_2}$$

The speed of light

Light is an electromagnetic (EM) wave like ultraviolet, infrared, and radar waves. All EM waves travel at the same speed, $c = 3 \times 10^8$ m/s in vacuum. However, each wave and each color in the spectrum of light has its own wavelength λ and frequency f. So, the speed v is determined using the wave equation,

$$v = \lambda f$$

Example

A light wave has a frequency of 5×10^{14} Hz in air. Find the wavelength of this wave in glass (n = 1.5).

Solution

When a wave travels from one medium into another, the frequency of the wave is **unchanged**, but the wavelength changes according to the equation $v = \lambda f$.

Thus, air (approximated to vacuum), $\lambda_1 = c/f$ and in a material medium, $\lambda_2 = v/f$.

We are given, $f_1 = f_2 = 5 \times 10^{14}$ Hz

$n_1 = 1$ (air)

$n_2 = 1.5$ (glass)

Applying the equation, $n = c/v$, we have

$$n = \frac{c}{v} = \frac{\lambda_1 f}{\lambda_2 f} = \frac{\lambda_1}{\lambda_2} \text{ or } \left\| \frac{n_2}{n_1} = \frac{\lambda_1}{\lambda_2} \right\| \text{-----------} \quad (\bigstar)$$

The wavelength in the first medium (air) is,

$$\lambda_1 = \frac{c}{f} = \frac{3 \times 10^8 \text{ m/s}}{5 \times 10^{14} \text{ Hz}} = 6 \times 10^{-7} \text{m}$$

and the wavelength in the second medium (water) is, therefore,

$$\frac{\lambda_1}{\lambda_2} = \frac{n_2}{n_1}$$

$$\lambda_2 = \frac{n_1}{n_2} \times \lambda_1 = \frac{1 \times 6 \times 10^{-7} \text{m}}{1.5} = 4 \times 10^{-7} \text{m}$$

Exercises

The index of refraction for water is 4/3. What happens to light when it travels from air into water?

(a) Its speed increases to 4c/3 and its frequency decreases.

(b) Its speed decreases to 3c/4, and its wavelength decreases by a factor of 3/4.

(c) Its speed decreases to 3c/4, and its wavelength increases by a factor of 4/3.

(d) Its speed decreases to 3c/4 and its frequency increases.

Apparent and real depth

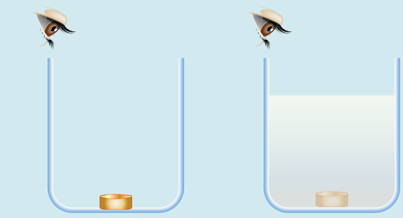
Refraction is responsible for several common "optical illusions." For example, a pencil placed in a glass of water appears to be bent, though it is still perfectly straight. The cause of this illusion is shown in Figure 10. The figure shows that rays leaving the water bend away from the normal and make the pencil appear to be above its actual position.

Figure 10.

- *This phenomenon is an example of what is known as* ***apparent depth****, in which an object appears to be closer to the water's surface than it really is.*

Activity 2

To observe apparent depth due to refraction of light, you need the following materials: A cup, a coin, a glass, and water.



(a) *(b)*

Figure 11.

Place the coin into the glass. Move your head slightly back until the coin is just out of sight (Figure 11 (a)). Now, without moving your head, fill the glass with water (Figure 11 (b)). As the water rises, do you see the coin?

As water fills the glass, the coin comes into sight. How does this happen? The light rays from the coin bend away from the normal as they pass into the air. Therefore, the observer sees the coin at the point where the extensions of the refracted rays cross.

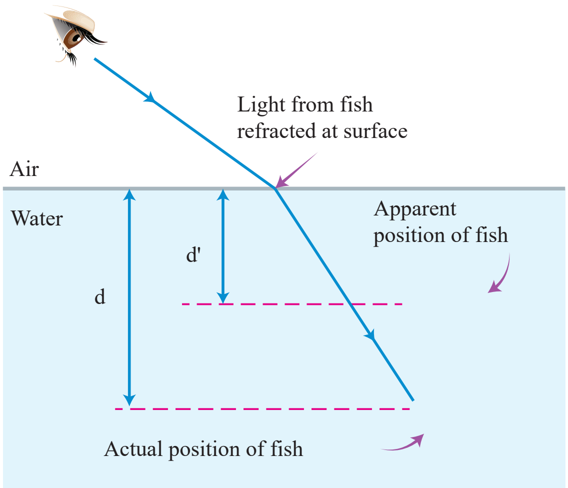


Figure 12.

Consider viewing a fish underwater, as shown in Figure 12. Because of refraction, light from the fish doesn't go straight to your eye, i.e., the position you see isn't the actual position of the fish. The relation between the true depth, d, and apparent depth, $d^!$ is given by the equation:

$$\text{Apparent depth} = \frac{\text{real depth}}{\text{index of refraction}}$$

$$d^! = \frac{d}{n}$$

Example

A coin is lying at the bottom of a pool of water that is 1.50 m deep. But the coin doesn't look that deep. How deep does the coin appear to be when you look straight down into the water?

Solution

The real depth of the coin is, d = 1.50 m, and the index of refraction of water is, n = 1.33. The apparent depth of the coin is, therefore,

$$d^! = \frac{d}{n} = \frac{1.60\text{ m}}{1.33} = 1.20\text{ m}$$

Notice that the apparent depth, $d^!$ is less than the real depth d. Because of refraction the coin appears to be closer than it actually is.

Exercises

A coin is resting on the bottom of an empty container. The container is then filled to the brim three times, each time with a different liquid. With liquid A in the container, the apparent depth of the coin is 7 cm, with liquid B it is 6 cm, and with liquid C it is 5 cm. Which one of the following is correct about the indices of refraction of the liquids?

(a) $n_A > n_B > n_C$

(b) $n_A > n_C > n_B$

(c) $n_C > n_A > n_B$

(d) $n_C > n_B > n_A$

Mirage

The mirage is another phenomenon of nature produced by refraction in the atmosphere. A mirage can be observed when the ground is so hot that the air directly above it is warmer than the air at higher elevations. These layers of air at different heights above Earth have different refractive indices.

One of the most common mirages, often seen in hot weather, makes a stretch of road looking like the surface of a lake (Figure 13). The blue color that so resembles water to our eyes is actually an image of the sky, refracted by the hot, low-density air above the road.

Figure 13.

- A mirage is produced when light bends upward due to the low index of refraction of heated air near the ground.

Exercises

1. What is the relationship between the speed of light and the index of refraction of a transparent substance?
2. What are the three conditions that must be met for refraction to occur?
3. A ray of light has an angle of incidence of 30° on a block of unknown material and an angle of refraction of 20°. What is the index of refraction of the material?

4. You notice that when a light ray enters a certain liquid from water, it is bent toward the normal, but when it enters the same liquid from crown glass, it is bent away from the normal. What can you conclude about the liquid's index of refraction?
5. What is the speed of light in chloroform (n = 1.51)?
6. As light travels from one medium to another, does its wavelength change? What about its frequency? Its speed?

KEY TERMS

- Critical angle
- Mirage

An interesting phenomenon occurs when light travels from a medium with a higher index of refraction into a medium with a lower one, such as when light goes from water into air. If you've ever looked upward from the bottom of a swimming pool, you've probably noticed an interesting effect. Directly overhead you see the ceiling or the sky. As you look farther away from the vertical, however, you can no longer see out of the pool. Instead, you see a reflection of the bottom of the pool. In this section you will learn how this comes about.

Critical angle

When light passes from a medium of larger refractive index into one of smaller refractive index, for example, from water to air, the refracted ray bends away from the normal, as in Figure 14 (a). As the angle of incidence increases, the angle of refraction increases as well. When the angle of incidence reaches a certain value, called the **critical angle** θ_c, the refracted ray no longer enters the air but instead is parallel to the water-air boundary. This is shown in Figure 14 (c). In this case the angle of refraction is 90°. For angles of incidence greater than the critical angle, as in Figure 14 (d), all of the light is reflected back into the water.

- When light is completely reflected back into the original medium in which it was traveling, we say that it has undergone **total internal reflection**.

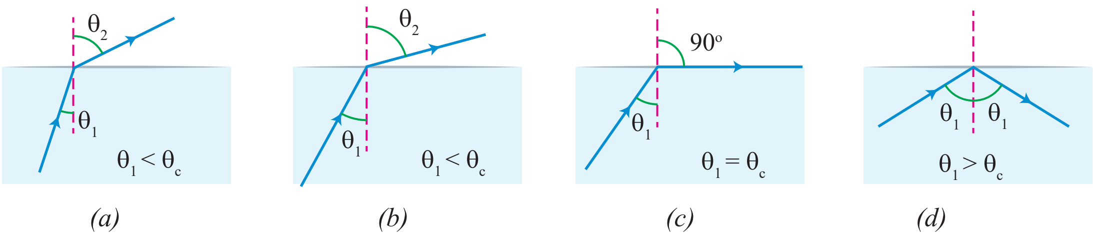


Figure 14.

We can find the critical angle for total internal reflection by setting θ_2 = 90° and applying Snell's law:

$$n_1 \sin\theta_c = n_2 \sin 90^o \qquad (\sin 90^o = 1)$$

$$n_1 \sin\theta_c = n_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Example

Find the critical angle for light passing from glass (n = 1.50) into (a) air (n = 1.00) and (b) water (n = 1.33).

Solution

(a) From glass into air: n_1 = 1.50, n_2 = 1.00, θ_c =?

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.00}{1.50} = 0.67$$

$$\theta_c = \sin^{-1}(0.67) = 42^o$$

(b) From glass into water: n_1 = 1.50, n_2 = 1.33, θ_c =?

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1.33}{1.50} = 0.887$$

$$\theta_c = \sin^{-1}(0.887) = 62.5^o$$

Example

A beam of light is propagating through diamond (n_1 = 2.42) and strikes a diamond–air interface at an angle of incidence of 28°. (a) Will part of the beam enter the air or will the beam be totally reflected at the interface? (b) Repeat part (a), assuming that the diamond is surrounded by water (n_2 = 1.33) instead of air.

Solution

Total internal reflection occurs only when the beam of light has an angle of incidence that is greater than the critical angle, θ_c. The critical angle is different in parts (a) and (b), since it depends on the ratio n_2/n_1 of the refractive indices of the incident (n_1) and refracting (n_2) media.

(a) The critical angle θ_c for total internal reflection at the diamond–air interface is:

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{1}{2.42} = 0.41$$

$$\theta_c = \sin^{-1}(0.41) = 24.4^o$$

Because the angle of incidence of 28° is greater than the critical angle, there is no refraction, and the light is totally reflected back into the diamond.

(b) If water, rather than air, surrounds the diamond, the critical angle for total internal reflection becomes larger:

$$\sin \theta_c = \frac{n_2}{n_1} = \frac{1.33}{2.42} = 0.55$$

$$\theta_c = \sin^{-1}(0.55) = 33.3^\circ$$

Now a beam of light that has an angle of incidence of 28° (less than the critical angle of 33.3°) at the diamond–water interface is refracted into the water.

Example

The critical angle for total internal reflection for a certain material/air boundary is 30°. What is the index of refraction of the material?

Solution

$\theta_c = 30°$ and $n_2 = 1$

$$\text{Sin } \theta_c = \frac{n_2}{n_1}$$

$$n_1 = \frac{n_2}{\sin \theta_c} = \frac{1}{\sin 30^\circ} = \frac{1}{0.5} = 2$$

Fiber Optics

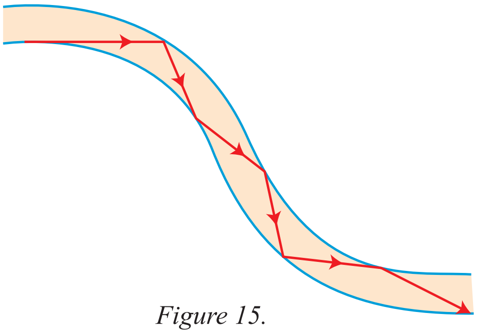


Figure 15.

An interesting application of total internal reflection is the use of glass or transparent plastic rods, like the ones shown in Figure 15, to transfer light from one place to another. As indicated in the figure, light is confined to traveling within the rods, even around gentle curves, as a result of successive internal reflections. Such a light pipe can be flexible if thin fibers rather than thick rods are used. If a bundle of parallel fibers is used to construct an optical transmission line, images can be transferred from one point to another.

This technique is used in a technology known as fiber optics. Fiber-optic devices are particularly useful for viewing images produced at inaccessible locations. For example, a fiber-optic cable can be threaded through the esophagus (endoscopy) and into the stomach to look for ulcers.

Fiber-optic cables are used in telecommunications because the fibers can carry much higher volumes of telephone calls and computer signals than can electrical wires.

Example

A ray of light in an optically dense liquid of refractive index n = 1.4 approaches the boundary between the liquid and the air at an angle of incidence whose sine is 0.8. Will the ray be refracted?

Solution

$n_1 = 1.4$ $\quad$ $\sin\theta_1 = 0.8$

$n_2 = 1$ $\quad$ $\sin\theta_2 = ?$

We use Snell's law to determine θ_2

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{1.4 \times 0.8}{1} = 1.12$$

Since sine of any angle cannot be greater than 1, the ray will not pass to the next medium; instead it will be totally internally reflected.

Example

Light passes from a medium whose refractive index n = 1.6 to air. What is observed when the angle of incidence $\theta_1 = 53°$?

Solution

$n_1 = 1.6$ $\quad$ $\theta_1 = 53^o$

$n_2 = 1.0$ $\quad$ $\theta_2 = ?$

First, we determine the critical angle θ_c for this medium.

$$\text{Sin } \theta_c = \frac{n_2}{n_1} = \frac{1}{1.6} = 0.625$$

$\theta_c = \sin^{-1}(0.625) = 38.7°$

We see that the angle of incidence, $\theta_1 = 53°$, is greater than the critical angle and at this angle total internal reflection will take place.

Activity 3

Align two prisms ($45^o - 45^o - 90^o$) side by side as shown in Figure 16. Note that this configuration can be used like a periscope to see an object above your line of sight if the configuration is oriented vertically and to see around a corner if it is oriented horizontally. How would you arrange the prisms to see behind you? Draw your design on paper and test it.

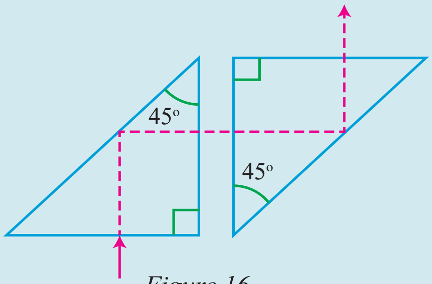


Figure 16.

Note: This activity is meant to demonstrate an application of total internal reflection. Point out that the critical angle for glass is about 41° and that the angle of incidence from glass to air in this arrangement is 45°.

Exercises

1. What is the angle of refraction when the angle of incidence is equal to the critical angle for total internal reflection?
2.
   (e) For total internal reflection to occur, should the light be directed from (i) air to water or (ii) water to air? Explain.

   (f) What is the critical angle for water/air boundary?
3. Find the critical angle for light passing from glass (n = 1.50) into (a) air (n = 1.00) (b) water (n = 1.33).
4. The critical angle for total internal reflection for light traveling from a particular type of glass to air is 37°. What is the index of refraction of this glass?

Key Terms

- lens
- convex lens
- concave lens
- Thin-lens equation
- nearsightedness
- farsightedness
- Dispersion
- pigments

When you look through a microscope, things appear larger than they actually are. Why? How does a simple piece of glass magnify objects? The wonderful behavior of lenses is the subject of this section.

Lens

A ray of light is refracted, or bent, as it passes from one medium to another. A device that takes advantage of refraction and uses it to focus light is referred to as a **lens**. A lens is made of transparent material, such as glass or plastic. Each of the Lens's two faces is part of a sphere and can be convex, concave, or flat. Generally convex and concave lenses come in a variety of shapes (Figure 17). Generally, convex lenses are thicker in the middle than at the edges, and concave lenses are thinner in the middle. We will use double convex and double concave lenses when we present examples of converging and diverging lenses, respectively.

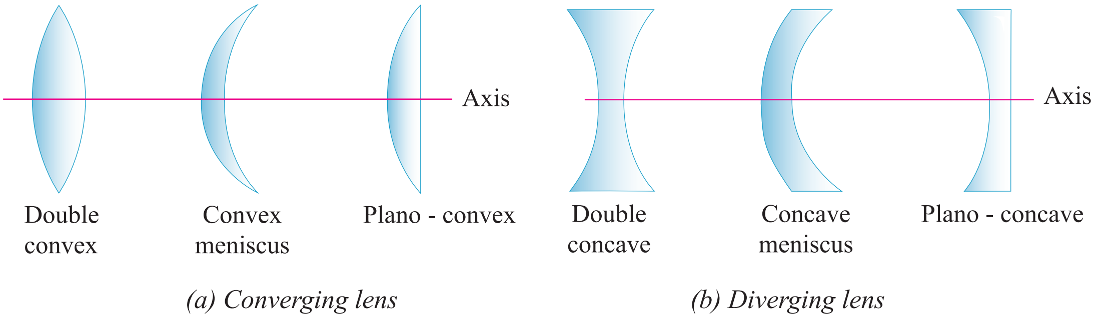


Figure 17.

Convex lenses are converging lenses because they refract parallel rays of light and bring them together at a focus, F, as shown in Figure 18 (a). A concave lens is called a diverging lens because parallel rays passing through it spread out as if diverging from a point, F, as shown in Figure 18 (b).

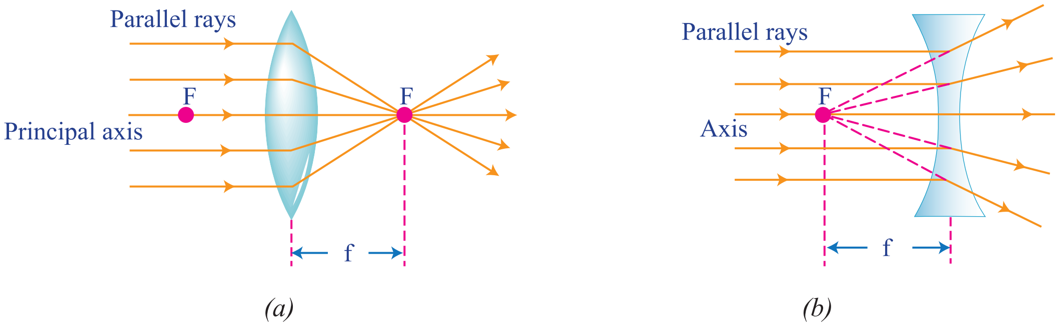


Figure 18.

Note: The most important parameter of a lens is its focal length f, which is the same on both sides of the lens. For a converging lens, f can be measured by finding the image position for the Sun or other distant objects. Once f is known, the image position can be determined using ray diagrams for any object.

Ray diagrams: We can determine the position and size of an image of an object formed with a thin lens by using a graphical method similar to the one used for spherical mirrors. For this purpose we use three principal rays that emanate from the tip of the object. The intersection of these rays, after they pass through the lens, determines the position and size of the image (Figure 19).

Convex lens (converging lens)

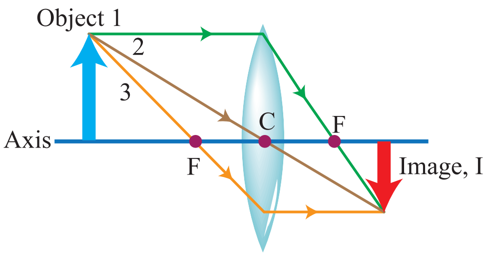


Figure 19.

- Ray 1 - Parallel ray: This ray initially travels parallel to the principal axis. In passing through a converging lens, the ray is refracted toward the axis and travels through the focal point on the right side of the lens.
- **Ray 2 – A ray through the center of the lens, C** (or **central ray**): This ray travels directly through the center, C, of the thin lens without any bending.
- **Ray 3 – A ray through the focal point (focal ray):** This ray first passes through the focal point, F, on the left and then is refracted by the lens and travels parallel to the axis – basically the reverse of a parallel ray.

Concave (Diverging) lens

By drawing the same three rays from a single point on the object, we can determine the image position formed by a diverging lens, as shown in Figure 20.

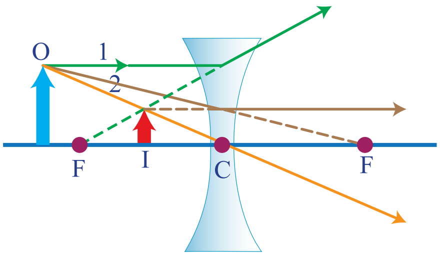


Figure 20.

- **Ray 1 – Parallel ray:** This ray is drawn parallel to the axis, but does not pass through the focal point F (right) behind the lens. Instead it seems to come (dashed line) from the focal point F (left) in front of the lens.
- **Ray 2 – A ray Through the center of the lens, C (central ray):** This ray passes directly through the center of the lens.
- **Ray 3 – A ray through the focal point (focal ray):** This ray is directed toward F (right) and is refracted parallel to the lens axis by the lens.

Finding the image location for a lens - the thin lens equation

Ray tracing enables us to describe qualitatively the image formed by a lens. It is also possible to describe the image formed by a lens quantitatively using the thin-lens equation. This equation, also known as the lens maker's equation, relates the image distance to the object distance and the focal length of a thin lens.

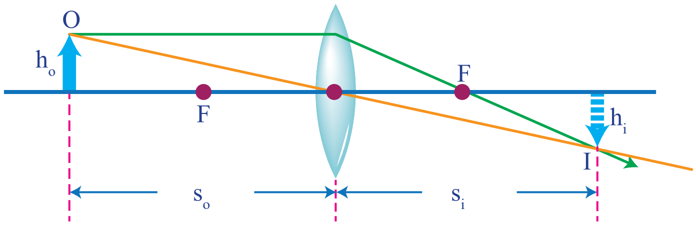


Figure 21.

Let s_o be the object distance – the distance of the object from the center of the lens, and s_i be the image distance – the distance of the image from the center of the lens (Figure 21). We can easily derive the thin-lens equation from the geometry of the figure.

Thin-lens equation

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

The magnification equation is

$$m = \frac{\text{Image height}}{\text{Object height}} = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

Note

(i) Diverging lenses always produce an upright virtual image for any real object, no matter where that object is.

(ii) Converging lenses can produce real (inverted) images, or virtual (upright) images, depending on object position.

(iii) Real images are those on the opposite side of the lens from the object.

(iv) The focal length of the lens is half of the radius of curvature, R, of the lens: R = 2f.

Sign conventions for lenses

Focal Length: f is +ve for convex (converging) lens

f is -ve for concave (diverging) lens.

Image distance: s_i is +ve for real images

s_i is -ve for virtual images.

Magnification: m is +ve for upright images (same orientation as the object).

m is –ve for inverted images (opposite orientation of object).

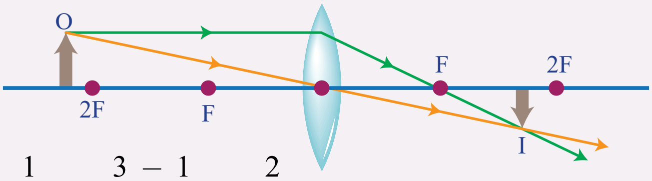
Example

A converging lens has a focal length of 20 cm. Find the image location for an object placed at each of the following distances from the lens: (a) 60 cm, (b) 40 cm, (c) 30 cm, (d) 30 cm, (e) 10 cm. Determine the characteristics of the image and the magnification in each case.

Solution

For a converging lens, f is positive: f = 20 cm

(a) When s_o = 60 cm, the object is beyond 2F. To find the image location, we use the thin lens equation.



$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{20\text{ cm}} - \frac{1}{60\text{ cm}} = \frac{3-1}{60\text{ cm}} = \frac{2}{60\text{ cm}}$$

s_i = 30 cm

The image is between F and 2F (see the ray tracing at the right). Since s_i is positive, the image is real. The magnification gives the amount by which its size is enlarged or reduced compared with the object.

$$m = -\frac{s_i}{s_o} = -\frac{30\text{ cm}}{60\text{ cm}} = -\frac{1}{2}$$

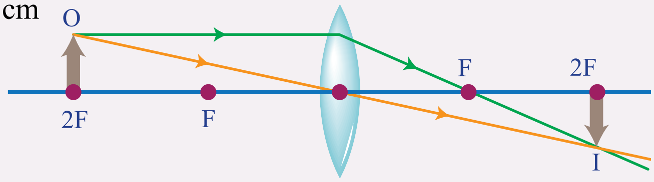
The image is reduced (smaller than the object).

(b) When s_o = 40 cm, the object is placed at 2F (i.e., at the center of curvature, C).

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{20\text{ cm}} - \frac{1}{40\text{ cm}}$$

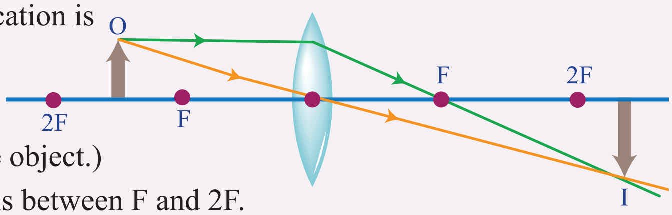
$$\frac{1}{s_i} = \frac{2-1}{40\text{ cm}} = \frac{1}{40\text{ cm}}$$

s_i = 40 cm



The image is formed at 2F (i.e., the same position as the object), as shown in the drawing. It is real and inverted. Its magnification is

$$m = -\frac{s_i}{s_o} = -\frac{40\text{ cm}}{40\text{ cm}} = -1$$



(The image is the same size as the object.)

(c) When s_o = 30 cm, the object is between F and 2F.

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{20\text{ cm}} - \frac{1}{30\text{ cm}} = \frac{3-2}{60\text{ cm}} = \frac{1}{60\text{ cm}}$$

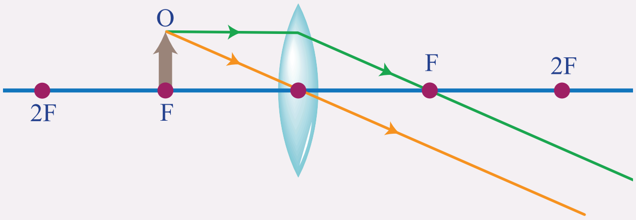
$s_i = 60$ cm

Comparing this with the position of the object in part (a), we see that as the object is moved toward the lens, the image moves away from the lens (with an increase in size).

$$m = -\frac{s_i}{s_o} = -\frac{60\text{ cm}}{30\text{ cm}} = -2$$

The image is enlarged or magnified.

(d) When s_o = 20 cm, the object is at F. From the ray diagram shown, we see that the image formed is at infinity.



(e) When s_o = 10 cm, the object is between F and the lens.

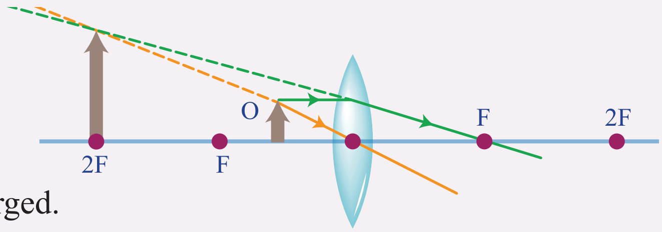
$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{20\text{ cm}} - \frac{1}{10\text{ cm}} = \frac{1-2}{20\text{ cm}} = \frac{-1}{20\text{ cm}}$$

$s_i = -20$ cm

Negative image distance implies that the image is virtual (i.e., on the same side as the object).

Its magnification is:

$$m = -\frac{s_i}{s_o} = -\frac{-20\text{ cm}}{10\text{ cm}} = 2$$



The image is magnified and enlarged.

Converging lenses can produce real or virtual images of real objects

An object infinitely far away from a converging lens will create a point image at the focal point, as shown in the first diagram in Figure 18. This image is **real**, which means that it can be projected on a screen.

As a distant object approaches the focal point, the image becomes larger and farther away, as shown in the diagrams of **Worked Example 1.15** (a) through (e). When the object is at the focal point, as shown in the fourth diagram, the light rays from the object are refracted so that they exit the lens parallel to each other.

When the object is between a converging lens and its focal point, the light rays from the object diverge when they pass through the lens, as shown in the fifth diagram. This image appears to an observer in back of the lens as being on the same side of the lens as the object. In this case the image is upright, **virtual**, and magnified.

Applications of concex lenses: We have seen the characteristics of the images formed by convex lenses. They can form images that are upright or inverted, behind the lens or in front of the lens, and either enlarged or reduced in size depending on the position of the object in front of the lens.

These characteristics enables converging lenses to serve a variety of purposes. Some of them are: magnifying glass, lens of a camera, human eyeball lens, objective lens of a refracting telescope, inverting lens of a field telescope, slide projector, objective lens in a compound microscope, searchlights, eyepiece lens of microscope, binoculars, and telescope.

Discuss 4: What is the difference between a **virtual image** and a **real image**? (Hint: Compare the way the ray diagrams are drawn to locate the position of the images in relation to the object in both lenses.)

Exercises

A certain lens forms a real image of a very distant object on a screen that is 10 cm from the lens. As the object is moved closer to the lens, at what distance of the object from the lens are the object and image the same height?

(a) Only at 20 cm.
(b) Between 20 cm and 40 cm.
(c) At 40 cm.
(d) Only at 10 cm.

Activity 4

A magnifying glass is basically a converging lens used in biology lab. On a sunny day, hold the magnifying glass above a nonflammable flat surface, such as a concrete slab so that a round spot of light is formed on the surface. Move the magnifying glass up and down to find the height at which the spot formed by the lens is most distinct, or smallest. Use the ruler to measure the distance between the magnifying glass and the surface. This distance is the approximate focal length of the lens.

Diverging lenses produce virtual images from real objects

A diverging lens creates a virtual image of a real object placed anywhere with respect to the lens. The image is upright, and the magnification is always less than one; that is, the image size is reduced. Furthermore, the image appears between the focal point and the lens for any position of the object.

Example

An object is 6 cm from a lens, which produces a virtual image between the object and the lens. This image is 2 cm from the lens. What type of lens is it, and what is its focal length?

Solution

$s_o = 6$ cm

$s_i = -2$ cm (The image is virtual)

The magnification equation gives,

$$m = -\frac{s_i}{s_o} = -\left(\frac{-2\text{ cm}}{6\text{ cm}}\right) = \frac{1}{3}$$

The lens is a diverging (concave) lens because m < 1. Note that convex lenses also form virtual images, however, the images are enlarged (or magnified).

$$\frac{1}{f} = \frac{1}{6\text{ cm}} + \frac{1}{-2\text{ cm}} = \frac{1-3}{6\text{ cm}} = -\frac{2}{6\text{ cm}} = -\frac{1}{3\text{ cm}}$$

Notice that the negative focal length confirms that the lens is a diverging lens.

Example

An object is placed 12 cm in front of a diverging lens with a focal length of 8 cm. Find (a) the image distance and (b) the magnification.

Solution

(a) Given the focal length and object distance, we can use the thin-lens equation to find the image distance. (For concave lens, f is negative)

$$\text{or } \frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = -\frac{1}{8\text{ cm}} - \frac{1}{12\text{ cm}}$$

$$= \frac{-6 - 4}{48\text{ cm}} = \frac{-10}{48\text{ cm}}$$

$$s_i = -\frac{48\text{ cm}}{10} = -4.8\text{ cm}$$

(b) Once the image and object distances are known, we can use them to find the magnification.

$$m = -\frac{s_i}{s_o} = \frac{-4.8\text{ cm}}{12\text{ cm}} = -0.4$$

Because the image distance is negative, it follows that the image is virtual and on the same side of the lens as the object, as expected for a concave (diverging) lens. In addition, the fact that the magnification is positive means that the image is upright.

- No matter where an object is placed in front of a diverging lens, the emerging rays appear to be coming from in front of the lens and so form a virtual, erect image. This image is always smaller than the object.

Discuss 5: A ray from an object passes through a thin lens, as shown in Figure 22 (a), and (b). What can we conclude about the lens from these rays? Answer the following question for each figure.

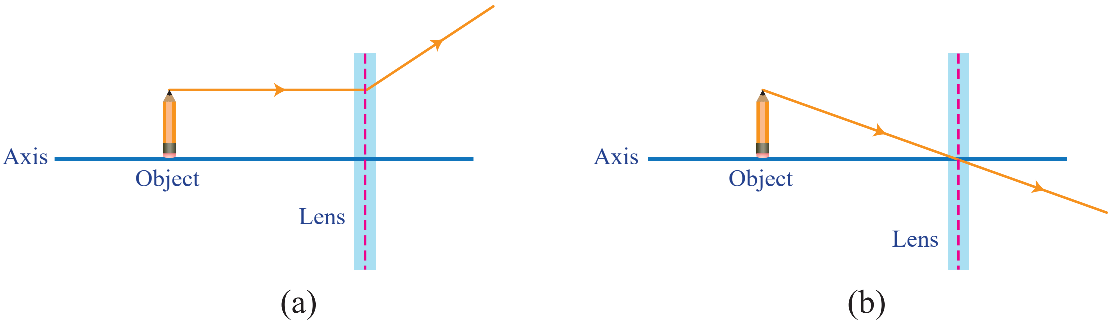


Figure 22.

A. It must be a converging lens.

B. It must be a diverging lens.

C. It could be either a converging or a diverging lens.

Example

An object is located at a distance s_o in front of a lens. The lens has a focal length f and produces an upright image that is twice as tall as the object. (a) What kind of lens is it, and (b) what is the object distance?

Solution

(a) Converging lens (convex lens).

(b) First we use the magnification equation and express s_o in terms of s_i.

The image is upright and is twice as tall as the object. So, m = + 2.

$$m = -\frac{s_i}{s_o}$$

$s_i = -ms_o$

Now we use the thin-lens equation to find the object distance, s_o.

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{s_o} + \frac{1}{-ms_o} = \frac{1}{s_o} - \frac{1}{ms_o} = \frac{m-1}{ms_o}$$

$$ms_o = (m-1)f$$

$$s_o = \frac{(m-1)f}{m} = \frac{(2-1)f}{2} = \frac{1}{2}f$$

When the object is placed between F and the lens, the image is virtual (upright), and magnified. A simple magnifier can be used in this way.

Example

A convex lens of focal length 5 cm produces an erect image which is five times the size of the object. Find the distance of the object from the lens.

Solution

f = 5 cm and m = 5 (m is + ve for erect or virtual image)

From the magnification equation,

$$m = -\frac{s_i}{s_o}$$

$$s_i = -m\,s_o = -5s_o$$

Now, we use the thin - lens equation to find s_o

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{s_o} + \frac{1}{-5s_o} = \frac{5-1}{5s_o} = \frac{4}{5s_o}$$

$$\frac{1}{5\text{ cm}} = \frac{4}{5s_o}$$

$$s_o = 4\text{ cm}$$

Example

An object 6 cm high is 10 cm from a concave lens. The image formed is 3 cm high. Find the focal length of the lens and the distance of the image from the lens.

Solution

$h_o = 6$ cm $s_o = 10$ cm $f = ?$

$h_i = 3$ cm $s_i = ?$

The magnification equation can be used to find the image distance, s_i.

$$m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$$\frac{3\text{ cm}}{6\text{ cm}} = -\frac{s_i}{10\text{ cm}}$$

$s_i = -5$ cm

The image is virtual because concave lenses always form virtual images.

$$\frac{1}{f} = \frac{1}{10\text{ cm}} + \frac{1}{-5\text{ cm}} = \frac{1-2}{10\text{ cm}} = -\frac{1}{10\text{ cm}}$$

$f = -10$ cm

Notice that for a concave lens, f is negative.

Example

A lens of focal length 12 cm forms an erect image of an object. If the image is three times bigger than the object, what is the distance between the object and the image?

Solution

The lens is convex (converging) because the virtual image is an enlarged image. Thus, m = 3, and f = 12 cm.

Using the magnification equation, we get

$$m = -\frac{s_i}{s_o}$$

$s_i = -m\, s_o = -3s_o$

Now, we use the thin - lens equation.

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{s_o} - \frac{1}{3s_o} = \frac{2}{3s_o}$$

$$s_o = \frac{2}{3}f = \frac{2}{3} \times 12\text{ cm} = 8\text{ cm}$$

$s_i = -3s_o = -3 \times 8$ cm $= -24$ cm

The virtual image is formed on the same side of the object. The distance between object and image is therefore: 24 cm – 8 cm = 16 cm

Example

A converging lens can produce both real and virtual images depending on where the object is placed relative to the focal point of the lens. For what object distances will it produce a real image? Virtual image?

Solution

Whether the image is real or virtual is determined by the image distance, s_i. From the thin lens equation, we have

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{s_o - f}{f\,s_o}$$

$$s_i = \frac{f\,s_o}{s_o - f}$$

- If $s_o > f$, then s_i is positive and the image is real.
- If $s_o < f$, then s_i is negative and the image is virtual.

Example

A magnifying glass uses a converging lens whose focal length is 15 cm. The magnifying glass produces a virtual and upright image that is 3 times larger than the object. (a) How far is the object from the lens? (b) What is the image distance?

Solution

$f = 15$ cm (Converging lens has + ve focal lens)

$m = +3$ (The image formed is virtual)

(a) The object distance can be found by using both magnification equation and the lens equation.

$$m = -\frac{s_i}{s_o}$$

$$3 = -\frac{s_i}{s_o} \text{ or } s_i = -3s_o$$

Now, $$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{s_o} - \frac{1}{3s_o} = \frac{2}{3s_o}$$

$$\frac{1}{15\text{ cm}} = \frac{2}{3s_o}$$

$$3\,s_o = 15\text{ cm} \times 2 = 30\text{ cm}$$

$$s_o = 10\text{ cm}$$

(b) We are given that $s_i = -3s_o$

$$s_i = -3(10\text{ cm}) = -30\text{ cm}$$

The image is formed on the same side as the object.

The human eye

The fundamental parts of an eye are shown in Figure 23. Light enters the eye through a variable aperture called the **pupil**. This light is converged by the **cornea**, with assistance from the lens, to form an image on the **retina**, which has a film of nerve fibers covering the back surface of the eye. The shape of the crystalline lens can be altered slightly by the action of the **ciliary muscle**.

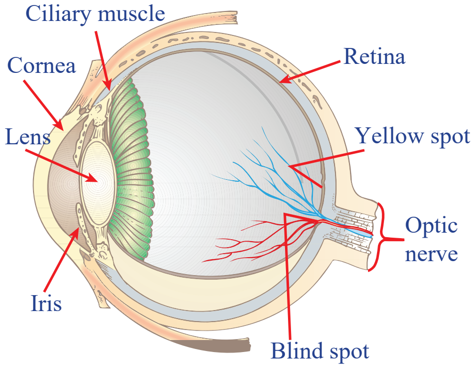


Figure 23.

- The human eye uses the cornea and lens to form an image on the retina.

Focusing and accommodation

When the eye is viewing a distant object, the ciliary muscles are relaxed allowing the lens to be relatively flat, as shown in Figure 24 (a). As a result, it causes little refraction and its focal length is at its greatest. When the eye is focusing on a near object, the ciliary muscles are tensed to give the lens a greater curvature thereby changing the shape and reducing the focal length of the lens.

The process of changing the shape of the lens, and hence adjusting its focal length, is referred to as **accommodation**. Producing the proper accommodation is not easy for a newborn, but is automatic for an adult. If the object is too close to the eye, the lens cannot focus the light on the retina and the image is blurred (Figure 24 (b)).

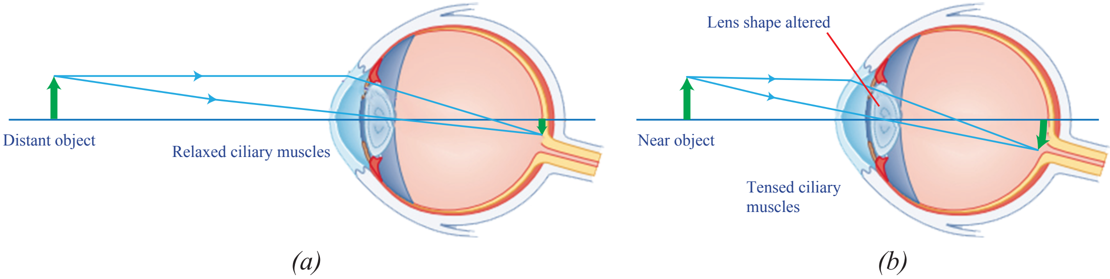


Figure 24.

- The human eye focuses light by changing the shape of its lens, which changes the focal length, rather than by moving the lens back and forth as in a camera.

The Near and Far Points

The closest point for which the lens can focus the image on the retina is called the **near point**. The distance from the eye to the near point varies greatly from one person to another and changes with age. At 10 years, the near point may be as close as 18 cm whereas at 60 years it may reach to 500 cm because of the loss of flexibility of the lens. The standard value taken for the near point is 25 cm.

The **far point** of the eye represents the farthest distance for which the lens of the relaxed eye can focus light on the retina. A person with normal vision is able to see very distant objects, such as the moon, and so has a far point at infinity.

Exercises

Your eye is focusing on a person. As he walks toward you, what must be done to keep him in focus?

A. The pupil of your eye must open-up.

B. The distance from the eye lens to the retina must decrease.

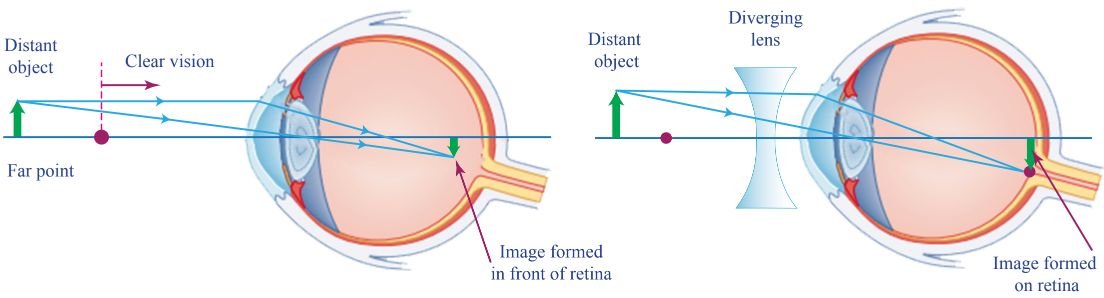
C. The distance from the eye lens to the retina must increase.

D. The focal length of your eye must decrease.

- The fact that the ciliary muscles must be tensed to focus on nearby objects means that our eyes can tire from muscular strain. That's why it's beneficial to pause occasionally from reading and look off into the distance. Viewing distant objects allows the ciliary muscles to relax, thus reducing the strain on our eyes.

Eye defects – nearsightedness or farsightedness

The eyes of many people do not focus sharp images on the retina. Instead, images are focused either in front of the retina or behind it. External lenses, in the form of eyeglasses or contact lenses, are needed to adjust the focal length and move images to the retina.



(a) *(b)*

Figure 25.

Figure 25 (a) shows the condition of **nearsightedness**, or **myopia**, whereby the focal length of the eye is too short to focus light on the retina. Images are formed in front of the retina. As shown in Figure 25 (b), concave lenses correct this by diverging light, thereby increasing image's distance from the lens, and forming images on the retina.

Farsightedness, or hyperopia, refers to an eye that cannot focus on nearby objects. Although distant objects are usually seen clearly, the near point is somewhat greater than the "normal" 25cm, which makes reading difficult. In the hyperopic (farsighted) eye, the eyeball is too short or the cornea is not curved enough, and the image of an infinitely distant object is behind the retina (Figure 26 (a)). It is corrected by a converging lens (Figure 26 (b)).

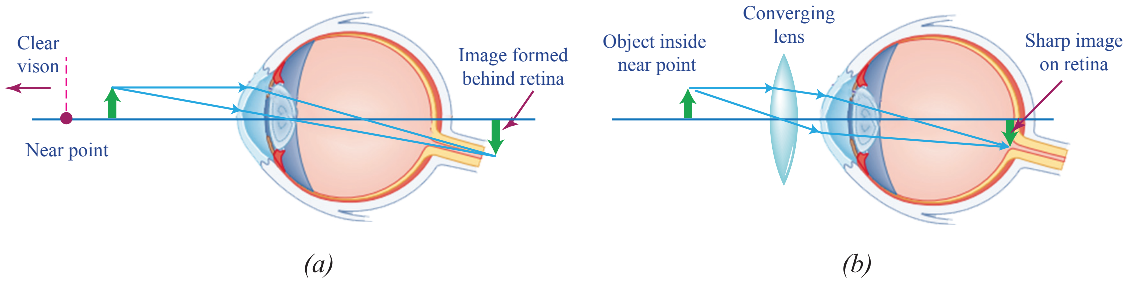
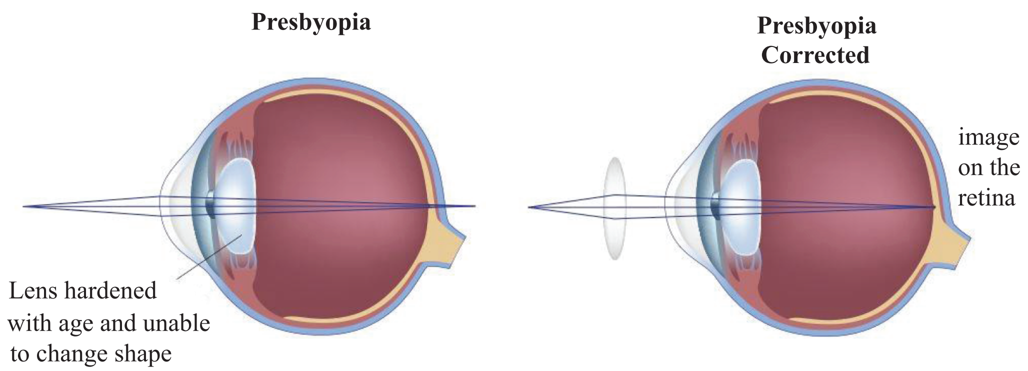


Figure 26.

Presbyopia

Presbyopia is the condition where eye becomes hard, mostly because of ageing. As the lens of the human eye becomes less flexible, it is difficult for the lens to focus on a particular nearby object. As a result, the object appears out of focus. Presbyopia can be corrected with the use of convex lens which converges light rays and helps in the formation of the image at the retina as shown in the following diagram:



ACTIVITY 5

Try to get a pair of glasses prescribed by an optometrist. Hold the glasses at various distances from your eye, and look at different objects through the lenses. Try this with different types of glasses, such as those for farsightedness and nearsightedness, and describe what effect the differences have on the image you see. Discuss in groups.

Exercises

Which of the following statements is true about the lenses used in eyeglasses to correct nearsightedness and farsightedness?

(a) They produce a real image.

(b) They produce a virtual image.

(c) Both nearsightedness and farsightedness are corrected with a converging lens.

(d) Both nearsightedness and farsightedness are corrected with a diverging lens.

Example

The near point of a given individual's eye is 55 cm. What focal length corrective lens should be prescribed so that an object can be clearly seen when placed 25 cm in front of the eye?

The corrective lens must form an upright, virtual image located 55 cm in front of the lens (s_i = −55 cm) when the object is 25 cm in front of the lens (s_o = 25 cm). The thin-lens equation, then gives the required focal length.

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{25\text{ cm}} + \frac{1}{-55\text{ cm}} = \frac{55-25}{25 \times 55\text{ cm}}$$

$$f = 46\text{ cm}$$

Optometrists and ophthalmologists, instead of using the focal length, use the reciprocal of the focal length to specify the strength of eye glass (or contact) lenses. This is called the **power,** P, of the lens:

$$P = \frac{1}{f}$$

Its unit is the diopter (D): 1 D = 1 m^{-1}.

For instance, a 50 cm focal length lens has a power P = 1/50 cm = 1/0.50 m = 2.0 D.

Astigmatism refers to a defect in which the surface of the cornea is not spherical but is more sharply curved in one plane than in another. Light from a point source produces a line image on the retina. Astigmatism may make it impossible, for example, to focus clearly on the horizontal and vertical bars of a window at the same time. Astigmatism can be corrected with lenses having different curvature in two mutually perpendicular directions.

Visible light that contains all the component colors is termed as white light. For example, sunlight is white light. White light is composed of red, orange, green, blue, indigo, yellow and violet light. These seven distinct colors differ from one another in their frequencies and wavelengths.

Figure 27 shows a ray of white light passing through a glass prism. When the light enters the prism at the left face, the refracted ray is bent toward the normal. Snell's law indicates that incident light of different wavelengths is bent at different angles as it moves into a refracting material, such as the prism. When the light leaves the prism at the right face, it is refracted away from the normal. Thus, the net effect of the prism is to change the direction of the ray, causing it to bend downward upon entering the prism, and downward again upon leaving.

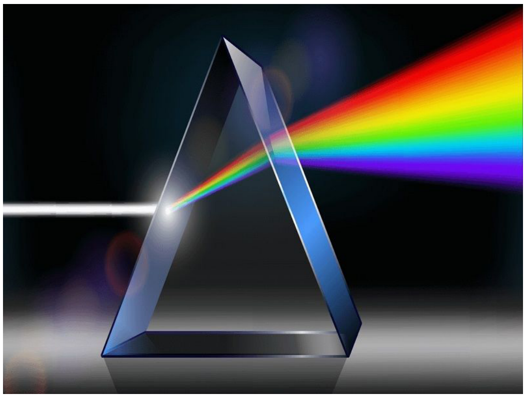


Figure 27.

Perhaps the well known and striking example of dispersion is provided by the rainbow, which is caused by the dispersion of light in droplets of rain (see front page). The physical situation shows that a single drop of rain and an incident beam of sunlight. When sunlight enters the drop, it is separated into its red and violet components by dispersion, as shown. The light then reflects from the back of the drop and finally refracts and undergoes additional dispersion as it leaves the drop.

To understand how dispersion can affect light, consider what happens when light strikes a prism, as in Figure 28. Because of dispersion, the violet component of the incoming ray is bent

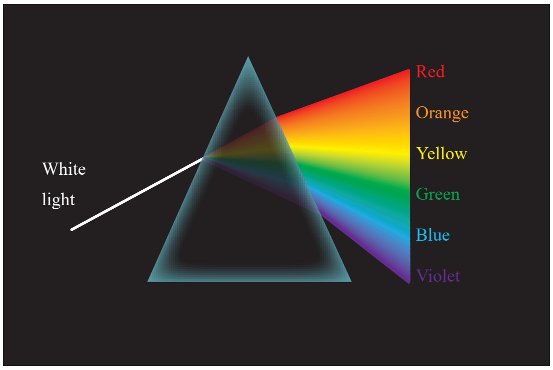


Figure 28.

more than the red component, and the rays that emerge from the second face of the prism spread out in a series of colors known as a ***visible spectrum***. These colors, in order of increasing wavelength, are Violet, Indigo, Blue, Green, Yellow, Orange, and Red (VIBGYOR).

Exercises

Red light is bent less when it passes from air into glass prism than blue light is. Is the index of refraction of the glass for red light greater than, less than, or equal to the index of refraction for blue light? Explain.

Explanation The index of refraction decreases with increasing wavelength. Therefore, blue light ($\lambda \approx 470$ nm) bends more than red light ($\lambda \approx 650$ nm) when passing into the glass prism.

Primary and Secondary Colors: White light can be formed from colored light in a variety of ways. For example, when the correct intensities of red, green, and blue light are projected onto a white screen, as in Figure 29 (a), the region where these three colors overlap on the screen will appear to be white. Thus, red, green, and blue lights form white light when they are combined. This is called the additive color process, which is used in color-television tubes.

A color-television tube contains tiny, dot-like sources of red, green, and blue light. When all three colors of light have the correct intensities, the screen appears to be white. For this reason, red, green, and blue are each called a **primary color**. The primary colors can be mixed in pairs to form three additional colors, as shown in Figure 29 (b). Red and green light together produce yellow light, blue and green light produce cyan, and red and blue light produce magenta. The colors yellow, cyan, and magenta are each called a **secondary color**, because each is a combination of two primary colors.

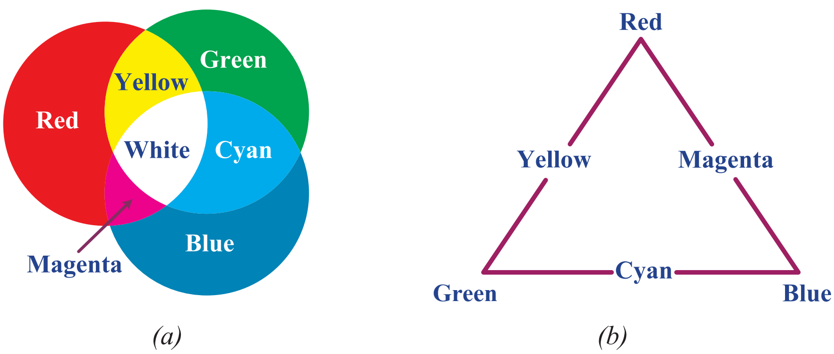


Figure 29.

Primary Pigment: A dye is a molecule that absorbs certain wavelengths of light and reflects others. For example, a red shirt is red because the dyes in it reflect red light to our eyes. When white light falls on the red object, the dye molecules in the object absorb the blue and green light and reflect the red light. When only blue light falls on the red object, very little light is reflected and the object appears to be almost black.

The difference between a dye and a pigment is that pigments usually are made of crushed minerals, rather than plant or insect extracts. Pigment particles can be seen with a microscope. A pigment that absorbs only one primary color and reflects two from white light is called a **primary pigment**. Yellow pigment absorbs blue light and reflects red and green light.

- Yellow, cyan, and magenta are the colors of primary pigments. A pigment that absorbs two primary colors and reflects one color is called a secondary pigment.

The colors of secondary pigments are red (which absorbs green and blue light), green (which absorbs red and blue light), and blue (which absorbs red and green light). Note that the primary pigment colors are the secondary colors. In the same way, the secondary pigment colors are the primary colors.

Figure 29 (b) also shows the results of combining pigments: the colors at any two sides of the triangle combine to form the color at the vertex between them. For example, pure magenta pigment mixed with pure cyan pigment will only reflect blue light. The pigment whose name is on one side reflects the light composed of the colors that are indicated at the vertices on either end of that side. This same pigment absorbs the light with the color whose name is on the opposite vertex.

The primary and secondary pigments are shown in Figure 30. When the primary pigments yellow and cyan are mixed, the yellow absorbs blue light and the cyan absorbs red light. Thus, Figure 30 shows yellow and cyan combining to make green pigment. When yellow pigment is mixed with the secondary pigment, blue, which absorbs green and red light, all of the primary colors are absorbed, so the result is black. Thus, yellow and blue are complementary pigments. Cyan and red, as well as magenta and green, are also complementary pigments.

Figure 30.

Note: In general, the primary pigments are magenta, cyan, and yellow. Mixing these in pairs produces the secondary pigments: red, green, and blue.

- *A color printer uses yellow, magenta, and cyan dots of pigment to make a color image on paper.*

Exercises

What color of light must be combined with blue light to obtain white light?

Exercises

What primary pigment colors must be mixed to produce red? Explain how red results using color subtraction for pigment colors.

Explanation: Yellow and magenta pigments are used to produce red. Yellow pigment subtracts blue and magenta pigment subtracts green, neither subtracts red so the mixture would reflect red.

Exercises

What color will a yellow banana appear to be when illuminated by each of the following? (a) White light, (b) Green and red light, (c) Blue light.

Exercises

1. An object is placed 12 cm in front of a diverging lens with a focal length of 8 cm. Find (a) the image distance and (b) the magnification.
2. An object that is 24 cm from a convex lens produces a real image that is 13 cm from the lens. What is the focal length of the lens?
3. Which type of lens can focus the sun's rays?
4. An object is 185 cm from a convex lens with a focal length of 25 cm. If the inverted image formed is 12 cm tall, how tall is the associated object?

Most of the optical technology we use is not as simple as a single lens or mirror. For example, a camera may contain several lenses, a prism, and a mirror. The telephoto function in a camera is created by a pair of lenses. When you zoom in and out, the camera changes the separation between the lenses. As the separation changes, the magnification also changes. This section is about optical instruments, such as cameras, microscopes, and telescopes. Optical instruments are built from lenses, mirrors, and prisms. They collect light and may use refraction and reflection to form an image, or may process light in other ways.

The Camera

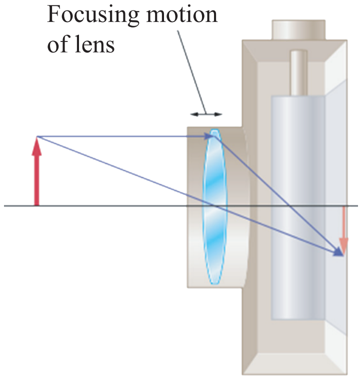


Figure 31.

Cameras come in many types and sizes, from the small and simple camera on your cell phone to the large and complex video camera used to film a Hollywood motion picture. Most cameras have at least one lens, and more complex cameras may have 30 or more lenses and may even contain mirrors and prisms. Up until relatively recently, cameras used film to record an image. The film would undergo a chemical change when exposed to light.

Today, however, most cameras are digital and no longer require film. Instead, they use a charged-coupled device (CCD), an array of tiny electronic sensors that can sense light. The CCD lies on the wall opposite the lens and creates an electrical impulse when hit by incoming photons. A microchip in the camera then translates these data into an image that is then stored on a memory storage devi ce like a hard drive.

A simple camera, such as the one illustrated in Figure 31, operates in much the same way as the eye. In particular, the lens of the camera forms a real, inverted image on a light-sensitive material—which in this case is usually a charge-coupled device (CCD) in a digital camera. The focusing mechanism is different, however. To focus a camera at different distances the convex lens is moved either toward or away from the CCD. Thus, the eye focuses by changing the shape of a stationary lens; the camera focuses by moving a lens of fixed shape.

Example

A simple camera uses a thin lens with a focal length of 6 cm. How far, and in what direction, must the lens be moved to change the focus of the camera from a person 20 m away to a person only 3 m away?

Solution

We can use the thin-lens equation to find the image distance for each case. The difference between these image distances is the distance the lens must be moved.

First, calculate the image distance for an object distance of 20 m = 2000 cm:

$$\frac{1}{s_{i1}} = \frac{1}{f} - \frac{1}{d_{o1}} = \frac{1}{6\text{ cm}} - \frac{1}{2000\text{ cm}} = \frac{2000 - 6}{(6)(2000)}$$

$$s_{i1} = 6.02\text{ cm}$$

Next, calculate the image distance for an object distance of 3 m = 300 cm:

$$\frac{1}{s_{i2}} = \frac{1}{f} - \frac{1}{d_{o2}} = \frac{1}{6\text{ cm}} - \frac{1}{300\text{ cm}} = \frac{300-6}{(6)(300)}$$

$s_{i2} = 6.12$ cm

Finally, find the difference in image distance:

$s_{i2} - s_{i1} = 6.12\text{ cm} - 6.02\text{ cm} = 0.10\text{ cm} = 1.0\text{ mm}$

The image distance is greater for the person at 3 m, and hence the lens must be moved away from the film by 1.3 mm to change the focus the desired amount.

Example

How far must a 50 mm focal length camera lens be moved from its infinity setting to sharply focus an object 3.00 m away?

Solution

For an object at infinity, the image is at the focal point because $1/s_o$ is zero. Thus,

$s_{i1} = f = 50$ mm

For an object distance of 3.00 m, we use the thin lens equation,

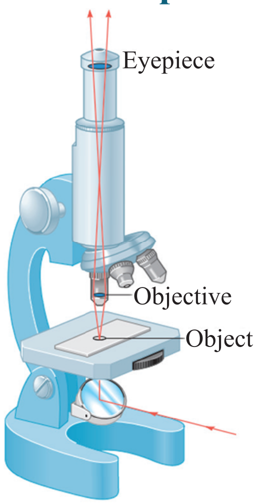
$s_{i2} = 50.8$ mm

So, the lens needs to move 0.8 mm away from the film or digital sensor.

Note

For proper focusing, which leads to sharp images, the lens-to-film distance depends on the object distance as well as on the focal length of the lens.

The Compound Microscope



Although a magnifying glass is a useful device, higher magnifications and improved optical quality can be obtained with a microscope. The simplest microscope consists of two converging lenses fixed at either end of a tube. Such an instrument, illustrated in Figure 32, is sometimes referred to as a **compound microscope**.

The basic optical elements of a microscope are the **objective** and the **eyepiece**. The objective is a converging lens with a relatively short focal length that is placed near the object to be viewed. It forms a real, inverted, and enlarged image. The precise location of the image is adjusted when the microscope is focused by moving the objective up or down. This image serves as the object for the second lens, the eyepiece, in the microscope. In fact, the eyepiece is simply a magnifier that views the image of the objective, giving it an additional enlargement. The final magnification of the microscope, then, is the product of the magnification of the objective and the magnification of the eyepiece. For example, a microscope might have a 10× eyepiece (meaning it magnifies 10 times) and a 50× objective. When these two lenses are used together, the magnification of the microscope is (10)(50) = 500× (or 500 times).

In a typical situation, the object to be examined is placed only a small distance beyond the focal point of the objective, which means that $s_o \approx f_{objective}$.

The magnification produced by the objective is:

$$m_{objective} = -\frac{s_i}{s_o} \approx -\frac{s_i}{f_{objective}}$$

Next, the image formed by the objective is essentially at the focal point of the eyepiece. This means that the eyepiece forms a virtual image at infinity that the observer can view with a relaxed eye. The angular magnification of the eyepiece is given by the equation (M = N/f),

$$M_{eyepiece} = \frac{N}{f_{eyepiece}} \qquad \text{(a)}$$

Multiplying these magnifications, we find the total magnification of the microscope:

$$M_{total} = m_{objective} M_{eyepiece} = \left(-\frac{s_i}{f_{objective}}\right)\left(\frac{N}{f_{eyepiece}}\right)$$

The minus sign indicates that the image is inverted.

Example

In biology class, a student with a near-point distance of N = 25 cm uses a microscope to view an amoeba. If the objective has a focal length of 1.0 cm, the eyepiece has a focal length of 2.5 cm, and the amoeba is 1.1 cm from the objective, what is the magnification produced by the microscope?

Solution

$f_{objective} = 1.0$ cm	N = 25 cm	Near-point distance	M_{total} =?
$F_{eyepiece} = 2.5$ cm	$s_o = 1.1$ cm	object distance	

Calculate s_i using the thin-lens equation:

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o} = \frac{1}{1.0\text{ cm}} - \frac{1}{1.1\text{ cm}}$$

$$s_i = 11\text{ cm}$$

Total magnification is:

$$M_{total} = m_{objective} M_{eyepiece} = \left(-\frac{s_i}{f_{objective}}\right)\left(\frac{N}{f_{eyepiece}}\right)$$

$$= -\frac{(11\text{ cm})(25\text{ cm})}{(1.0\text{ cm})(2.5\text{ cm})} = -110$$

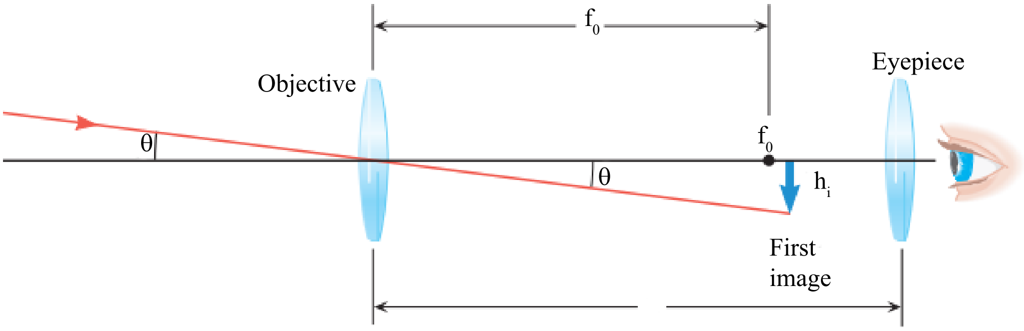
The image of the amoeba is inverted and 110 times larger than life size.

The Telescope

A telescope is an instrument for magnifying distant objects, such as stars and planets. Like a microscope, a telescope consists of an objective and an eyepiece (also called the ocular). Both instruments use two converging lenses to produce a magnified image of an object. In the case of a microscope, the object is small and close at hand. In the case of the telescope, the object is large, such as a planet, but its apparent size can be very small because of its great distance. The major difference between these instruments is that the telescope must deal with an object that is essentially infinitely far away.

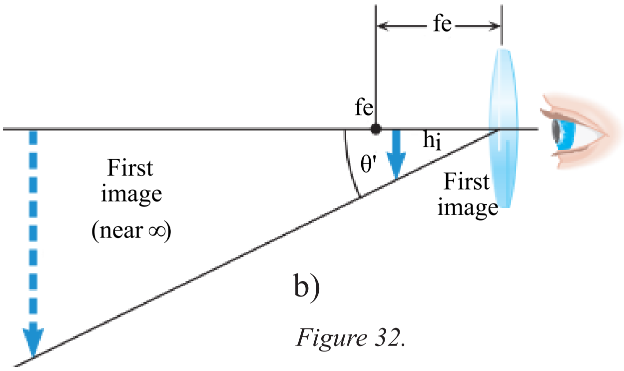
Refracting Telescopes The most common type of telescope for nonscientific work is the refracting telescope, which has two converging lenses that function just as those in a compound microscope. The refracting telescope has an objective lens that forms a real image of the object; the eyepiece (ocular) is used to view this real image. The microscope is used to view tiny objects placed close to the objective lens; the purpose of the objective is t o form an enlarged image. The telescope is used to view objects whose angular sizes are small because they are far away; the objective forms an image that is tiny compared with the object, but the image is available for closeup viewing through the eyepiece.

Astronomical Telescope In an astronomical refracting telescope, the object is so far away that the rays from a point on the object can be assumed to be parallel; the object distance is taken as infinity. The objective forms a real, diminished image at its focal point. By placing this image at the secondary focal point of the eyepiece, the final image is at infinity for ease of viewing. Thus, the focal point of the objective must coincide with the secondary focal point of the eyepiece, in contrast to the microscope in which the two are separated by a distance L (the tube length) Figure 33 (a).



a)

The angular magnification M of a telescope, like that of a magnifying glass or a microscope, is the angular size θ′ subtended by the final image of the telescope divided by the reference angular size θ of the object. For an astronomical object, such as a planet, it is convenient to use as a reference the angular size of the object seen in the sky with the unaided eye. Since the object is far away, the angular size seen by the unaided eye is nearly the same as the angle θ subtended at the objective of the telescope in Figure 33 (a). Moreover, θ is also the angle subtended by the first image, so $\theta \approx -h_i/f_{objective}$, where h_i is the height of the first image and f_o is the focal length of the objective. To obtain an expression for θ′, refer to Figure 33 (b) and note that the first image is located very near the focal point $F_{eyepiece}$ of the eyepiece, which has a focal length f_e. Therefore, $\theta' = -h_i/f_e$. To find the total angular magnification of the telescope, we take the ratio of θ′/θ:



b)

Figure 32.

Figure 33.

$$M = -\frac{f_{objective}}{f_{eyepiece}}$$

Thus, for example, a telescope with an objective whose focal length is 1500 mm and an eyepiece whose focal length is 10.0 mm produces an angular magnification of 150.

Example

The telescope shown in Figure 34 has the following specifications: f_o = 985 mm and f_e = 5 mm. From these data, find (a) the angular magnification of the telescope and (b) the approximate length of the telescope.

Solution

$f_{objective}$ = 985 mm (a) M =?

$F_{eyepiece}$ = 5 mm (b) L=?

(a) The angular magnification is approximately

$$M = -\frac{f_{objective}}{f_{eyepiece}} = -\frac{985 \text{ mm}}{5 \text{ mm}} = -197$$

(b) The approximate length of the telescope is

$L = f_o + f_e$ = 985 mm + 5.00 mm = 990 mm

Example

In the construction of a telescope, one of two lenses is to be used as the objective and one as the eyepiece. The focal lengths of the lenses are (a) 3 cm and (b) 45 cm. Which lens should be used as the objective?

Solution

The image of the object at infnity must be clearly visble to the normal relaxed eye. Therefore, the focal length of the objective should be greater than that of the eyepiece.

The answer is: (b).

Example

Two refracting telescopes have identical eyepieces, but telescope A is twice as long as telescope B. Which telescope has the greater magnification?

Solution

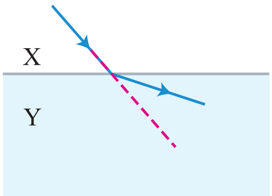
The total length of the telescope is $f_{objective} + f_{eyepiece}$. Thus, because the telescopes have identical eyepieces, it follows that telescope A must have a greater objective focal length than telescope B. We know, then, from the equation $M = f_o/f_e$, that the magnification of telescope A is greater than the magnification of telescope B.

The answer is: Telescope A has a greater magnification than telescope B.

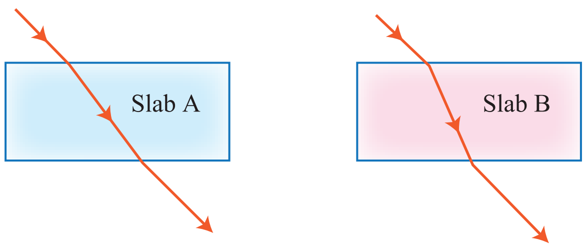
Review Exercises

I Conceptual Questions

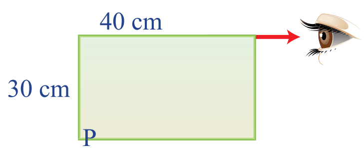
1. Does a light ray traveling from one medium into another always bend toward the normal?
2. The index of refraction of glass is 1.5. This means that a given wavelength of light travels
   (a) 1.5 times faster in air than it does in glass.
   (b) 1.5 times faster in glass than it does in air.
   (c) 1.5 times faster in vacuum than it does in glass.
   (d) 1.5 times faster in glass than it does in vacuum.
3. Light travels through a material with an index of refraction equal to 1.20. How does the speed of light through this material compare to its speed through a material with an index of refraction equal to 1.33?
4. Why does a clear stream of water appear to be shallower than it actually is?
5. A light ray traveling in medium 1 strikes medium 2 at an angle of incidence 30^o. If the angle of refraction is 90^o, which statement is incorrect?
   (a) Medium 1 is denser than medium 2.
   (b) The critical angle for this pair is 30^o.
   (c) If the angle of incidence is greater than 30^o, the light will be totally reflected.
   (d) The index of refraction of medium 1 is less than that of medium 2.
6. Light travels from medium X to medium Y as shown in the figure below. Which one of the following is correct?
   (a) Speed decreases and the wavelength increases.
   (b) Both the speed and the wavelength decrease.
   (c) Both the speed and the wavelength increase.
   (d) Both wavelength and speed are unchanged.



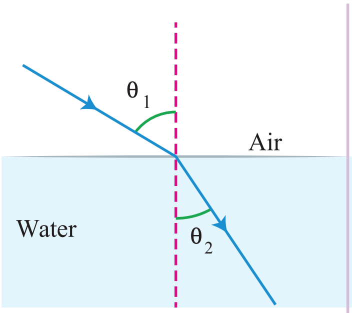
7. On a hot day, what is it that we are seeing when we observe a "water on the road" mirage?
8. Two slabs with parallel faces are made from different types of glass. A ray of light enters each slab at the same angle of incidence as shown in the figure. Which slab has the greater index of refraction?



9. What happens to a ray of light with an angle of incidence greater than the critical angle?
10. If a glass converging lens is submerged in water, will its focal length be longer or shorter than when the lens is in air?
11. An object is placed 10 cm in front of a converging lens that has a focal length of 6 cm. The image formed is
    (a) real, upright, and enlarged.
    (b) real, inverted, and enlarged.
    (c) real, upright, and reduced.
    (d) real, inverted, and reduced
12. A convex lens has focal length F. An object is located at infinity. The image formed is located
    (a) at 2F.
    (b) between F and 2F.
    (c) at F.
    (d) between the lens and F.
13. White light incident on glass prism is split into a spectrum within the glass. Which color light has the greatest angle of refraction?
14. Which of the following light color combinations is incorrect?
    (a) Red plus green produces yellow.
    (b) Red plus yellow produces magenta.
    (c) Blue plus green produces cyan.
    (d) Blue plus yellow produces white.
15. A ray of light, which is traveling in air is incident on a glass slab at an angle of 60^o. The angle of refraction in the glass is
    (a) 60^o
    (b) less than 60^o
    (c) greater than 60^o
    (d) 90^o
16. Light traveling at a non-zero angle into a denser medium is refracted
    (a) toward the normal.
    (b) away from the normal.
    (c) parallel to the normal.
    (d) parallel to the boundary.
17. The rectangular metal tank shown in the figure is filled with an unknown liquid. The observer, whose eye is level with the top of the tank, can just see corner P. What is the index of refraction of this liquid?
    (a) 1.67
    (b) 1.33
    (c) 1.50
    (d) 1.25



18. A light beam is incident upon a still water surface as it moves from air into water, as shown in the figure. The critical angle at the air-water interface is 48.8°. What is the maximum possible value for the angle of refraction θ_2?



    (a) 90°
    (b) 48.8°
    (c) Less than 48.8°
    (d) Between 48.8° and 90°

19. The speed of light in a certain medium is two-third of the speed of light in air. What is the index of refraction of the medium?
    (a) 2.25 (c) 1.5
    (b) 2.0 (d) 0.67

20. Light travels in a block of plastic that has an index of refraction of 2. What is the speed of light in the plastic?
    (a) 3×10^{-8} m/s (c) 1.5×10^{8} m/s
    (b) 6×10^{8} m/s (d) 4×10^{8} m/s

21. An object is placed at a distance of 24 cm from a diverging lens of focal length 12 cm. Where from the lens is the image formed?
    (a) 8 cm (c) –8 cm
    (b) 24 cm (d) –24 cm

22. A thin convex lens of focal length 10 cm forms an image 2 cm high of an object 5 cm high. The distance of the object from the lens is
    (a) 79 cm (c) 30 cm
    (b) 35 cm (d) 15 cm

23. What is the magnification of the image of an object placed at a distance of 15 cm from a converging lens of focal length 10 cm?
    (a) –2 (c) ½
    (b) –½ (d) 2

24. If a lens produces a real image that is four times as large as the object and is located 24 cm from the lens, what is the focal length of the lens?
    (a) 4 cm (c) 6 cm
    (b) 4.8 cm (d) 8 cm

25. Where is the image formed if an object is placed 25 cm from the eye of a nearsighted person? What kind of a corrective lens should the person wear?

(a) Behind the retina. Converging lenses.

(b) In front of the retina. Converging lenses.

(c) Behind the retina. Diverging lenses.

(d) In front of the retina. Diverging lenses.

26. White light incident on a glass prism is split into a spectrum within the glass. Which color light has the greatest angle of refraction?

(a) red light

(b) yellow light

(c) blue light

(d) The angle is the same for all wavelengths.

27. What is the relationship between the speed of light and the index of refraction of a transparent substance?

28. You notice that when a light ray enters a certain liquid from water, it is bent toward the normal, but when it enters the same liquid from crown glass, it is bent away from the normal. What can you conclude about its index of refraction comparing to that of water and the crown glass?

29. Is it possible to have an index of refraction of a medium to be less than 1? What would that imply about the velocity of light?

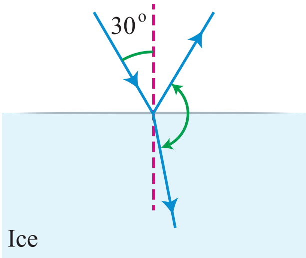
II Problems

30. A ray of light enters the top of a glass of water at an angle of 37° with the vertical. What is the angle between the refracted ray and the vertical?

31. An object is placed in front of a diverging lens with a focal length of 20 cm. For each object distance, find the image distance and the magnification. Describe the characteristics of each image. (a) 40 cm, (b) 20 cm, (c) 10 cm.

32. The angle of incidence and the angle of refraction for light going from air into a material with a higher index of refraction are 60° and 45°, respectively. What is the index of refraction of this material?

33. A ray of light is incident on the surface of a block of ice (n = 1.31) at an angle of 30° to the normal as in the figure. A portion of the light is reflected from the surface, and a portion is refracted into the ice. Determine the angle between the reflected and refracted light.



34. What is the index of refraction of a material in which the speed of light is 1.85×10^8 m/s?

35. The critical angle for a certain type of glass is determined to be 40°. What is the index of refraction of the glass?

36. Light passes from material A, which has an index of refraction of 4/3 into material B, having an index of refraction of 5/4. Find the ratio of the speed of light in material B to that in material A.
37. Where will the image be if you place an object exactly at the focal point of a concave lens? How big will the height of the image be compared to the height of the object?
38. An image seen through a 10 cm focal length convex lens is exactly the same size as the object but inverted and on the opposite side of the lens. Where must the object and image be located?
39. An object is 45 cm to the left of a lens, and the image is 25 cm to the left of the lens. What is the magnification?
40. An object is placed in front of a lens. The image is upright, 30 cm to the left of the lens, and is half as tall as the object. What is the focal length of the lens?
41. A fly appears to be upside down and twice its actual size when viewed through a converging lens with a focal length of 5 cm. How far is the fly from the lens?
42. A lens forms an image of an object. The object is 16 cm from the lens. The image is 12 cm from the lens on the same side as the object. (a) What is the focal length of the lens? Is the lens converging or diverging? (b) If the object is 8 mm tall, how tall is the image? Is it erect or inverted?
43. What color of light has the shortest wavelength?
44. Of what colors does white light consist?
45. Why does an object appear to be black?

Sample Test

1. The speed of light in a certain medium is 2.2×10^8 m/s. What is the refractive index of the medium if the speed of light in air is 3.0×10^8 m/s?
   (a) 1.2 (c) 1.4
   (b) 1.3 (d) 1.53
2. How is light affected by an increase in the index of refraction?
   (a) Its frequency increases. (c) Its speed increases.
   (b) Its frequency decreases. (d) Its speed decreases
3. If the speed of light through an unknown liquid is measured at 1.80×10^8 m/s, what is the index of refraction of this liquid? ($c = 3.00 \times 10^8$ m/s)
   (a) 1.80 (c) 1.20
   (b) 1.67 (d) 1. 00
4. Light with wavelength of 500 nm in air passes into benzene, which has an index of refraction of 1.5. What is the wavelength of the light within the benzene?
   (a) 0.0013 nm (c) 333 nm
   (b) 0.0030 nm (d) 750 nm
5. A block of flint glass with an index of refraction of 1.66 is immersed in oil with an index of refraction of 1.33. How does the critical angle for a refracted light ray in the glass vary from when the glass is surrounded by air?
   (a) It remains unchanged.
   (b) It increases.
   (c) It decreases.
   (d) No total internal reflection takes place when the glass is placed in the oil.
6. If an object in air is viewed from beneath the surface of water below, where does the object appear to be?
   (a) The object appears above its true position.
   (b) The object appears exactly at its true position.
   (c) The object appears below its true position.
   (d) The object cannot be viewed from beneath the water's surface.
7. A beam ray is incident upon a water/air interface. What is the maximum possible value for the angle of refraction? ($n_{water} = 1.333$)
   (a) 76.2° (c) 54.4°
   (b) 67.5° (d) 48.6°
8. The critical angle for light passing from a certain material into air is 45°. What is the index of refraction of this material?

(a) 1.62 (c) 1.41
(b) 1.56 (d) 1.37

9. What is the angle of refraction when the angle of incidence is equal to the critical angle for total internal reflection?
   (a) 0^o (c) 0^o
   (b) 45^o (d) 180^o

10. Which of the following conditions is not necessary for refraction to occur?
    (a) Both the incident and refracting substances must be transparent.
    (b) Both substances must have different indices of refraction.
    (c) The light must have only one wavelength.
    (d) The light must enter at an angle greater than 0^o with respect to the normal.

11. A block of glass with an index of refraction of 1.5 is immersed in water with an index of refraction of 1.33. How does the critical angle for a refracted light ray in the glass vary from when the glass is surrounded by air?
    (a) It remains unchanged.
    (b) It increases.
    (c) It decreases.
    (d) No total internal reflection takes place when the glass is placed in the oil.

12. When light of color A and light of color B are sent through a prism, color A is bent more than color B. Which color of light travels more rapidly in the prism?
    (a) Color A
    (b) Color B
    (c) They travel at the same speed
    (d) The index of refraction must be known.

13. Which statement describes what happens when green light passes from air into a new material with an index of refraction of 1.6?
    (a) Its speed increases by a factor of 1.6.
    (b) Its speed decreases by a factor of 1.6.
    (c) Its direction must change.
    (d) Its color changes.

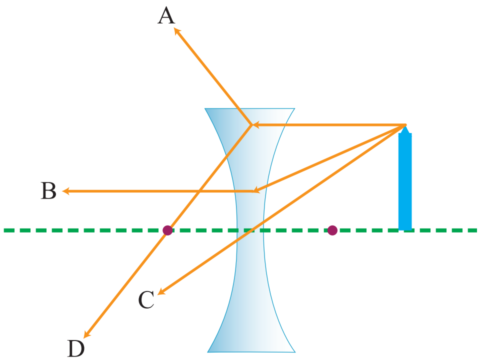
14. A light ray strikes the boundary between two transparent media. What is the angle of incidence for which there is no refraction?
    (a) 0^o (c) 90^o
    (b) 45^o (d) 180^o

15. A ray of light in a tank of water has an angle of incidence of 55°. What is the angle of refraction in air?
    (a) Less than 55°
    (b) Greater than 55°
    (c) 90°
    (d) Total internal reflection occurs
16. The critical angle for total internal reflection at a diamond/air boundary is 24.4°. For which one of the following angles will total internal reflection occur?
    (a) 22.5° (c) 24.4°
    (b) 23.4° (d) 25. 4°
17. If an object is 10 cm from a converging lens that has a focal length of 5 cm, how far from the lens will the image be?
    (a) 10 cm (c) 5 cm
    (b) 7.5 cm (d) 2.5 cm
18. An object is 30 cm in front of a converging lens of focal length 10 cm. The image is
    (a) real and larger than the object
    (b) real and the same size than the object
    (c) real and smaller than the object
    (d) virtual and the same size than the object
19. A convex lens is needed to produce an image that is 0.75 times the size of the object and located 24 cm from the lens on the other side. What focal length should be specified?
    (a) 32 cm (c) 14 cm
    (b) 24 cm (d) 7.5 cm
20. An object is located in front of a converging lens at a distance less than the focal length from the lens. Its image is
    (a) virtual and larger than the object.
    (b) real and smaller than the object.
    (c) virtual and smaller than the object.
    (d) real and larger than the object.
21. A diverging lens has a focal length of 15.0 cm. An object placed near it forms a 2.0 cm high image at a distance of 5.0 cm from the lens. The object position and object height, respectively, are
    (a) 5 cm, 2.5 cm (c) 10 cm, 4.5 cm
    (b) 7.5 cm, 3 cm (d) 15 cm, 6 cm

22. Where must an object be placed in front of a converging lens in order to obtain a virtual image?
    (a) At the focal point.
    (b) At twice the focal length.
    (c) Greater than the focal length.
    (d) Between the focal point and the lens.

23. If the diverging lens is in the previous problem is now replaced by a converging lens with the same focal length. The image position and height of image, respectively, are
    (a) 5 cm, 2.5 cm
    (b) −7.5 cm, 3.0 cm
    (c) 10 cm, 4.5 cm
    (d) −15 cm, 6.0 cm

24. Which of the rays labeled A through D in the figure is not correct?
    (a) A
    (b) B
    (c) C
    (d) D



25. What is the focal length of a lens that forms a virtual image 30 cm from the lens when a real object is placed 15 cm from the lens?
    (a) 10 cm
    (b) 15 cm
    (c) 30 cm
    (d) 45 cm

26. A concave lens has a focal length of -32 cm. what is the image distance when an object is placed 23 cm in front of the lens?
    (a) 14.2 cm
    (b) 16.0 cm
    (c) 23.3 cm
    (d) 31.5 cm

27. Which one of the items below is not involved in the formation of rainbows?
    (a) diffraction
    (b) dispersion
    (c) reflection
    (d) refraction

P12CH02

CHAPTER 2

DIRECT CURRENT ELECTRICITY

Chapter Contents

Chapter Outcomes

Learners will able to:

- construct and analyze electric circuit in determining the resistance, potential difference and the current in accordance with Ohm's and Kirchhoff' Laws.
- identify factors affecting the resistance of a conductor.

Chapter Objectives

After completing this chapter,you will be able to:

- describe the basic properties of electric current, and solve problems relating current, charge and time;
- differentiate between the different sources of direct current;
- describe the energy conversions that occur in the various sources of electric energy;
- describe the methods of detecting the presence of an electric current in a circuit;
- describe the concept of a simple electric circuit;
- explain how voltage, current, and resistance are related in an electric circuit.
- distinguishes between open and closed circuits;
- explain Ohm's law and solve problems using ohm's law;
- describe the factors that affect the resistance of a conductor (wire);
- calculate internal resistance of batteries or cells;
- distinguish between emf and terminal voltage;
- describe the effects of closed circuit and open circuit in measuring emf;
- Investigate the difference between series and parallel circuits and solve circuit problems;
- relate electric power to the rate at which electrical energy is converted to other forms of energy;
- calculate electric power and the cost of running electrical appliances;
- explain the two Kirchhoff's rules - the junction rule and the loop rule;
- solve circuit problems using the loop rule, the junction rule or both.

Introduction

Although many practical applications and devices are based on the principles of static electricity, electricity did not become an integral part of our daily lives until scientists learned to control the movement of electric charge, known as current. Electric currents power our lights, radios, television sets, air conditioners, and refrigerators. Currents also are used in automobile engines, travel through miniature components that make up the chips of computers, and perform a number of invaluable tasks.

This unit explores the properties of batteries and how they cause current and energy transfer in a circuit. This analysis includes the concepts of current, potential difference ("voltage"), resistance, electromotive force, dc circuits, and electrical energy.

All electric circuits have one thing in common they depend on the flow of electric charge. In general, electric charge is carried through a circuit by electrons. This section explores the properties of moving electric charges and what a battery does to the charges.

KEY TERMS

- electric current
- charge
- conventional current
- primary cell
- secondary cell

Direct current

A current is maintained in a closed circuit by a source of **emf**. Among such sources are batteries and generators that increase the potential energy of the moving charges. Within a battery, a chemical reaction occurs that transfers electrons from one terminal (+) to another terminal (–). Figure 1 (a) shows the two terminals of a flashlight battery. Because of the positive and negative charges on the battery terminals, an electric potential difference exists between them. Figure 1 (b) shows the symbol used to represent a battery in a circuit.

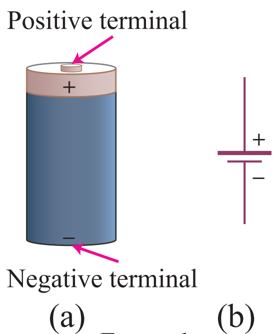


Figure 1.

- The maximum potential difference is called the electromotive force (emf) of the battery, for which the symbol ε is used.

The purpose of the battery is, therefore, to produce a potential difference, which can then make charges move. When a continuous conducting path is connected between the terminals of a battery, we have an electric circuit (Figure 2). When such a circuit is formed, charge can move (or flow) through the wires of the circuit, from one terminal of the battery to the other.

Electric Current: Suppose an amount of charge Q flows past a given point in the wire in a time t. The electric current, I, in the wire is simply defined as ***the amount of charge divided by the time.***

$$I = \frac{Q}{t}$$

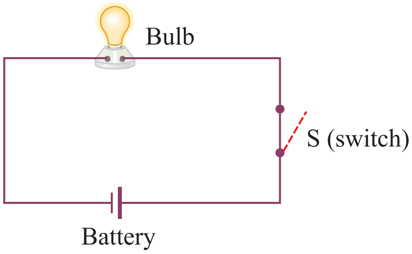


Figure 2.

The SI unit of current is the ampere, A.

(Coulomb per second, C/s = Ampere, A or 1 A = 1 C/s).

Smaller units of current are often used, such as the milliampere (mA) and microampere (μA).

(1 mA = 10^{-3} A and 1μ m = 10^{-6}A)

- Conventionally, the direction of an electric current is the direction in which positive charges would move. Thus, the current in the external circuit is directed from the positive terminal to the negative terminal of the battery. **Such current is known as direct current (DC).**

Example

A steady current of 2.5 A flows in a wire connected to a battery. But after 4 minutes, the current suddenly ceases because the wire is disconnected from the battery. (a) How much charge passed through the circuit in this time? (b) How many electrons does this represent?

Solution

I = 2.5 A (a) Q =?

t = 4 min = 4 × 60 s = 240 s (b) N =?

(a) From the definition of current, I = Q/t, we solve for the charge Q.

$Q = I\,t = 2.5\ C/s \times 240\ s = 600\ C$

(b) The amount of charge Q contains N number of elections. That is,

$Q = Ne$

$$N = \frac{Q}{e} = \frac{600\ C}{1.6 \times 10^{-19}\ C} = 3.75 \times 10^{21}\ \text{electrons}$$

Note: Current exists in a circuit only if there is a continuous conducting path. We then have a complete circuit or **closed circuit**. If there is a break in the circuit, say, a cut wire, we call it an **open circuit** and no current flows.

Example

A copper wire carries a current I to the right. In each second 1×10^{20} electrons pass through a cross-section of the wire. (a) In what direction are the electrons moving? (b) What is the value of I?

Solution

(a) According to our choice, I is the conventional current which is given to be to the right. Thus, the elections are moving to the left – opposite to the current I.

(b) The charge Q can be obtained from the equation,

$Q = Ne = (1 \times 10^{20})\ (1.6 \times 10^{-19} C) = 16\ C$

$$I = \frac{Q}{t} = \frac{16\ C}{1\ s} = 16\ A$$

Example

How long would it take for a net charge of 2.4 C to pass through a cross-sectional area of wire so as to produce a steady current of 8 mA?

Solution

$Q = 2.4\ C$ $\quad t = ?$

$I = 8\ mA = 8 \times 10^{-3}\ A$

From the definition of current, $I = Q/t$, we solve for the time t.

$$t = \frac{Q}{I} = \frac{2.4\ C}{8 \times 10^{-3} A} = 300\ \text{seconds}$$

or $t = 300/60 = 5$ minutes

Activity 1

For this activity the following materials are needed: a dry cell, a flashlight bulb, a switch and conducting wires as in Figure 3. Show all the procedures for connecting them to get a complete circuit for switching on and off the bulb. Notice that the wires are insulated. When do you say that the circuit is open? and closed?

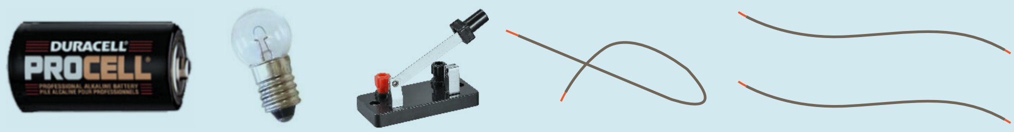


Figure 3.

Exercises

Which of the following situations has (a) the greatest, and (b) the smallest electric current? (a) the flow of 6 C in 3 s, (b) the flow of 3 C in 1 s, (c) the flow of 1 C in 2 s.

Sources of Direct Current

Every complete circuit with a steady current must include some device that provides electric energy. Such a device is called a source of emf. Dry cells, generators, solar cells, and fuel cells are all examples of sources of emf. All such devices convert energy of some form (mechanical, chemical, thermal, and so on) into electric energy and transfer it into the circuit to which it is connected.

Primary and secondary cells: One familiar source of electric energy is a voltaic cell (a common dry cell), which converts chemical energy to electric energy. Such types of sources are used only once and must be replaced with another new ones. We call this type of cells primary cells. Several of those cells connected together is called a battery. To get more voltage, for example, we connect two or more dry cells as shown in Figure 4. The cells are said to be connected in series (+ to –). The symbol of a dry cell is: 

Batteries produce electricity by converting chemical energy into electrical energy. The simplest batteries contain two dissimilar metals called electrodes. The electrodes are immersed in a solution, such as a dilute acid, called the **electrolyte**. Such a device is called an electric cell, or a dry cell.

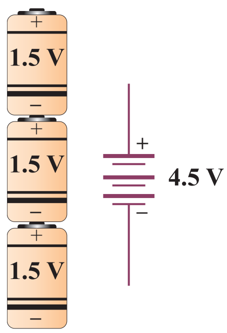


Figure 4.

The dry cell is the most common form of portable cell (Figure 5). The positive electrode is a carbon rod and the negative electrode a zinc can, which encloses the carbon rod. The electrolyte is a paste containing zinc chloride, ammonium chloride, and manganese dioxide. The whole is sealed at the top with a layer of wax. As the cell is discharged, the zinc cathode dissolves and zinc ions are formed. The cell is worn out when the manganese dioxide becomes exhausted.

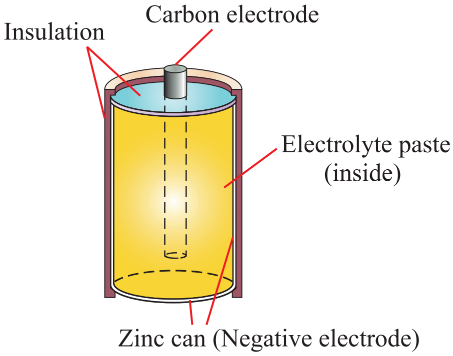


Figure 5.

Note:

(i) The voltage that exists between the terminals of a battery depends on what the electrodes are made of and their relative ability to be dissolved or give up electrons.

(ii) Batteries come with various emf's (12 V, 9 V, 1.5 V, etc.) as well as in various sizes.

(iii) Common battery sizes AAA, AA, A, C, D all provide the same emf (1.5V).

Defects of Primary cells and their correction: Simple cell is a type of primary cell which consists of four components (copper anode, zinc cathode, electrolyte solution (dilute sulphuric acid), and glass container).

There are two defects in the simple cell.

1. Local action: The electrolyte attacks the impurities in the zinc forming small cells and corrodes the electrode. This can be prevented by applying coating of mercury on zinc cathode (amalgamation).

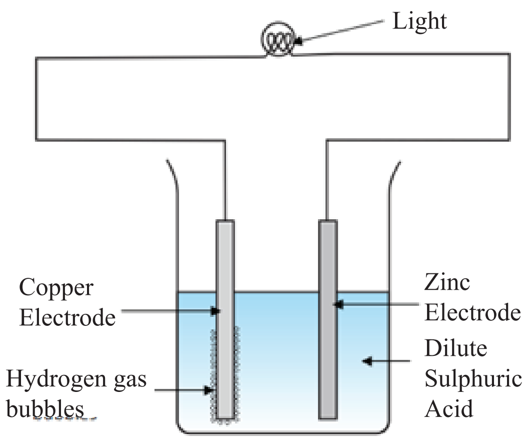


Figure 6.

2. Polarization: Bubble formation of hydrogen gas (an insulator) around the copper plate when the cell is in use resisting the flow of current.

This can be prevented by brushing the plates or by using depolarizer (potassiumchromate, which acts as an oxidising agent). This oxidizes hydrogen to form water and helps in removing the hydrogen bubbles.

The other group of cells, known as **secondary cells**, can be used again and again by recharging them or replacing the chemical inside the cell. The lead storage cell (car battery) is a widely used secondary cell. The recharging process is passing a current through it and reversing the chemical reactions. In the fully charged condition, the anode of this cell is a lead plate coated with the lead peroxide and the cathode is metallic lead. The electrolyte is sulfuric acid. The emf of the cell is 2.0 volts.

Research: dry-cell batteries and lead-acid batteries. Several chemicals are used to make batteries. Make a table listing the chemicals and electrodes used in these batteries and their specific purpose.

A fuel cell is a device that uses a chemical reaction to convert a fuel directly into electric current. Hydrogen is currently used in fuel cells that power cars and other vehicles. When two hydrogen atoms bond to an oxygen atom, the chemical reaction releases a great deal of energy. Hydrogen-powered fuel cells convert this energy into electrical energy. Hydrogen is also appealing as a fuel because the only product of the fuel-cell reaction is H_2O, water!

Solar cell (also called photovoltaic cells): The amount of solar energy that strikes the Earth in one hour could power the world's energy consumption for an entire year. Yet solar energy is not directly usable; it has to be converted. It has proven difficult to capture sunlight and transform it into usable forms. Solar cells are able to convert solar energy into electrical energy. These cells are serving as the source of potential difference in artificial satellites and calculators.

Solar cells are made of layers of two types of semiconductors called n-type and p-type. These semiconductors are made of pure silicon mixed with various chemicals. N-type semiconductors have an element, often phosphorus, which makes them electron rich. They have an overall negative charge. P-type semiconductors are mixed with an element like boron that makes them electron poor. They have an overall positive charge. When these two semiconductors are sandwiched on top of each other they create an electric field that can, under the right conditions, produce an electric current.

The solar cell is unable to produce a current by itself; it requires energy to cause its electrons to move. This energy comes from sunlight (Figure 7). When photons from the sun hit silicon atoms on the surface of the solar cell, they dislodge electrons.

These photons need to have enough energy to release electrons, as described by the photoelectric effect. The electrons are then free to move through the semiconductor. Because of the arrangement of the semiconductors, the electrons can only move in a very specific way - from the n-type to the p-type material. Metal wires that run between the two materials capture these moving electrons and lead them away from the cell. The current that leaves the cell is direct current (DC).

Figure 7.

Activity 2

Take two electrodes made of different metals and insert them into a lemon to construct a chemical cell as in Figure 8. The difference in electric potential creates an electric current that powers a bulb.

For example, a paper clip and a copper wire can be used to produce small current. Touching the ends of both wires with a tongue gives a slight tingling sensation on the tongue.

- **Key idea:** A battery is a device that uses chemical energy to separate positive and negative charges, producing a potential difference between its terminals. In this case, the chemical energy comes from reactions that take place between the metal electrodes and the acid in the lemon juice. The potential difference causes a current to flow in the wires and the bulb gives light.

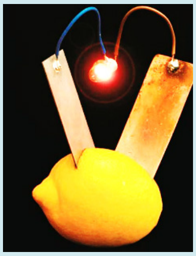


Figure 8.

Exercises

1. A wire carries a steady current of 0.1 A over a period of 20 s. What total charge passes through the wire in this time interval?
2. A flashlight bulb carries a current of 0.18 A. How much time is required for 14 C of charge to pass through the bulb?
3. In 2.0 s, 3 C of charge passes a certain point in a wire. What is the current I in the wire?
4. What is an “open” circuit? What is a “closed” circuit?
5. Give some sources of electromotive force, emf.

A potential difference is needed to cause electric current to flow through a resistor. The greater the resistance, the greater the required potential difference. Wires come in a variety of shapes, sizes, and lengths. In addition, they can be made from different metals and operate over a wide range of temperatures. In this section, you will study the relationships between voltage, current, and resistance in simple circuits and the factors that affect the resistance of a conductor (wire).

KEY TERMS

- Ohm's law
- ohmic and non-ohmic
- resistance
- resistivity

Ohm's law

The current that a battery can push through a wire is analogous to the water flow that a pump can push through a pipe. Greater pump pressures lead to larger water flow rates, and, similarly, greater battery voltages lead to larger electric currents. For example, a 9.0 V battery connected to a light bulb generates a greater current than a 6.0 V battery connected to the same bulb. For a specific case, the current I is directly proportional to the voltage V; that is, $I \sim V$.

However, the current in a wire depends not only on the voltage between its ends, but also on the resistance the wire offers to the flow of electrons. Resistance is the opposition to the motion of charge through a conductor. Mathematically, electrical resistance R is defined as the proportionality factor between the voltage V and the current I:

$$V = IR$$

The relation $R = V/I$ is referred to as **Ohm's law**, after the German physicist Georg Simon Ohm who discovered it. The SI unit of the resistance is: volt per ampere (V/A) which is called an **ohm.** It is represented by the Greek letter omega, **Ω**. (That is, $1\ \Omega = 1$ V/A.)

Note:

(i) A device for measuring resistance is called an ohmmeter.

(ii) In an electric circuit, a resistor is represented by a zigzag line: -WW-.

Example

An electric heater carries a current of 14 A when operating at a voltage of 120 V. What is the resistance of the heater?

Solution

I = 13 A R =?

V = 120 V

Applying Ohm's law, we have

$$R = \frac{V}{I} = \frac{120\ V}{14\ A} = 8.6\ \Omega$$

Example

The resistance of a steam iron is 20 Ω. What is the current in the iron when it is connected across a potential difference of 120 V?

Solution

R = 20 Ω I =?

V = 120 V

Solving Ohm's law for the current, we find

$$I = \frac{V}{R} = \frac{120\ V}{20\ \Omega} = 6\ A$$

Example

What voltage is required to produce a current of 0.60 A in a 250 Ω resistor?

Solution

I = 0.60 A V =?

R = 250 Ω

Solving Ohm's law for the voltage, we find

$V = IR = (0.60\ A)\ (250\ \Omega) = 150\ V$

Research: The unit for electrical resistance was named in honor of the German physicist Georg Simon Ohm. Ohm is credited for discovering the relationship between current, voltage, and resistance. Research and find out more about Georg Simon Ohm. Write a brief biography of him to share with the class.

Ohmic and Non-Ohmic Materials Ohm's law is an empirical relationship valid only for certain materials. Materials that obey Ohm's law, and hence have a constant resistance over a wide range of potential differences, are said to be **ohmic**. Materials having resistance that changes with potential difference or current are **Non-ohmic**.

A graph of current versus potential difference for an ohmic material is linear, as shown in Figure 9 (a). The slope of such a graph (I/ΔV) is inversely proportional to resistance. When resistance is constant, the current is proportional to the potential difference and the resulting graph is a straight line.

Non-ohmic materials have a non-linear current–voltage relationship (Figure 9 (b)). For example, a common semiconducting device that is non-ohmic is the diode - a circuit element that acts like a one-way valve for current. Its resistance is small for currents in one direction and large for currents in the reverse direction.

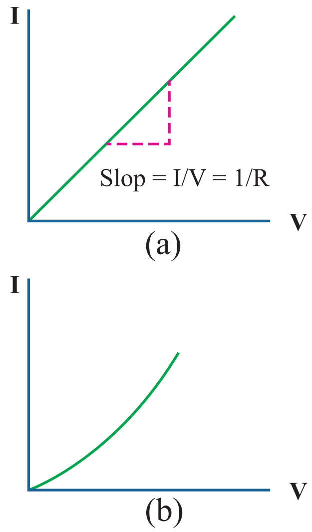


Figure 9.

Example

The current in a certain resistor is 0.50 A when it is connected to a potential difference of 110 V. What is the current in this same resistor if the operating potential difference is

(a) 90 V?

(b) 130 V? (Assume that the resistor obeys Ohm's law)

Solution

First, we determine the resistance of the resistor Using Ohm's law.

$$R = \frac{V}{I} = \frac{110\ \text{V}}{0.50\ \text{A}} = 220\ \Omega$$

(a) Since the resistor is ohmic, its value is constant: R = 220 **Ω**. Solving Ohm's law for the current, we get

$$I = \frac{V}{R} = \frac{90\ \text{V}}{220\ \Omega} = 0.41\ \text{A}$$

(b) Similarly, $I = \frac{V}{R} = \frac{130\ \text{V}}{220\ \Omega} = 0.6\ \text{A}$

Note: The resistance of the conductor does not depend on voltage and current

Exercises

Figure 10 shows a graph of current versus voltage for two different materials, A and B. Which of these materials obeys Ohm's law?

Explanation: Ohm's Law states that current is directly proportional to voltage, $V \propto I$. Wire A satisfies this linear relationship (or wire A obeys Ohm's Law).

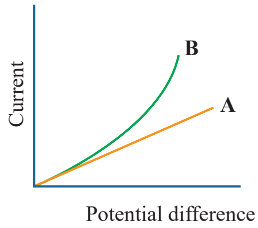


Figure 10.

Note: Material A is an ohmic material and material B is non-ohmic.

Exercises

A lamp draws a 60 mA current when connected to a 6 V battery. When a 9 V battery is used, the lamp draws 75 mA. Does the lamp obey Ohm's law?

Explanation: If the lamp obeys Ohm's law, $V \propto I$, or the ratio V/I must be constant. This constant is the resistance of the lamp.

In the 1st case: $V/I = 6\text{ V}/60\text{ mA} = 100\ \Omega$

In the 2nd case: $V/I = 9\text{ V}/75\text{ mA} = 120\ \Omega$

Clearly, R is not constant for these two measurements. This implies that the lamp doesn't obey Ohm's law.

Example

Figure 11 shows part of a circuit. Find the potential at point B.

Solution

The potential difference across the 3 Ω resistor is

$V_{AB} = IR = 2\text{ A} \times 3\ \Omega = 6\text{ V}$

This is the potential difference between points A and B. Therefore,

$V_{AB} = V_A - V_B$

$6\text{ V} = 10\text{ V} - V_B$

$V_B = 10\text{ V} - 6\text{ V} = 4\text{ V}$

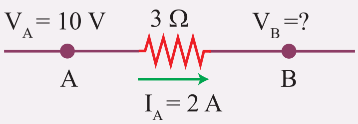


Figure 11.

Note that I is conventional current and its direction is from +ve to -ve terminal or from higher potential to lower potential. Therefore, $V_A > V_B$.

Factors affecting the resistance of a conductor (resistor)

Wires (conductors) come in a variety of shapes, sizes, and lengths. In addition, they can be made from different metals and operate at different temperatures. The resistance of a wire is affected by these factors.

A. The resistance of a wire depends on the material from which it is made. If a wire is made of copper, for instance, its resistance is less than if it is made from iron. The quantity that characterizes the resistance of a given material is its resistivity, ρ.

B. A wire's resistance also depends on its length, L, and its cross-sectional area, A. These two factors are purely geometrical. It is not difficult to see that a longer length of wire provides more resistance than a shorter length of wire does. Similarly, a wider wire allows charges to flow more easily than a thinner wire does, much as a larger pipe allows water to flow more easily than a smaller pipe does.

Combining these observations, we can write the resistance of a wire of length L, area A, and resistivity ρ in the following way:

$$R = \rho \frac{L}{A}$$

The unit of resistivity, ρ (Greek rho), is Ω.m (ohm-meter). The reciprocal of resistivity is conductivity. Good conductors have larger conductivity than insulators.

The magnitude of resistivity varies greatly with the type of material. Insulators have large resistivities, typically in the range of 10^{10} Ω·m, and conductors have low resistivities, in the range of 10^{-8} Ω·m. Thus, insulators are characterized by large resistance, and conductors have low resistance.

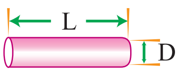
Exercises

Two wires that are made from the same material have the following dimensions. Wire 1 has a length L and a cross-section of diameter D. Wire 2 has a length 2L, and a diameter 2D. Which wire has the greater resistance? By what factor is it greater?

Explanation First, we determine the resistance of each wire.

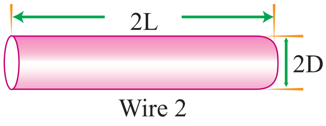
For wire 1: $R_1 = \frac{\rho L}{A} = \frac{\rho L}{\pi D^2/4}$ where the area $A = \pi r^2 = \pi(D/2)^2 = \pi D^2/4$

$$R_1 = \frac{\rho L}{A} = \frac{\rho L}{\pi D^2/4}$$



Wire 1

For wire 2, replace L with 2L and D with 2D:



$$R_2 = \frac{\rho(2L)}{\pi\frac{(2D)^2}{4}} = \frac{2}{4}\left(\frac{\rho L}{\pi D^2/4}\right)$$

$$R_2 = \frac{1}{2}\left(\frac{\rho L}{\pi D^2/4}\right) \qquad \text{But} \quad R_1 = \frac{\rho L}{\pi D^2/4}$$

$$R_2 = \frac{1}{2}R_1$$

Notice that increasing the length by a factor of 2 increases the resistance by a factor of 2; on the other hand, increasing the diameter by a factor of 2 increases the area 4 times and this in turn decreases the resistance by a factor of 4. The overall result is that the resistance of wire 2 is half that of wire 1. Therefore, wire 1 has a greater resistance than wire 2 by a factor of 2.

Exercises

The conductors shown in Figure 12 are all made of copper and are at the same temperature. Which conductor would have the least resistance to the flow of charge entering from the left?

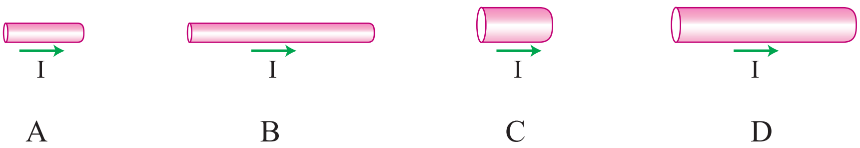


Figure 12.

Example

A current of 2 A flows through a copper wire 3.14 m long and 2 mm in diameter. What is the potential difference between the ends of the wire? (For copper, $\rho = 1.7 \times 10^{-8}$ Ω.m)

Solution

$I = 2$ A $\qquad D = 2$ mm $= 2 \times 10^{-3}$m

$L = 3.14$ m $\qquad V = ?$

First, determine the resistance of copper wire from its geometry.

$$R = \rho\frac{L}{A} = \frac{\rho L}{\pi r^2} \quad \text{(Area, } A = \pi r^2 \text{ where } r = D/2)$$

$$R = \frac{1.7 \times 10^{-8}\Omega.m \times 3.14\ m}{3.14 \times (1 \times 10^{-3} m)^2} = \frac{1.7 \times 10^{-8}\ \Omega.m^2}{1 \times 10^{-6}\ m^2} = 1.7 \times 10^{-2}\ \Omega$$

Now, apply Ohm's law to find the potential difference.

$V = IR = (2\ A)\ (1.7 \times 10^{-2}\ \Omega) = 0.034\ V$

Discuss 1: A metal wire is made thinner. How would you change the length of the wire to keep the electric resistance of the wire from changing?

Example

A 12 m length of wire 1.80 mm in diameter has a resistance of 4 Ω. What is the resistance of a 25 m length of wire 3.00 mm in diameter made of the same material?

Solution

Let the length, diameter and resistance of the first wire be

$L_1 = 12$ m, $D_1 = 1.80$ mm and $R_1 = 4\ \Omega$

Its resistance is: $R_1 = \rho_1 L_1 / A_1$ where $A_1 = \pi(D_1)^2/4$

Similarly, for the second wire, $L_1 = 25$ m, and $D_2 = 3.00$ mm

Its resistance is: $R_2 = \rho_2 L_2 / A_2$ where $A_2 = \pi(D_2)^2/4$

Both wires are made of the same material and they have the same resistivity ($\rho_1 = \rho_2$). Solving each equation for ρ, we get

$$\rho_1 = \frac{R_1A_1}{L_1} \quad \text{and} \quad \rho_2 = \frac{R_2A_2}{L_2}$$

Since the wires are made of the same material, $\rho_1 = \rho_2$. Thus,

$$\frac{R_1A_1}{L_1} = \frac{R_2A_2}{L_2}$$

$$R_2 = \frac{L_2R_1A_1}{A_2L_1} = \frac{L_2A_1}{A_2L_1}R_1 = \frac{L_2\pi D_1^2/4}{L_1\pi D_2^2/4}R_1 = \frac{L_2}{L_1} \times \left(\frac{D_1}{D_2}\right)^2 R_1$$

$$R_2 = \frac{25\ m}{12\ m} \times \left(\frac{1.80\ mm}{3.00\ mm}\right)^2 \times 4\ \Omega = \frac{25}{12} \times \left(\frac{3}{5}\right)^2 \times 4\ \Omega$$

$$R_2 = \frac{25\ m}{12\ m} \times \frac{9}{25} \times 4\ \Omega = 3\ \Omega$$

Example

When a potential difference of 12 V is applied to a wire 7 m long and cross-sectional area 8.4×10^{-8} m^2, the result is an electric current of 2 A. What is the resistivity of the wire?

Solution

V = 12 V	L = 7 m	ρ =?
I = 2 A	A = 8.4×10^{-8} m^2	

To find the resistivity first determine the resistance using ohm's law.

R = V/I = 12 V/2 A = 6 Ω

Now, solve the equation R = ρL/A for the resistivity, ρ.

$$\rho = \frac{RA}{L} = \frac{6\,\Omega \times 8.4 \times 10^{-8}\,m^2}{7\,m} = 7.2 \times 10^{-8}\ \Omega m$$

Example

A silver wire and a copper wire have the same length and the same resistance. Find the ratio of their radii, r_{silver} / r_{copper} (or r_s/r_c).

Solution

For copper: $\rho_c = 1.7 \times 10^{-8}$ Ω.m	and	$A_c = \pi r_c^2$
For silver: $\rho_s = 1.6 \times 10^{-8}$ Ω..m	and	$A_s = \pi r_s^2$
We are given that: $L_c = L_s = L$	and	$R_c = R_s$

$R_c = R_s$

$$\rho_c \frac{L}{\pi r_c^2} = \rho_s \frac{L}{\pi r_s^2}$$

$$\left(\frac{r_s}{r_c}\right)^2 = \frac{\rho_s}{\rho_c} \text{ or } \frac{r_s}{r_c} = \sqrt{\frac{\rho_s}{\rho_c}} = \sqrt{\frac{1.6 \times 10^{-8}\ \Omega.m}{1.7 \times 10^{-8}\ \Omega.m}}$$

$r_s / r_c = 0.97$

Temperature Dependence of Resistance: Over a limited temperature range, the resistivity of most metals increases linearly with increasing temperature according to the equation

$$\rho = \rho_o\left[1 + \alpha\left(T - T_o\right)\right]$$

where ρ is the resistivity at a given temperature T (in Celsius degrees), ρ_o is the resistivity at some reference temperature T_o (usually taken to be 20°C), and α is a constant called the **temperature coefficient of resistivity**. Since $R = \rho L / A$, the temperature variation of resistance can be written as

$$R = R_o\left[1 + \alpha\left(T - T_o\right)\right]$$

Example

A certain light bulb has a tungsten filament with a resistance of 20 Ω when cold (20°C) and 110 Ω when hot. Find the temperature of the filament when it is hot.

Solution

$R_o = 20\ \Omega$ $\quad T_o = 20°C$ $\quad \alpha = 4.5 \times 10^{-3}/°C$ (for tungsten)

$R = 110\ \Omega$ $\quad T = ?$

The resistance at any temperature T is given by

$R = R_o[1 + \propto (T - T_o)]$

$110\ \Omega = 20\ \Omega[1 + 4.5 \times 10^{-3}/°C(T - 20°C)]$

$110\ \Omega / 20\ \Omega = 1 + 4.5 \times 10^{-3}/°C(T - 20°C)$

$5.5 - 1 = 4.5 \times 10^{-3}/°C(T - 20°C)$

$4.5 / (4.5 \times 10^{-3}/°C) = (T - 20°C)$

$1000°C = (T - 20°C)$

$T = 1000°C + 20°C = 1020°C$

Example

A certain wire has a resistance of 40.0 Ω at 20°C. If its resistance increases to 41.4 Ω at 30°C, what is the temperature coefficient of resistivity?

Solution

$R_o = 40.0\ \Omega$ $\quad T_o = 20°C$ $\quad \alpha = ?$

$R = 41.4\ \Omega$ $\quad T = 30°C$

The temperature coefficient of resistivity α of the wire is

$R = R_o[1 + \alpha (T - T_o)]$

$R / R_o = 1 + \alpha (T - T_o)$

$(R / R_o) - 1 = \alpha (T - T_o)$

$$\alpha = \frac{\frac{R}{R_o} - 1}{T - T_o} = \frac{\frac{41.4\ \Omega}{40\ \Omega} - 1}{30°C - 20°C} = \frac{1.035 - 1}{30°C - 20°C} = \frac{0.035}{10°C}$$

$\alpha = 3.5 \times 10^{-3}/°C$ $\quad$ (or $3.5 \times 10^{-3}\ °C^{-1}$)

Exercises

1. If the voltage V were plotted against current I for two ohmic conductors with different resistances on the same graph, how could you tell the less resistive one?
2. A student argues that because R = V/I, if he increases the voltage, the resistance will increase as well. Is he correct?
3. A 1.5 V battery is connected to a small light bulb with a resistance of 3.5 Ω. What is the current in the bulb?
4. The current in a certain resistor is 0.50 A when it is connected to a potential difference of 110V. What is the current in this same resistor if the operating potential difference is 120 V?
5. A metal conductor has a resistance R. If both its length and its diameter are tripled, what will be its new resistance?
6. A wire has resistance R. What is the resistance of a wire of the same substance that has the same length but twice the cross-sectional area?

Electric circuits often contain several resistors connected in various ways. In this section we consider simple circuits containing resistors and batteries. For each type of circuit, you'll learn how to calculate the net effect of a group of resistors.

Key Terms

- series circuit
- parallel circuit
- Ammeter
- Voltmeter

Direct current (DC) circuits

A simple DC circuit consists of a source of potential difference, such as a battery, a resistor or load (such as a bulb), switch, and wire. An electric circuit is a complete path through which charges can flow. A schematic diagram for a circuit is sometimes called a **circuit diagram.**

Note: Batteries produce direct currents (DC).

In the circuit diagram shown in Figure 13, electrons flow from the negative terminal of the battery to the positive terminal. The direction of the current, I, is just the opposite: from the positive terminal to the negative terminal.

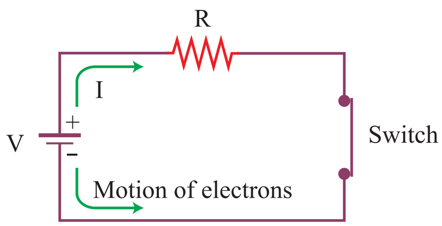


Figure 13.

Resistors in series

Thus far, we have dealt with circuits that include only a single device, such as a light bulb. In this case, the potential difference across the bulb equals that of the battery. The total current in the circuit can be found using the equation V = IR. There are, however, many circuits in which more than one device (resistor) is connected to a voltage source, as shown in Figure 14.

When moving through this circuit, charges that pass through one resistor must also move through the second and third resistors. Because all charges in the circuit must follow the same conducting path, these resistors are said to be connected in **series**. In a series circuit,

(i) the same current I passes through each resistor. I is the current from the battery.

(ii) the total voltage of the source, V, is equal to the sum of the voltage drops across the separate resistors.

$V = V_1 + V_2 + V_3$

Using Ohm's law, V = IR, we can write

$V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$.

For resistors in series, we have:

$IR_{eq} = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$

$R_{eq} = R_1 + R_2 + R_3$

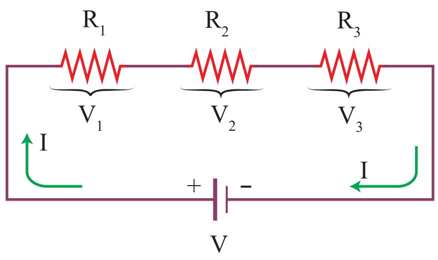


Figure 14.

When we put several resistances in series, the total or equivalent resistance, R_{eq}, is the sum of the separate resistances.

Exercises

A series circuit has four resistors. The current through one resistor is 200 mA. a) How much current is supplied by the source? b) If the four resistors are identical and the battery has a voltage V, what will be the voltage drop across each resistor?

Exercises

Figure 15 shows three identical light bulbs, each with a resistance R connected in series. When the switch S is closed, what happens to the brightness of (a) bulb 2, and (b) bulbs 1 and 3?

Explanation

(a) The light of bulb 2 goes out when the switch S is closed, because the current follows a path with least resistance - a case known as short -circuit

(b) When S is closed, the equivalent resistance of the circuit decreases from $R_{eq} = R + R + R = 3R$ to $R_{eq} = R + R = 2R$. As a result, the current increases and this in turn increases the 'power'. Therefore, the intensities of bulbs 1 and 3 increases (i.e., bulbs 1 and 3 become brighter).

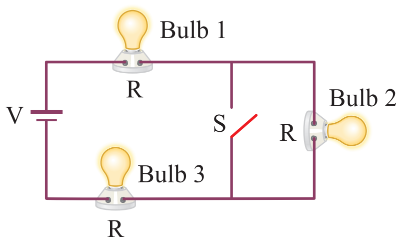


Figure 15.

Example

Four resistors are connected as shown in Figure 16. Find (a) the total resistance of the circuit and (b) the current in the circuit if the voltage of the battery is 6 V.

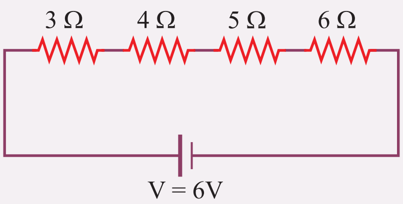


Figure 16.

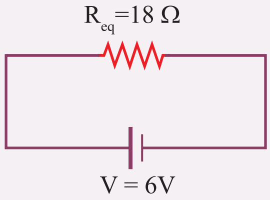
Solution

(a) The four resistors are connected in series. Their equivalent resistance is

$R_{eq} = 3\ \Omega + 4\ \Omega + 5\ \Omega + 6\ \Omega$

$R_{eq} = 18\ \Omega$

(b) Apply Ohm's law and find the current through the equivalent resistor, R_{eq}, shown at the right.



$$I = \frac{V}{R_{eq}} = \frac{6\ V}{18\ \Omega} = 0.5\ A$$

Example

A circuit consists of three resistors connected in series to a 24 V battery. The current in the circuit is 0.5 A. If $R_1 = 16\ \Omega$ and $R_2 = 24\ \Omega$, find (a) the value of R_3 and (b) the voltage across each resistor.

Solution

$V = 24\ V$ $\quad R_1 = 16\ \Omega$

$I = 0.5\ A$ $\quad R_2 = 24\ \Omega$

(a) Use ohm's law to find the equivalent resistance of the circuit

$$R_{eq} = \frac{V}{I} = \frac{24\ V}{0.5\ A} = 48\ \Omega$$

Now, use this value of R_{eq} to find R_3.

$R_{eq} = R_1 + R_2 + R_3$

$48\ \Omega = 16\ \Omega + 24\ \Omega + R_3 = 40\ \Omega + R_3$

$R_3 = 48\ \Omega - 40\ \Omega = 8\ \Omega$

(b) Use ohm's law to determine the voltage across each resistor.

$V_1 = IR_1 = 0.5\ A \times 16\ \Omega = 8\ V$

$V_2 = IR_2 = 0.5\ A \times 24\ \Omega = 12\ V$

$V_3 = IR_3 = 0.5\ A \times 8\ \Omega = 4\ V$

Notice that 8 V + 12 V + 4 V = 24 V, as expected.

Example

A string of holiday lights has ten bulbs with equal resistances connected in series. When the string of lights is connected to a 120 V outlet, the current through the bulb is 0.6 A. (a) What is the equivalent resistance of the circuit? (b) What is the resistance of each bulb?

Solution

$V = 120\ V$

$I = 0.6\ A$

(a) Applying Ohm's law, the total resistance (R_{eq}) is:

$$R_{eq} = \frac{V}{I} = \frac{120\ V}{0.06\ A} = 2\ 000\ \Omega$$

(b) Since each bulb has the same resistance,

$R_{eq} = R_1 + R_2 + R_3 + \ldots R_n = R + R + R + \ldots = \Sigma R = nR$

If there are n bulbs, the equivalent resistance equals n times R_1, and the resistance of one bulb is:

$R_{eq} = nR$ but $n = 10$ bulbs

$2000\ \Omega = 10R$

$$R = \frac{2000\ \Omega}{10} = 200\ \Omega$$

Resistors in Parallel: Another way to connect resistors is in parallel, so that the current from the source splits into separate branches or paths. Parallel wiring means that the devices are connected in such a way that the same voltage is applied across each device. Figure 17 shows three resistors connected in parallel between the terminals of a battery. In a parallel circuit,

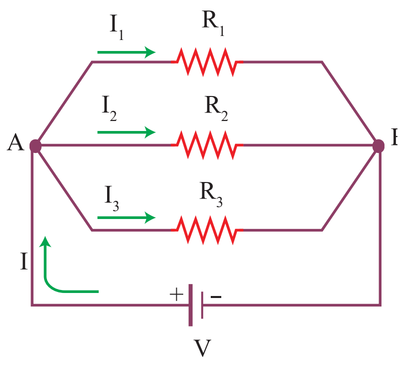


Figure 17.

(i) the total current I that leaves the battery in Figure 17 splits into three separate paths. Because electric charge is conserved, the current I flowing into junction A (where the different wires or conductors meet) must equal the current flowing out of the junction, B. Thus

$I = I_1 + I_2 + I_3$

(ii) each has the same voltage across it. Hence the total voltage of the battery is applied to each resistor. Applying Ohm's law to each resistor, we have

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \text{ and } I_3 = \frac{V}{R_3}$$

The total current from the battery is: $I = V/R_{eq}$. Therefore,

$$I = I_1 + I_2 + I_3$$

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{(Resistors in parallel)}$$

Note

(i) With parallel wiring, if you disconnect one resistor (say, R_1 or R_1), the current to the other devices is not interrupted. Compare to a series circuit, where if one resistor (say, R_1) is disconnected, the current is stopped to all others.

(ii) The equivalent (or net) resistance is less than each single resistance.

Exercises

Two identical light bulbs are connected to a battery, either in series or in parallel. Are the bulbs in series (a) brighter, (b) dimmer, (c) the same brightness as the bulbs in parallel?

Explanation: Both bulbs are connected to the same potential difference. If we let the resistance of each bulb as R, then R_{eq} is twice the resistance of one bulb in the series circuit,

$$R_{eq} = R + R = 2R$$

And half the resistance of a bulb in the parallel circuit: $R_{eq} = ½\,R$

Therefore, the bulbs connected in series are dimmer than the bulbs connected in parallel, and the correct answer is choice (b).

Example

Three resistors, $R_1 = 60\ \Omega$, $R_2 = 30\ \Omega$, and $R_3 = 20\ \Omega$, are connected in parallel across a 90V battery (Figure 18). Find (a) the equivalent resistance of the circuit (b) the current through each branch of the circuit (c) the current through the battery.

Solution

(a) For a parallel combination, the equivalent resistance is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{60\ \Omega} + \frac{1}{30\ \Omega} + \frac{1}{20\ \Omega}$$

$$= \frac{1 + 2 + 3}{60\ \Omega} = \frac{6}{60\ \Omega} = \frac{1}{10\ \Omega}$$

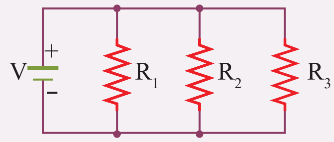


Figure 18.

$$R_{eq} = 10\ \Omega$$

(b) The voltage across each resistor is the same, so we use I = V/R for each branch.

$$I_1 = \frac{V}{R_1} = \frac{90\ V}{60\ \Omega} = 1.50\ A, \qquad I_2 = \frac{V}{R_2} = \frac{90\ V}{30\ \Omega} = 3.00\ A, \text{ and}$$

$$I_3 = \frac{V}{R_3} = \frac{90\ V}{20\ \Omega} = 4.50\ A$$

(c) The total current is

$$I = \frac{V}{R_{eq}} = \frac{90\ V}{10\ \Omega} = 9\ A$$

Example

Consider a circuit with two identical resistors R connected in parallel. What is the equivalent resistance?

Solution

Since the two resistors are identical, $R_1 = R_2 = R$.

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} = \frac{2}{R}$$

$$R_{eq} = \frac{1}{2} R$$

If we connect three such resistors in parallel, the corresponding result is:

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} = \frac{3}{R}$$

$$R_{eq} = \frac{1}{3} R$$

We see that the more resistors we connect in parallel, the smaller the equivalent resistance. Each time we add a new resistor in parallel with the others, we give the current a new path to flow; this means that the equivalent resistance will be reduced.

Example

Two identical 100 Ω Resistors are joined together as shown in Figure 19. What is the current through each resistor when the switch S is (a) open? (b) closed?

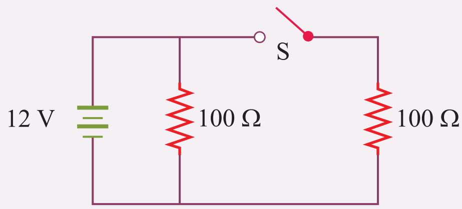


Figure 19.

Solution

(a) When the switch S is open, there is no current through the resistor at the right side. So the current through the left resistor is

$$I = \frac{V}{R} = \frac{12\ V}{100\ \Omega} = 0.12\ A$$

(b) When S is closed, current flows through both resistors.

$$I_1 = \frac{12\ V}{100\ |} = 0.12\ A \quad \text{and} \quad I_2 = \frac{12\ V}{100\ |} = 0.12\ A$$

Example

Two resistors are connected in parallel across a 12 V battery. Resistor A has a value of 24Ω and resistor B carries a current of 0.25 A. Determine (a) the potential difference across each resistor, (b) the current in resistor A, and (c) the resistance of B.

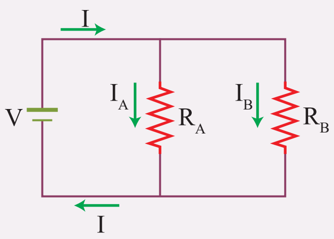
Solution

$R_A = 24\ \Omega$ $\qquad I_B = 0.25$ A

V = 12 V

(a) The figure below illustrates the situation. We know that for parallel connection of resistors,

$V = V_A = V_B = 12$ V



(b) Use ohm's law to find I_A

$$I_A = \frac{V}{R_A} = \frac{12\ V}{24\ \Omega} = 0.5\ A$$

(c) Since $V_B = 12$ V and $I_B = 0.25$ A,

$$R_B = \frac{V}{I_B} = \frac{12\ V}{0.25\ A} = 48\ \Omega$$

Example

Four resistors, 5 Ω, 8 Ω, 12 Ω, and 15 Ω, are connected in parallel. The current through the 15 Ω resistor is 4 A. Determine the currents in the other three resistors.

Solution

First, determine the voltage across the 15 Ω resistor.

$V = IR = 4\ A \times 15\ \Omega = 60$ V

This voltage is the same across the other three resistors because all are connected in parallel. The current through the 5 Ω resistor is then,

$$I_5 = \frac{V}{R} = \frac{60\ V}{5\ \Omega} = 12\ A$$

Similarly, $I_8 = \frac{60\ V}{8\ \Omega} = 7.5A$ and $I_{12} = \frac{60\ V}{12\ \Omega} = 5\ A$

Example

Figure 20 represents an electric circuit consisting of a 12 V battery, a 3 Ω resistor (R_1) and a variable resistor (R_2). At what value must the variable resistor be set to produce a current of 1.0 A through R_1?

Solution

R_1 and the variable resistor R_2 are in series.

$V = IR_{eq} = I(R_1 + R_2)$

$12\ V = (1.0\ A)\ (3\ \Omega + R_2)$ or $R_2 = 9\ \Omega$

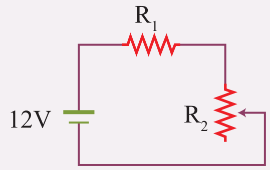


Figure 20.

ACTIVITY 3

For this activity you need the following materials: Two flashlight bulbs with bulb holders, battery, and four short pieces of wire (Figure 21). Show all the procedures needed to connect the bulbs (a) in series, and (b) in parallel with the battery. Notice that the wires are insulated.

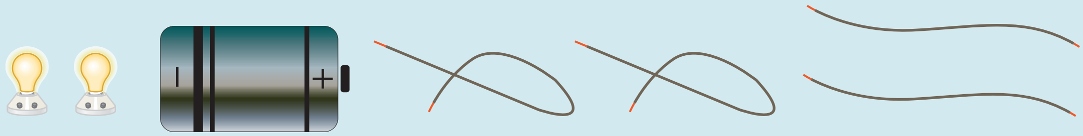


Figure 21.

Discuss 2: What will happen to one bulb if the other bulb is removed (unscrewed) from the circuit. Is the brightness of the bulbs the same in each case? Explain.

Series-parallel resistors

The rules we've developed for series and parallel resistors can be applied to a variety of complex circuits that aren't purely series or parallel.

Example

In the circuit shown in Figure 22, the voltage of the battery is 12 V, and all the resistors have a resistance of 150 Ω. Find (a) the equivalent resistance of the network, and (b) the current supplied by the battery.

Solution

(a) To find the equivalent resistance R_{eq}, we first note that the upper two resistors are in series, giving a total resistance of

$R + R = 2R$

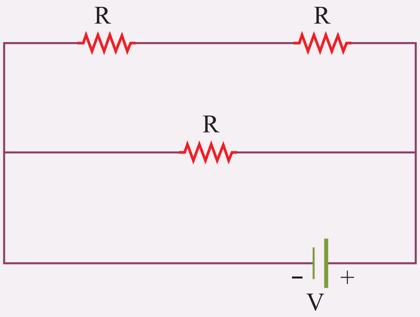
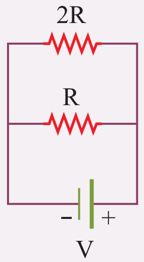


Figure 22.

Next, the lower resistor, R, is in parallel with 2R. The equivalent resistance of the given circuit is, therefore,

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} = \frac{2+1}{2R} = \frac{3}{2R}$$

$$R_{eq} = \frac{2}{3}R = \frac{2}{3} \times 150\ \Omega = 100\ \Omega$$



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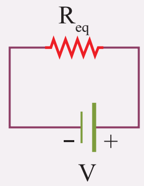


(b) The current I is given by ohm's law, $I = V/R_{eq}$

$$I = \frac{12\ V}{100\ \Omega} = 0.12\ A$$

Example

Compute the equivalent resistance of the network shown in Figure 23 and find the current supplied by the battery.

Solution

In this circuit the two resistors, R_2 and R_3, are in parallel. So, their combined resistance is

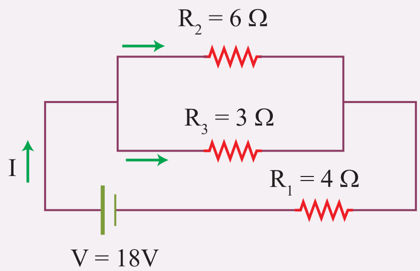


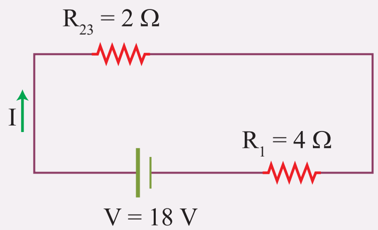
Figure 23.

$$\frac{1}{R_{23}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6\ \Omega} + \frac{1}{3\ \Omega}$$

$$\frac{1}{R_{23}} = \frac{1+2}{6\ \Omega} = \frac{3}{6\ \Omega} = \frac{1}{2\ \Omega}$$

$R_{23} = 2\ \Omega$

Now, R_1 is in series with R_{23} (see the circuit diagram at the right). The equivalent resistance of this circuit is



$R_{eq} = R_1 + R_{23} = 4\ \Omega + 2\ \Omega = 6\ \Omega$

The current delivered by the battery is:

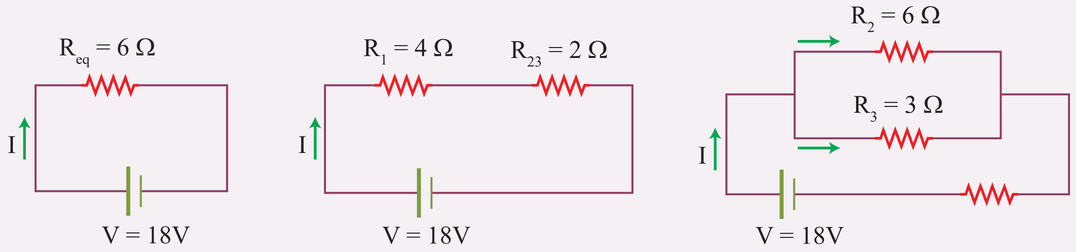
$I = V/R_{eq} = 18\ V/6\ \Omega = 3\ A$

Example

For the circuit diagram shown in Worked Example: 2.25 (above), find the current in each resistor.

Solution

To find the current in each resistor of the original network, we reverse the steps by which we reduced the network. In the figure below, the last diagram is (c), the middle step is (b), and the original circuit diagram is (a).



(c) (b) (a)

In the circuit shown in part (c), the current is 3 A. So, the current in the two series resistors, 4 Ω and 2 Ω, is also 3 A (see part (b)). The voltage across the 2 Ω resistor is therefore,

$$V = 3\ \text{A} \times 2\ \Omega = 6\ \text{V}$$

This voltage must also be 6 V across the two parallel resistors, 6 Ω and 3 Ω, (see part (a)). Finally, the current through the 6 Ω resistor is:

$$I = \frac{V}{R} = \frac{6\ \text{V}}{6\ \Omega} = 1\ \text{A},$$

and through the 3 Ω resistor:

$$I = \frac{V}{R} = \frac{6\ \text{V}}{3\ \Omega} = 2\ \text{A}$$

Note that for the two resistors in parallel (part (a)), more current goes through the path of least resistance.

Example

For the circuit shown in Figure 24, determine (a) the current through the 4 Ω resistor, and (b) the potential difference across the 3.5 Ω resistor.

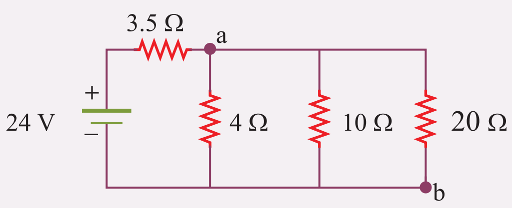


Figure 24.

Solution

First, determine the equivalent resistance of the circuit. The 4 Ω, 10 Ω, and 20 Ω resistors are in parallel. So, their equivalent resistance is R_{ab} (total resistance b/n points a & b).

$$\frac{1}{R_{ab}} = \frac{1}{4\ \Omega} + \frac{1}{10\ \Omega} + \frac{1}{20\ \Omega} = \frac{5+2+1}{20\ \Omega} = \frac{8}{20\ \Omega}$$

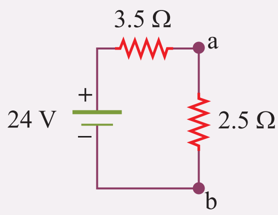
$R_{ab} = 2.5\ \Omega$

The equivalent resistance of the **circuit** is therefore,

$R_{eq} = 3.5\ \Omega + 2.5\ \Omega = 6\ \Omega$

The total current I is

$$I = \frac{V}{R_{eq}} = \frac{24\ V}{6\ \Omega} = 4\ A$$



The voltage drop across R_{ab} is: $V_{ab} = IR_{ab} = 4\ A \times 2.5\ \Omega = 10\ V$

Note that this voltage, V_{ab}, is the same across the three parallel resistors whose equivalent resistance is 2.5 Ω.

(a) The current through the 4 Ω resistor is then

$$I_4 = \frac{V}{R} = \frac{10\ V}{4\ \Omega} = 2.5A$$

(b) The potential difference across the 3.5 Ω resistor is:

$V = IR = 4\ A \times 3.5\ V = 14\ V$

Example

Compute the equivalent resistance of the network shown in Figure 25 and also calculate the current in the 3 Ω resistor.

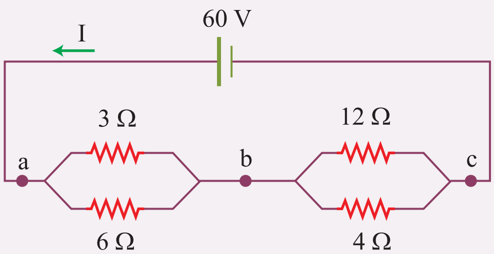


Figure 25.

Solution

We can reduce the network to a single equivalent resistance. Note that the 3 Ω and 6 Ω resistors are in parallel with each other, and the 12 Ω and 4 Ω resistors are in parallel with each other.

$$\frac{1}{R_{ab}} = \frac{1}{3\ \Omega} + \frac{1}{6\ \Omega} = \frac{2+1}{6\ \Omega} = \frac{3}{6\ \Omega}$$

$R_{ab} = 2\ \Omega$

$$\frac{1}{R_{bc}} = \frac{1}{12\ \Omega} + \frac{1}{4\ \Omega} = \frac{1+3}{12\ \Omega} = \frac{4}{12\ \Omega} = \frac{1}{3\ \Omega}$$

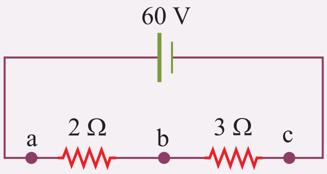
$R_{bc} = 3\ \Omega$

The equivalent resistance R_{eq} is therefore,

$R_{eq} = 2\ \Omega + 3\ \Omega = 5\ \Omega$

The total current I is

$$I = \frac{V}{R_{eq}} = \frac{60\ V}{5\ \Omega} = 12\ A$$



This current flows through $R_{ab} = 2\ \Omega$. The p.d. across R_{ab} is:

$V_{ab} = I\,R_{ab} = 12\ A \times 2\ \Omega = 24\ V$

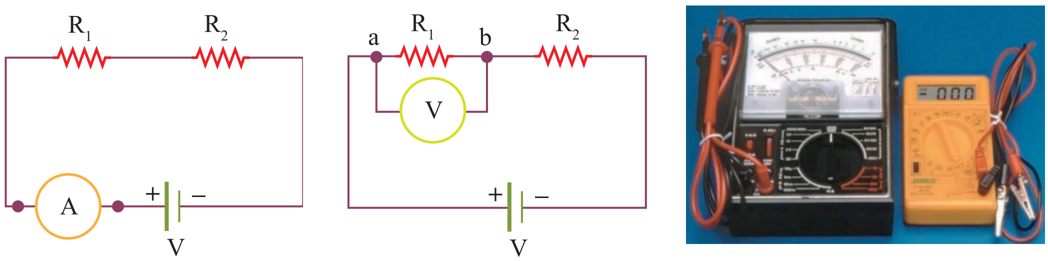
This voltage drop is also the voltage across the two original resistors between points a and b. Thus, the current through the 3 Ω resistor is:

$$I_3 = \frac{V_{ab}}{R_{ab}} = \frac{24\ V}{3\ \Omega} = 8\ A$$

Ammeters and voltmeters

Devices for measuring currents and voltages in a circuit are referred to as ammeters and voltmeters, respectively. In each case, the ideal situation is for the meter to measure the desired quantity without altering the characteristics of the circuit being studied. This is accomplished in different ways for these two types of meters.

Figure 26 (a) shows the connection of an ammeter in a circuit. Since an ammeter is an instrument that measures current, it must be inserted in the circuit so the current passes directly through it. When an ammeter is inserted into a circuit, the internal resistance of the ammeter adds to the circuit resistance. Any increase in circuit resistance causes a reduction in current, and this is a problem, because an ammeter should only measure the current, not change it. Therefore, an ideal ammeter would have zero internal resistance.



(a) (b) (c)

Figure 26.

A **voltmeter** is connected "externally," in parallel with the circuit across which the voltage is to be measured. It measures the potential difference between two points. Its two wire leads are connected to the two points a and b as shown in Figure 26 (b), where the voltage across R_1 is being measured.

A real voltmeter always allows some current to flow through it, which means that the current flowing through the circuit is less than before the meter was connected. As a result, the measured voltage is altered from its ideal value.

An ideal voltmeter, then, would be one in which the resistance is infinite, so that the current it draws from the circuit is negligible. In practical situations it is sufficient that the resistance of the meter be much greater than the resistances in the circuit.

An **ohmmeter** measures the electrical resistance of a circuit element when the element is disconnected from a circuit. Sometimes the functions of an ammeter, voltmeter, and ohmmeter are combined in a single device called a **multimeter** (Figure 26 (c)). Adjusting the settings on a multimeter allows a variety of circuit properties to be measured.

Example

For the circuit diagram shown in Figure 27, (a) what should the ammeter reading be? (b) what should the voltmeter reading be?

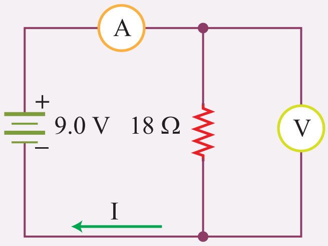


Figure 27.

Solution

In the ideal situation, both meters should give the values that are obtained by calculation. Thus,

(a) the ammeter should read the current in the circuit:

$I = V/R = 18\ V/9\ \Omega = 2\ A$

(b) the voltmeter (which is connected in parallel with the resistor) should read 9 V.

Exercises

1. A 5 Ω, a 10 Ω, and a 15 Ω resistor are connected in parallel. (a) Which resistor has the most current in it? (b) Which resistor has the largest potential difference across it?
2. Three 15 Ω resistors are connected in parallel and placed across a 30 V battery. What is (a) the equivalent resistance of the parallel circuit? (b) the total current through the circuit? (c) the current through each branch of the circuit?
3. Two identical resistors R are connected in parallel and their combination is connected in series to a 40 Ω resistor. If the equivalent resistance of the three resistors is 55 Ω, what is the value of R?

4. In an experiment 4 resistors of value 3 Ω, 4 Ω, 12 Ω, and 16 Ω are required at different times. But the laboratory technician told you to use only two resistors. Which two resistors do you choose to do the experiment?

Real sources of emf don't behave exactly like the ideal sources we've described because charge that moves through the material of any real source encounters resistance. We call this the internal resistance of the source. As a result, the potential difference between the terminals of a real battery is less than its ideal emf. In this section, we will discover how these parameters are related using Ohm's law.

KEY TERMS

- internal resistance
- terminal voltage
- emf

Electromotive force - emf

When a battery is disconnected from a circuit, and carries no current, the difference in electric potential between its terminals is referred to as its electromotive force (or emf, ε). The emf is the maximum possible potential difference between the battery terminals.

When a source (such as a battery) is connected to an external resistor R and current flows, the voltage across the battery (i.e., the **terminal voltage**, V) is slightly less than the emf, ε, of the source. The reason is that charge moving through the material of any real source encounters resistance – known as the **internal resistance** of the battery, denoted by r.

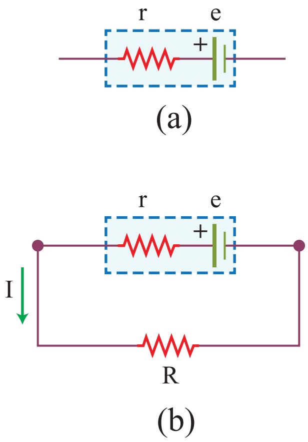


Figure 28.

The simplest way to model a real battery is to imagine it consists of an ideal battery of emf ε in series with an internal resistance r, as shown in Figure 28 (a). If this battery is then connected to an external resistance, R, the equivalent resistance of the circuit is r + R (Figure 28 (b)). As a result, the current flowing through the circuit is

$$I = \frac{\varepsilon}{(r + R)}$$

$$\text{or } \varepsilon = I(r + R) = Ir + IR$$

and the potential difference between the terminals of the battery is

$$V = \varepsilon - Ir$$

Thus, we see that the potential difference produced by the battery is less than ε by an amount that is proportional to the current I. Only when the current is zero (or open circuit) will the battery produce its full emf.

Example

Figure 29 shows a battery with an emf ε of 12 V and an internal resistance r of 2 Ω. The wires to the left of a and to the right of the ammeter A are not connected to anything. Determine the readings of the voltmeter and the ammeter. (Assume that the meters are ideal.)

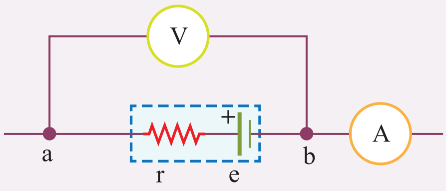


Figure 29.

Solution

There is no current because there is no complete circuit. Hence the ammeter A reads zero current, I = 0. Because there is no current through the battery, there is no potential difference across its internal resistance. Therefore, Ir = 0 and the voltmeter reads,

$\varepsilon = Ir + V = 0 + V$

$V = \varepsilon = 12\text{ V}$

Note: The terminal voltage of a real source equals the emf only if there is no current through the source.

Example

A 4 Ω resistor is now added to the battery shown in Figure 29 to get a complete circuit as in Figure 30. What are the voltmeter and ammeter readings now?

Solution

We now have a complete circuit. The current I through the resistor R is

$$I = \frac{\varepsilon}{R + r} = \frac{12\text{ V}}{4\,\Omega + 2\,\Omega} = 2\text{ A}$$

The ammeter A reads I = 2 A

The voltage across the resistor is then,

$V = IR = 2\text{ A} \times 4\,\Omega = 8\text{ V}$

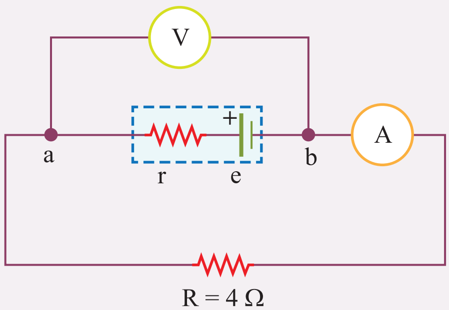


Figure 30.

This is equal to the terminal voltage V:

$V = \varepsilon - Ir = 12\ V - (2\ A \times 2\ \Omega) = 8\ V$

The terminal voltage is the voltage available for the external circuit. So, the voltmeter reads this value, 8 V.

Example

Four 1.5 V dry cells are connected in series to a 10 Ω light bulb. If the resulting current is 0.5 A, what is the internal resistance of each cell, assuming they are identical?

Solution

The cells are connected in series; so the total emf is

$\varepsilon = 4 \times 1.5\ V = 6\ V$

Each cell has a resistance r, and the total internal resistance of the four cells is

$r_{tot} = r + r + r + r = 4r$

We are given that I = 0.5 A and R = 10 Ω.

Solving the equation $\varepsilon = Ir_{tot} + IR$ for r_{tot}, we get

$Ir_{tot} = \varepsilon - IR$

$$r_{tot} = \frac{\varepsilon - IR}{r}$$

$$= \frac{6\ V - (0.5\ A)(10\ \Omega)}{0.5\ A} = 2\ \Omega$$

The internal resistance of each cell is then,

$r = 2\ \Omega / 4 = 0.5\ \Omega$

Example

A battery of emf 24 V and internal resistance 0.6 Ω is connected across a 9 Ω resistor. Determine (a) the current I through the resistor, and (b) the terminal voltage V.

Solution

(a) Solving the equation $\varepsilon = Ir + V = Ir + IR$ for I, we get

$\varepsilon = I(r + R)$

$$I = \frac{emf}{r + R} = \frac{24\ V}{0.6\ \Omega + 9\ \Omega} = \frac{24\ V}{9.6\ V} = 2.5\ V$$

(b) The terminal voltage, V, is

$V = \varepsilon - Ir = 24\ V - (2.5\ A)(0.6\ \Omega) = 24\ V - 1.5\ V$

$V = 22.5\ V$

Example

A battery of emf ε and internal resistance r is connected across a variable resistor. When the resistor is set at 21 Ω the current through it is 0.48 A; when it is set at 36 Ω the current is 0.30 A. Determine (a) r, and (b) ε.

Solution

A variable resistor is a resistor whose value we can vary as we wish. In a circuit its symbol is  or .

(a) To find r, we form two equations. When R = 21 Ω, the current I = 0.48 A. Then,

$\varepsilon = Ir + IR = 0.48r + 0.48\ \text{A} \times 21\ \Omega$

$\varepsilon = 0.48r + 10.08\ \text{V}$ ……………………………......... 1

When R = 36 Ω, I = 0.30 A. Then,

$\varepsilon = 0.30r + 0.30\ \text{A} \times 36\ \Omega$

$\varepsilon = 0.30r + 10.80\ \text{V}$ ……………………………......... 2

Since the emf of the battery is constant, we equate the two equations (1) and (2) to get r.

$$0.48r + 10.08\text{V} = 0.30r + 10.80\ \text{V}$$

$$0.48r - 0.30r = 10.80\ \text{V} - 10.08\ \text{V}$$

$$0.18r = 0.72\ \text{V}$$

$$r = 4\ \Omega$$

(b) Substituting the value of r in one of the two equations, we get

$\varepsilon = 0.48\ r + 10.08\ \text{V}$

$\varepsilon = 0.48 \times 4 + 10.08 = 1.92\ \text{V} + 10.08\ \text{V} = 12\ \text{V}$

Discuss 3: Which is greater, a battery's terminal voltage or the same battery's emf? Explain why these two quantities are not equal. Is there a situation where the emf and terminal voltage of a battery are equal?

Exercises

1. A 6.00 V battery is connected to a light bulb, and the resulting current in the circuit is 0.35 A. What is the internal resistance of the battery?
2. A complete series circuit consists of a 12 V battery, a resistor 4.70 Ω, and a switch. The internal resistance of the battery is 0.30 Ω. What does an ideal voltmeter read when placed across the terminals of the battery if the switch is (a) open, and (b) closed?

Many familiar household appliances convert electric energy to some other forms, such as light, kinetic energy, sound, or heat energy. When you turn on one of these appliances, you complete a circuit and begin converting electric energy. In this section, you will learn to determine the rate of energy conversion and the cost of using this converted energy.

KEY TERMS

- Energy (Work)
- Power
- kWh

Electric energy

When a source of voltage such as a battery is connected to a circuit, the source provides energy to the charge passing through it. This energy can then be transferred from the charge to other devices in the circuit, for example, to light a bulb. In other words, the charge that flows through a bulb or any resistor dissipates energy in it. It follows from the definition of potential difference ($V = W/Q$) that the energy dissipated is given by

$$W = VIt \qquad \text{(where } Q = It\text{)}$$

Notice that the energy gained by a charge Q from the voltage source is the same as the work done by the source on the charge; that is

Electrical energy = work done = VIt

The SI unit of energy is joule (J). 1J = 1VAs (volt-ampere-second).

Example

What electric energy is used when a current of 0.40 A is passed through a lamp for 1minute?

Solution

$I = 0.40$ A $\qquad$ $W = ?$

$t = 1\text{min} = 60$ s

Energy used = work (W)

$W = VIt$

$W = 6\text{ V} \times 0.40\text{ A} \times 60\text{ s} = 144\text{ J}$

Example

A 30 Ω resistor is connected across a 60 V battery. How much energy is used by the resistor in 5 minutes?

Solution

$R = 30\ \Omega \qquad t = 5\text{ min.} = 5 \times 60\text{ s} = 300\text{ s}$

V = 60 V W =?

We apply Ohm's law and express the current in terms of V and R.

W = VIt but I = V/R

$$W = \frac{V^2}{R} \bullet t = \frac{(60\ V)^2}{30\ \Omega} \times 300\ s$$

W = 36000 J = 36 KJ

Electric power

Electric power is the rate at which energy is delivered to an external circuit by the battery.

$$\text{Power} = \frac{\text{Energy}}{\text{time}}$$

$$P = \frac{W}{t} = \frac{VIt}{t}$$

$$P = VI$$

This general relation gives us the power transformed by any device, where I is the current passing through it and V is the potential difference across it. It also gives the power delivered by a source such as a battery. The SI unit of electric power is the same as for any kind of power, the **watt, W.**

1 W = J/s

Exercises

A battery that produces a potential difference V is connected to a 5 W light bulb. Later, the 5 W light bulb is replaced with a 10 W light bulb. (a) In which case does the battery supply the greatest current? (b) Which light bulb has the greatest resistance?

Explanation

(a) To compare the currents we need to consider only the relation P = VI. Solving for the current gives I = P / V. Since the voltage is the same, it follows that the greater the power, the greater the current. In this case, the current in the 10 W bulb is twice the current in the 5 W bulb.

(b) We now consider the relation $P = V^2/R$, which gives resistance in terms of voltage and power. Again, with V the same, it follows that the smaller the power the greater the resistance. Thus, the resistance of the 5 W bulb is twice that of the 10 W bulb.

Example

An electric lamp is labeled 12-volt, 60-watt. Given this information, determine (a) the current passing through the lamp, and (b) the energy the lamp will dissipate each second.

Solution

We are given, $P = 60$ W and $V = 12$ V

(a) To determine the current, use $P = VI$ and solve for I,

$$I = \frac{P}{V} = \frac{60\ \text{VA}}{12\ \text{V}} = 5\ \text{A}$$

(b) The energy W the lamp will dissipate in 1 s is:

$W = Pt = 60\ \text{W} \times 1\ \text{s} = 60$ Ws

or $W = 60$ J

Example

A light bulb is rated at 120 V /60 W. If the bulb is connected to a 120 V dc source, find (a) the current in the bulb, and (b) its resistance.

Solution

The light bulb is rated at 120 V/60 W, which means its operating voltage is 120 V and it has a power of 60 W. That is,

$P = 60$ W (a) $I =?$

$V = 120$ V (b) $R =?$

To find the current we use $P = VI$, from which

$$I = \frac{P}{V} = \frac{60\ \text{A}}{120\ \text{V}} = 0.5\ \text{A}$$

Ohm's law gives

$$R = \frac{V}{I} = \frac{120\ \text{V}}{0.5\ \text{A}} = 240\ \Omega$$

Example

A 3 V battery can transfer a total charge of 8000 C before it is exhausted. (a) What is the total energy the battery can deliver? (b) What is the maximum current the battery can maintain for 1 hour?

Solution

$V = 3$ V, and $Q = 8000$ C

(a) Charges are transferred from one region to another when there is a potential difference between these regions. In this process work is done and this is calculated as

$W = V. Q$

$W = 3\ V \times 8000\ C = 24000\ J$

$W = 24\ kJ$ (Energy delivered by the battery)

(b) Use $I = Q/t$ to find the current.

$$I = \frac{8000\ C}{1\ h} = \frac{8000\ C}{3600\ s} = 2.2\ A$$

Example

The heating element of a coffee maker operates at 120 V and carries a current of 2 A. Assuming that all of the heat generated is absorbed by the water, how long does it take to heat 0.5 kg of water from 20°C to boiling point (100°C)?

Solution

$V = 120\ V$ $m = 0.5\ kg$ $c = 4200\ J/kg.C^{\circ}$

$I = 2\ A$ $\Delta T = 100^{o}C - 20^{o} = 80\ C^{o}$ $t = ?$

Electrical energy I s converted to heat energy to boil water.

$W = Q$

$VIt = mc\Delta T$

$$t = \frac{mc\Delta T}{VI}$$

$$t = \frac{0.5\ kg \times 4200\ J/kg.C^{o} \times 80\ C^{o}}{120\ V \times 2\ A}$$

$t = 700\ s$ or $t = 11.7\ min$

The Kilowatt-hour (kWh): When consumers pay their home electric bills, they actually pay for electric energy, not power. The electric energy used by any device is its rate of energy consumption (in watts) times the time (in seconds) – which is watt- second or joule. The joule is too small for commercial sales use. For this reason, electric companies measure their energy sales in a unit called a **kilowatt-hour**, kWh.

$$1\ kWh = 1000\ J/s \times 3600\ s = 3.6 \times 10^{6}\ J$$

Example

A television set draws 2 A when operated on 120 V. (a) How much power does the set use? (b) If the set is operated for an average of 3 hours a day, what energy in kWh does it consume per month? (c) At $0.075 cents per kWh, what is the cost of operating the set per month?

Solution

I = 2 A (a) W =? (for t = 3 hours/day)

V = 120 V (b) cost =?

(a) We use P = VI to determine the power.

P = 120 V × 2 A = 240 W = 0.24 kW

(b) The electrical energy consumed in one month is: W= Pt.

But the time t = 3 hours / day for 30 days = 3 h/d × 30d = 90 h

W = Pt = 0.24 kW × 90 h = 21.6 kWh

(c) Cost of electrical energy = (energy consumed in kWh) × (price/kWh).

Cost = (21.6 kWh) × ($0.075 cents/kWh)

Cost = $1.62

Example

A 2.2 kW immersion heater is designed for use on 220 V mains. Determine (a) the current drawn from the mains, (b) the resistance of the heater, (c) the energy used in 10 min, and (d) the cost of using the heater for 5 hours at a cost of $0.075 per kWh.

Solution

P = 2.2 kW = 2200 W

V = 220 V

(a) From the equation P = VI, we solve for I.

$$I = \frac{P}{V} = \frac{2200\ W}{220\ V} = 10\ A$$

(b) Use ohm's law to determine R.

$$R = \frac{V}{I} = \frac{220\ V}{10\ A} = 22\ \Omega$$

(a) W = Pt (where t = 10 min = 10 × 60 = 600 seconds)

W = 2 200 W × 600 s = 1 320 000 J

(b) The energy consumed in 5 hours is

$W = Pt = 2.2\ kW \times 5\ h = 11\ kWh$

Cost = energy used in kwh × price/kWh

Cost = 11 kWh × \$0.075 /kWh

Cost = \$0.825

Example

What is the total cost of using four 100 W lamps, four 150 W lamps and a 3 kW immersion heater for a total of 10 hours at \$0.075 / kWh?

Solution

The total power of the electrical appliances is:

$P = (4 \times 100\ W) + (4 \times 150\ W) + 3\ kW$

$P = 0.4\ kW + 0.6\ kW + 3\ kW = 4\ kW$

$t = 10$ hours

We use $W = Pt$ to find the total energy consumed in kWh

$W = Pt = 4\ kW \times 10\ h = 40\ kWh$

Cost = 40 kWh × \$0.075/kWh

Cost = \$3.00

Research: Look for a label on the back or bottom of some electrical appliances in your vicinity. Record the power rating, which is given in units of watts (W). Use the billing statement to find the cost of energy per kilowatt-hour. Calculate the cost of running each appliance for 1 h. Estimate how many hours a day each appliance is used. Then calculate the monthly cost of using each appliance based on your daily estimate. Discuss how to minimize the monthly cost.

Exercises

1. What quantity is measured in kilowatt-hours? What quantity is measured in kilowatts?
2. How many joules are in a kilowatt-hour?
3. A 60 W light bulb and a 75 W light bulb operate from 120 V. Which bulb has a greater current in it?

4. Two conductors of the same length and radius are connected across the same potential difference. One conductor has twice as much resistance as the other. Which conductor dissipates more power?
5. The operating potential difference of a light bulb is 120 V. The power rating of the bulb is 75 W. Find the current in the bulb and the bulb's resistance.

The presence of electric current in a circuit can always be detected from its effects. The common effects of electric current are lighting effect, heating effect, magnetic effect, chemical effect, and physiological effect. In this section we will consider the two effects, namely, the heating effect and the chemical effect.

Key Terms

- heating effect
- electrolysis

Heating effects

When there is a continuous flow of charge in a conductor, heat is developed. This is one of the effects of an electric current called the **heating effect**. The electrons passing from atom to atom along the conductor cause agitation in these atoms, resulting in an increase in temperature. Electric heating devices, such as irons and heaters (Figure 31), contain heating elements of nickel-silver alloy or nichrome. These alloys have a high resistance and a high melting point. When connected across an emf source, the heating elements (resistors) become red hot.

Figure 31.

When resistors get hot, they dissipate power

The power dissipated by a resistor is the result of collisions between electrons moving through the circuit and atoms making up the resistor. Specifically, the potential difference produced by the battery causes these electrons to accelerate until they bounce off an atom of the resistor. These collisions transfer kinetic energy from the electrons to the atoms, causing the atoms to jiggle more rapidly. The increased kinetic energy of the atoms is reflected as an increased temperature of the resistor. After each collision the potential difference accelerates the electrons again, and the process repeats. The result is a continuous transfer of energy from the electrons to the atoms. This is why the filament in an incandescent lightbulb gets so hot.

The energy given off as heat and light by a lightbulb corresponds to an equivalent decrease in energy of the battery connected to the light. However, as is always the case, energy is conserved.

Exercises

When the device in that circuit is a resistor, the electrical power is dissipated in the form of heat. Show that the power dissipated in the resistor is given by

$$P = I^2R$$

Explanation The expression $P = IV$ applies to any electrical system. In the special case of a resistor, the electrical power is dissipated in the form of heat. Applying Ohm's law, $V = IR$, we can rewrite the power dissipated in the resistor as

$$P = VI = (IR)I = I^2R$$

The power lost as heat in a resistor of resistance R is referred to as I^2R loss.

Example

The nichrome wire in an electric heater has a total resistance of 8 Ω. If it operates on a 120 V source, find (a) the current carried by the wire and (b) the power dissipated in the heater.

Solution

$R = 8\ \Omega$ (a) $I = ?$

$V = 120\ \text{V}$ (b) $P = ?$

(a) Since $V = IR$, we have

$$I = \frac{V}{R} = \frac{120\ \text{V}}{8\ \Omega} = 15\ \text{A}$$

(b) The power P dissipated in the heater is:

$$P = I^2R = (15\ \text{A})^2 \times 8\ \Omega = 1800\ \text{W}$$

$$P = 1.8\ \text{kW}$$

Example

An electric heater has a resistance of 10 Ω. It operates on a 120 V power source. Determine (a) the current through the resistance, and (b) the heat energy supplied by the heater in 10 seconds.

Solution

$R = 10\ \Omega$ (a) $I = ?$

$V = 120\ V$ (b) $W = ?$

(a) Use Ohm's law to determine the current

$$I = \frac{V}{R} = \frac{120\ V}{10\ \Omega} = 12\ A$$

(b) Energy is changed from electric to heat in the heating element of the heater.

$W = Pt$ but $P = I^2R$

$W = I^2Rt$

$W = (12\ A)^2\ (10\ \Omega)\ (10\ s) = 14400\ J$

$W = 14.4\ kJ$

Note that energy values of heaters are large.

Electrolysis

When an electric current flows through a water solution of an acid, base, or salt, the solution is decomposed – broken down into simpler substances. This process is known as **electrolysis**. Negative ions in the solution go to the anode (+), where they become neutral atoms by losing electrons; positive ions go to the cathode (–), where they become neutral atoms by gaining electrons. The following examples are some of the uses of electrolysis.

- **Electrolysis of water -** The electrolysis of water is a common example of the effect of an electric current to decompose a compound. A direct current at a low voltage is sent through water containing a small amount of sulfuric acid. The water is decomposed into hydrogen and oxygen. Oxygen collects at the anode (+) and hydrogen collects at the cathode (–).

  Explanation of the electrolysis of water: When the sulfuric acid is dissolved in water, it dissociates into positively charged hydrogen ions and negatively charged sulfate ions. The hydrogen ions are attracted to the cathode, where they become neutral atoms by gaining electrons and the sulfate ions are attracted to the anode.

  At the anode, however, water molecules give up two electrons more readily than sulfate ions. When two water molecules have each lost two electrons, four hydrogen ions and one molecule of oxygen are formed. The oxygen molecules at the anode bubble off as a gas, leaving the sulfate and hydrogen ions in the electrolyte. Thus, the sulfuric acid content of the solution is unaffected when the water is dissolved.

- **Electroplating -** Nickel, chromium, cadmium, copper, and silver are often electroplated articles made of other metals, such as iron, in order to give the metal articles attractive appearance or to keep them from corroding. The material to be electroplated is made the cathode (+), the metal to be applied is made the anode (+), and the electrolyte is a solution of a salt of this metal. For example, if iron is to be plated with copper, iron is used as the cathode, pure copper is used as the anode, and the electrolyte is a solution of copper sulfate.

Electric circuits that contain several resistors can often be analyzed by combining individual groups of resistors in series and parallel. However, there are many circuits in which no two resistors are in series or in parallel. To deal with such circuits it is necessary to employ methods other than the series–parallel method. One alternative is to take advantage of **Kirchhoff's rules**, named after their developer Gustav Kirchhoff a German physicist. Kirchhoff's rules are simply ways of expressing charge conservation, and energy conservation in a closed circuit. In this section, we will study these laws and their applications in solving any circuit problems.

KEY TERMS

- loop rule
- junction rule

Kirchhoff's rules

There are many simple circuits, such as that shown in Figure 31 that cannot be analyzed by merely replacing combinations of resistors by an equivalent resistance. The two resistors R_2 and R_3 in this circuit look as if they might be in parallel, but they are not. The potential drop is not the same across both resistors because of the presence of the emf source ε_2 in series with R_3 and the emf source ε_3 in series with R_2. Nor are R_2 and R_3 in series, because they don't carry the same current.

Kirchhoff's two rules, the **junction rule** and **the loop rule**, apply to this and any other circuit.

- The junction rule is an application of the law **of conservation of electric charge** to the electric current in a circuit. The loop rule is an application of the **principle of conservation of energy** to the electric potential that exists at various places in a circuit.

Kirchhoff's first rule or junction rule states that:- at any junction point in a circuit where the current can divide, the sum of the currents into the junction must equal the sum of the currents out of the junction.

For example, at the junction point **a** in Figure 32, I_1 and I_2 are entering whereas I_3 is leaving. Thus, Kirchhoff's junction rule states that:

$$I_1 + I_2 = I_3.$$

In general, we associate a '+' sign with currents entering a junction and a '– ' sign with currents leaving a junction.

Note: A junction is any point in a circuit where three or more wires meet.

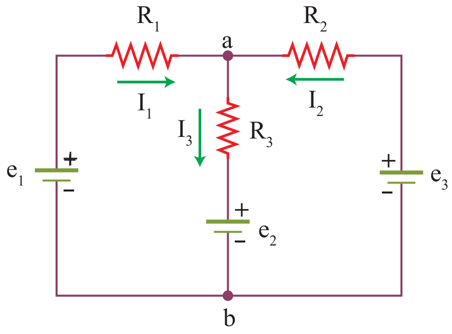


Figure 32.

Kirchhoff's second rule or loop rule states that:- the sum of the changes in potential around any closed loop of a circuit must be zero.

$$\Sigma V = 0$$

Another way of stating this rule is:- "around any closed-circuit loop, the sum of the potential drops equals the sum of the potential rises".

When applying Kirchhoff's rules, you must follow the following steps:

(i) Choose any closed loop in the network, and designate a direction (clockwise or counterclockwise) to go around the loop when applying the loop rule.

(ii) Assign symbols and directions to the currents in all branches of the circuit. Go around the loop in the assigned direction, adding potential differences as you cross them.

- An emf is counted as positive when you traverse it from – to + (Figure 33 (a)) and negative when you traverse it from + to – (Figure 33 (b)).
- An **IR** product is negative if your path passes through the resistor in the same direction as the assumed current (Figure 33 (c)) and positive if it passes through in the opposite direction (Figure 33 (d)).

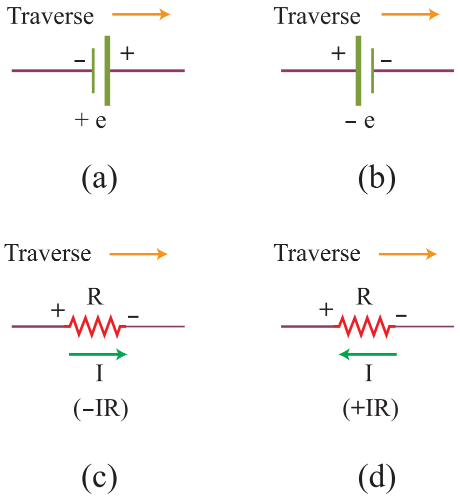


Figure 33.

Exercises

In the circuit shown in Figure 34, what is the value of the current I?

Explanation Four currents are shown out of which two are entering into the junction P and two leaving the junction.

Applying the junction rule, we get

$\Sigma I_p = 0$

$7\ A + 5\ A - 3\ A - I = 0$

$7\ A + 5\ A - 3\ A = I$

$I = 9\ A$

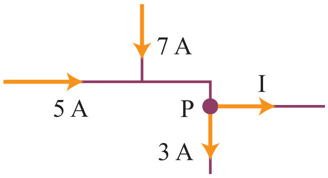


Figure 34.

Example

Figure 35 shows a circuit that contains two batteries and two resistors. Determine the current I in the circuit.

Solution

First, we draw the current I in the direction shown which we have chosen to be clockwise around the circuit. The choice of direction is arbitrary, and if it is incorrect, I will turn out to be negative. Next, mark the resistors with plus and minus signs, which serve as an aid in identifying the potential drops and rises for Kirchhoff's loop rule.

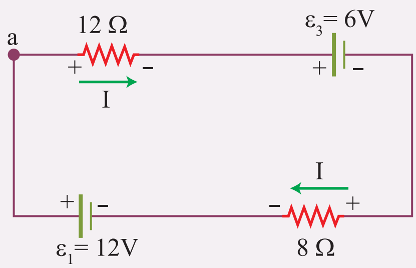


Figure 35.

Finally, apply Kirchhoff's loop rule to the circuit, starting at corner a, traveling clockwise around the loop, and identifying the potential drops and rises as we go.

$-I(12\ \Omega) - \varepsilon_2 - I(8\ \Omega) + \varepsilon_1 = 0$

$-12I - 6\ V - 8I + 12\ V = 0$

$-20I = -6\ V$

$I = 0.3\ A$

Example

For the circuit shown in Figure 36, find the current I.

Solution

Use Kirchhoff's loop rule, traversing counterclockwise

$-36\ V + (5\ \Omega)I + (4\ \Omega)I + 12\ V + (3\ \Omega)I = 0$

$(12\ \Omega)I - 24\ V = 0$

$(12\ \Omega)I = 24\ V$

$I = 24\ V/12\ \Omega$

$I = 2\ A$

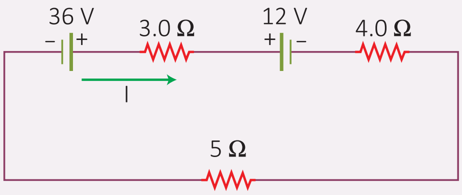


Figure 36.

Example

In the circuit diagram shown in Figure 37, if the emf of the two batteries and their internal resistances are: $\varepsilon_1 = 12$ V, $\varepsilon_2 = 18$ V, $r_1 = 1\ \Omega$, and $r_2 = 2\ \Omega$, find the terminal voltage of each battery. The external resistor has a value, $R = 5\ \Omega$.

Solution

Apply Kirchhoff's loop rule to the circuit, starting from the upper left corner in the clockwise direction, to calculate the current I.

$-Ir_2 + \varepsilon_2 - IR - \varepsilon_1 - Ir_1 = 0$

$-I(2) + 18\ V - I(5) - 12\ V - I(1) = 0$

$-8I = -6\ V$

$I = 0.75\ A$

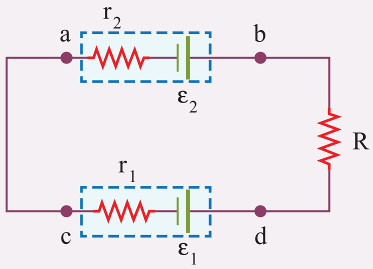


Figure 37.

The terminal voltage of the first battery is V_{cd}:

$V_{cd} = \varepsilon_1 + Ir_1 = 12\ V + (0.75\ A)(1\ \Omega) = 12.75\ V$

The terminal voltage of the second battery is V_{ab}:

$V_{ab} = \varepsilon_2 - Ir_2 = 18\ V - (0.75\ A)(2\ \Omega) = 16.5\ V$

Note that for the 12 V battery, there is voltage gain going across the internal resistance from left to right.

Example

The ammeter shown in Figure 38 reads 2 A. Find (a) I_1, (b) I_2, and (c) ε.

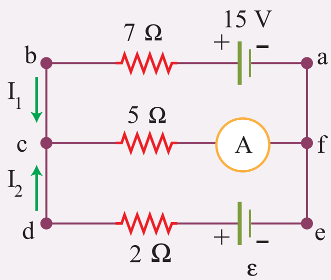


Figure 38.

Solution

(a) Applying Kirchhoff's loop rule to upper loop, going counterclockwise, gives

Loop abcfa: $+15\ V - (7\ \Omega)I_1 - (5\ \Omega)(2\ A) = 0$

$$I_1 = \frac{15V - 10V}{7\ \Omega} = 0.7\ A$$

(b) From Kirchhoff's junction rule, we get

$I_1 + I_2 - 2\text{ A} = 0$

$0.7\text{ A} + I_2 - 2\text{ A} = 0$

$I_2 = 1.3\text{ A}$

(c) Going around loop edcfe, we get

$+\varepsilon - (2\ \Omega)I_2 - (5\ \Omega)(2\text{ A}) = 0$

$\varepsilon = (2\ \Omega)(1.3\text{ A}) + (5\ \Omega)(2\text{ A}) = 12.6\text{ V}$

Example

Find the current in each part of the circuit shown in Figure 39.

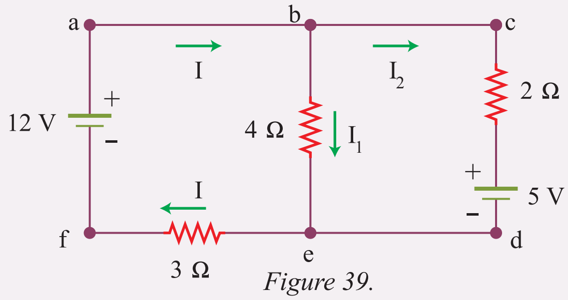


Figure 39.

Solution

(a) Apply the junction rule to point b:

$I = I_1 + I_2$

(b) Apply the loop rule to loop: abcdefa

$-(2\ \Omega)I_2 - 5\text{ V} - (3\ \Omega)I + 12\text{ V} = 0$..............................(ii)

(c) Again, apply the loop rule to the loop: abefa:

$-(4\ \Omega)I_1 - (3\ \Omega)I + 12\text{ V} = 0$

$-(4\ \Omega)I_1 - (3\ \Omega)(I_1 + I_2) + 12\text{ V} = 0$

$-4I_1 - 3I_1 - 3I_2 = -12\text{ V}$

$-7I_1 - 3I_2 = -12\text{ V}$...(iii)

The results for steps (ii) and (iii) can be combined to solve for I_1 and I_2:

$I_1 = 1.5\text{ A},\ I_2 = 0.5\text{ A}$

The value of the current I is: $I = 1.5\text{ A} + 0.5\text{ A} = 2.0\text{ A}$

Summary

- Electric current is the flow of electric charge: $I = Q/t$
- The unit of current is the Ampere, A. By definition, 1 A is 1 coulomb per second: 1 A = 1 C/s.
- A device that transforms another type of energy into electrical energy is called a source of emf.

- A battery is a source of direct current DC. Batteries produce a difference in potential between their terminals. This potential difference causes electrons in a wire to flow through a circuit from one terminal to the other. As the electrons flow, they convert electrical energy from the battery to other forms, such as light and heat energy.
- Primary and secondary cells are commonly used DC sources.
- When a battery is connected to a circuit, electrons move in a closed path from one terminal of the battery through the circuit and back to the other terminal of the battery.
- A battery uses chemical reactions to produce a difference in electric potential between its two terminals.
- The potential difference V necessary to produce a current I in a wire of resistance R is given by Ohm's law:

  $V = IR$
- The resistance of a wire depends on the material from which it is made. A wire's resistance also depends on its length, L, and its cross-sectional area, A.

  $$R = \rho\frac{L}{A}$$
- As a wire is heated, its resistivity tends to increase.
- Resistors connected end to end are said to be in series. Resistors connected across the same potential difference are said to be connected in parallel.

Series

- The equivalent resistance, R_{eq}, of resistors connected in series is equal to the sum of the individual resistances:

  $R_{eq} = R_1 + R_2 + R_3 + \ldots = \Sigma R$

  In general, the more resistors connected in series, the greater the equivalent resistance.

Parallel

- The equivalent resistance, R_{eq}, of resistors connected in parallel is given by the following:

  $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots = \sum\frac{1}{R}$$

  The more resistors connected in parallel, the smaller the equivalent resistance.
- The terminal voltage V of a battery is given by

  $V = \varepsilon - Ir$

where ε is the emf of the battery, I is the current, and r is the internal resistance of the battery. Generally, the internal resistance is small enough to be neglected.

- The electric power used by a device is equal to the current times the voltage.

  $P = IV$

Power dissipated in a resistor

- If a potential difference V produces a current I in a resistor R, the electrical power converted to heat is

  $P = I^2R = V^2/R$

- The electric company bills your family for the number of kilowatt-hours (kWh) of energy it uses.

Energy Usage and the Kilowatt-Hour

- The energy equivalent of 1 kilowatt-hour (kWh) is

  $1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$

- The presence of electric current in a wire (circuit) is detected by its effects.

  Heating effect is observed when the electric energy is converted into heat energy. Electric iron and electric toaster are common examples that convert the electric energy into heat energy as these devices are connected to an emf source.

- A current through acid solutions produce ions by the chemical reactions between the electrolyte and the electrodes.
- Complex circuits can be analyzed using **Kirchhoff's rules:**
  1. The sum of the currents entering any junction must equal the sum of the currents leaving that junction.
  2. The sum of the potential differences across all the elements around any closed circuit loop must be zero.
- The first rule, called the junction rule, is a statement of conservation of charge. The second rule, called the loop rule, is a statement of conservation of energy. Solving problems using these rules involves to generate as many equations as there are unknown currents. The equations can then be solved simultaneously.

Review Exercises

I Conceptual Questions

1. (a) List the appliances around you that depend on electricity for their operation.

   (b) List the appliances that do not use electrical energy.

2. Name three sources of an electric current.

3. Write some of the effects of an electric current.
4. Is the direction of an electric current from the positive terminal of a battery to the negative terminal or from the negative terminal to the positive terminal?
5. A wire carries a current of 3.0 A. How much charge passes a given point in the circuit in 30 seconds?

   (a) 0.10 C (b) 10 C (c) 33 C (d) 90 C
6. How does the resistance of a copper wire change when both the length and diameter of the wire are doubled?
7. A certain wire is to be doubled in length, but it is desired that its resistance remain the same. By what factor must the radius be changed?
8. When the voltage across a certain conductor is doubled, the current is observed to triple. What can you conclude about the conductor?
9. A motor with an operating resistance of 40 Ω is connected to a voltage source. The current in the motor is 2.75 A. What is the voltage of the source?

   (a) 0.069 V (b) 220 V (c) 110 V (d) 14.5 V
10. For a cylindrical resistor made of ohmic material, the resistance does NOT depend on

    (a) the current (c) the cross-sectional area

    (b) the length (d) the resistivity
11. A nichrome wire is 1 m long and 1×10^{-6} m^2 in cross-sectional area. When connected to a potential difference of 2 V, a current of 4 A exists in the wire. The resistivity of this nichrome wire is

    (a) 10−7 Ω.m (c) 4×10^{-7} Ω.m

    (b) 2×10^{-7} Ω.m (d) 5×10^{-7} Ω.m
12. Two copper wires have equal lengths, but the diameter of one is three times that of the other. What is the ratio of the resistance of the thicker wire to that of the thinner wire?

    (a) 1:3 (b) 3:1 (c) 1:9 (d) 9:1
13. Two identical resistors are connected in series across the terminals of a battery with a potential difference V and a current I through the circuit. If one of the resistors is removed from the circuit and the other one connected across the battery, what would be the current through the circuit?

    (a) 4I (b) 2I (c) I (d) I/2
14. A 10 Ω resistor and a 20 Ω resistor are connected in series to a voltage source. When the current through the 10 Ω resistor is 2.0 A, what is the current through the 20 Ω resistor?

    (a) 1.0 A (b) 2.0 A (c) 0.50 A (d) 4.0 A

15. parallel combination consisting of resistors A and B is connected across the terminals of a battery. The resistor A has twice the resistance of resistor B. If the current carried by resistor A is I, then what is the current carried by resistor B?

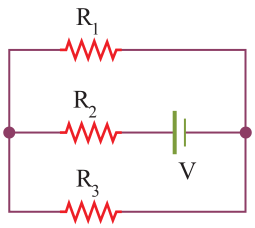
    (a) 2I (b) I (c) I/2 (d) I/4

16. A series combination consisting of resistors A and B is connected across the terminals of a battery. The resistor A has twice the resistance of resistor B. If the current carried by resistor A is I, then what is the current carried by resistor B?

    (a) 2I (b) I (c) I/2 (d) I/4

17. Which resistors in the figure below are connected in parallel?

    (a) R_1 and R_3
    (b) R_1 and R_2
    (c) R_2 and R_3
    (d) All three.



18. Which one of the following equations is **not** a valid expression for electric power P?

    (a) $P = \frac{I^2R}{V}$ (c) $P = \frac{V^2}{R}$
    (b) $P = I^2R$ (d) $P = IV$

19. Why do we connect electric devices in a home in parallel rather than in series?

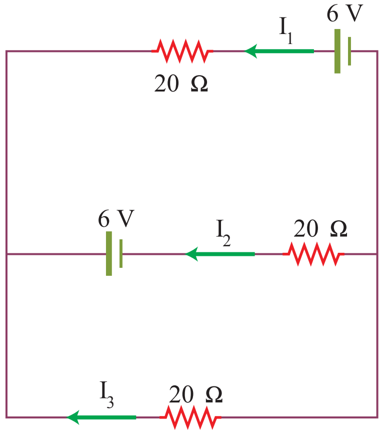
20. If electrical power is transmitted over long distances, the resistance of the wires becomes significant. Why?

21. Light A has four times the power rating of light B when operated at the same voltage. What is the ratio of the resistance of light A to the resistance of light B?

22. Explain how Kirchhoff's rules for circuit analysis are related to conservation principles.

23. Which one of the following equations is Kirchhoff's first rule as applied to the circuit shown in the figure?

    (a) $I_1 + I_2 + I_3 = 0$
    (b) $I_1 - I_2 + I_3 = 0$
    (c) $I_1 + I_2 - I_3 = 0$
    (d) $I_1 - I_2 - I_3 = 0$



24. Which one of the following is not the correct application of Kirchhoff's loop rule to above circuit?

(a) $6 - 20I_1 - 6 + 20I_2 = 0$
(b) $-6 - 20I_3 + 20I_1 = 0$
(c) $-20I_2 + 6 + 20I_3 = 0$
(d) $-6 - 20I_3 - 20I_1 = 0$

II Problems

25. The charge that passes through the filament of a certain light bulb in 5.0 s is 3.0 C. What is the current in the light bulb?

26. A copper wire and an aluminum wire with the same length and diameter carry the same current I. What is the ratio of the potential drops across these wires?

27. A certain resistor of resistivity ρ has resistance R. Suppose it is desirable to design a new resistor that has one-third the length and one-fourth the cross-sectional area of the existing resistor, but the same overall resistance. What should the resistivity ρ_n of the new resistor be, in terms of the resistivity ρ of the original device?

28. Aluminum and copper wires of equal length are found to have the same resistance. What is the ratio of their radii?

29. A potential difference of 12 V is found to produce a current of 0.40 A in a 3.2 m length of wire with a uniform radius of 0.40 cm. What is the resistivity of the wire?

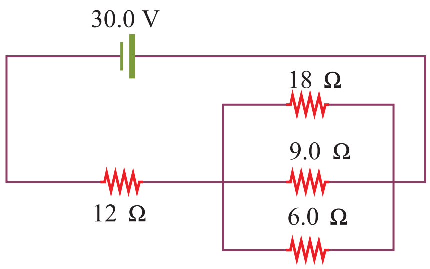
30. What is the equivalent resistance of two 2 Ω resistors in parallel connected to a third 2 Ω resistor in series?

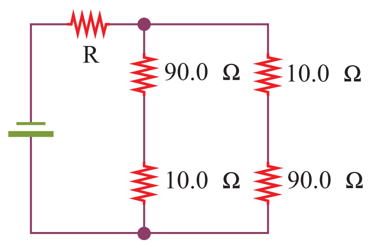
31. Two resistors, R_1 and R_2, are connected in parallel to a battery with a potential difference V. The voltage across R_1 is 12 V and the current through R_2 is 0.2 A. If the equivalent resistance is 20Ω, find (a) the resistances R_1 and R_2, (b) the current through R_1.

32. Two identical resistors R are connected in parallel and their combination is connected in series to a 40 Ω resistor. If the equivalent resistance of the three resistors is 55 Ω, what is the value of R?

33. Three resistors, 30 Ω, 60 Ω, and R, are connected in parallel with a 12 V battery. The total current through the battery is 1.2 A. (a) Find the value of resistance R. (b) Find the current through each resistor.

34. For the circuit diagram shown below (left), determine the current in each resistor and the potential difference across each resistor.



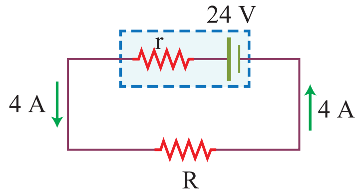


35. The equivalent resistance of the circuit shown above (right) is 60 Ω. Using the diagram, determine the value of R.

36. A resistor R is connected first in parallel and then in series with a 2 Ω resistor. A battery delivers five times as much current to the parallel combination as it does to the series combination. Determine the two possible values for R.

37. A battery with an emf of 6 V and an internal resistance of 0.3 Ω is connected to a variable resistor R. Find the current delivered by the battery when R is (a) 0, (b) 5 Ω, and (c) 10 Ω?

38. In the circuit diagram shown in the figure shown, the terminal voltage of the 24 V battery is 20.2V, and the current in the circuit is 4 A. What is (a) the internal resistance r of the battery; (b) the resistance R of the resistor?

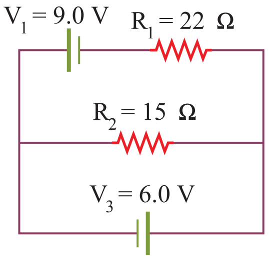


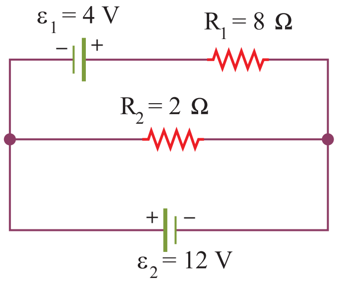
39. A small electronic device is rated at 0.25 W when connected to 120 V. What is the resistance of this device?

40. An electric heater is operated by applying a potential difference of 50 V across a wire of total resistance 8 Ω. Find the current in the wire and the power rating of the heater.

41. A welding machine draws 15 A of current at 220 V. If it operates for a period of 4 hours a day, (a) how much electrical energy does it consume in one month? (b) What is the cost of using the machine for a month if the cost of electricity is $0.08 per kWh?

42. What are the magnitudes and directions of the currents through R_1 and R_2 in the figure shown below (left).





43. Determine the current (magnitude and direction) in the 8 Ω and 2 Ω resistors shown in the figure above (right).

44. If a person has $5, how long could he or she play a200 W stereo if electricity costs $0.075 per kWh?

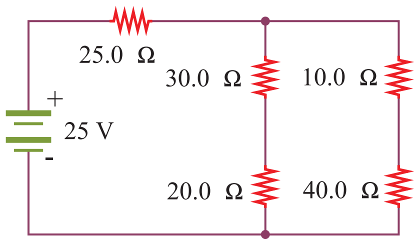
45. How much electric energy is delivered to a 60 W light bulb if the bulb is left on for 2.5 hours?

(a) 1.5×10^{-1} J (c) 2.5×10^{3} J

(b) 5.4×10^{5} J (d) 3.7×10^{4} J

46. What is a short circuit? Why is a short circuit dangerous?

47. For the circuit shown below, determine (a) the total resistance, (b) the current through the 25 Ω resistor.



Sample Test

1. The charge that passes through the filament of a certain light bulb in 5.00 s is 4.0 C. What is the current in the light bulb?

   (a) 20.0 A (c) 0.80 A

   (b) 1.25 A (d) 0.50 A

2. A 60 watt light bulb carries a current of 0.5 A. The total charge passing through it in one hour is:

   (a) 120 C (c) 2400 C

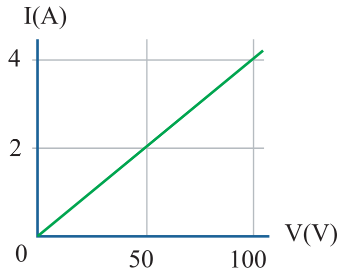
   (b) 1800 (d) 3600 C

3. Which of the following is the correct term for a circuit that does not have a complete path for charge flow?

   (a) Closed circuit (c) Dead circuit

   (b) Open circuit (d) Short circuit

4. The figure below is a graph of current versus potential difference for a material that obeys Ohm's law. What is the resistance of the material?



   (a) 400 Ω

   (b) 100 Ω

   (c) 50 Ω

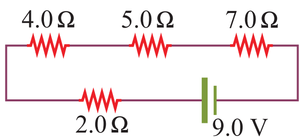
   (d) 25 Ω

5. How much current would a 10 Ω electric iron draw when connected to a 120 V outlet?

   (a) 12 A (c) 6 A

   (b) 10 A (d) 4 A

6. A 9.0 V battery is connected to four resistors as shown in the figure. What is the potential difference across the 5.0 Ω resistor?



   (a) 9.0 V (a) 2.5 V

   (a) 3.5 V (a) 2.0 V

7. Several light bulbs are connected in series across a 120 V source of emf. The current in the circuit is 2.0 A. If each light bulb has a resistance of 1.50 Ω, how many light bulbs are in the circuit?

   (a) 25 (c) 36

   (b) 30 (d) 40

8. A length of wire is cut into five equal pieces. The five pieces are then connected in parallel, with the resulting resistance being 2.00 Ω. What was the resistance of the original length of wire before it was cut up?

   (a) 10 Ω (c) 50 Ω

   (b) 30 Ω (d) 60 Ω

9. A student is provided with three 2 Ω resistors. How many different equivalent resistances can be produced if all three of the resistors are used in a circuit?

   (a) 3 (c) 5

   (b) 4 (d) 6

10. The emf of a battery is equal to its terminal potential difference

    (a) under all conditions

    (b) only when the battery is being charged

    (c) only when a large current is in the battery

    (d) only when there is no current in the battery

11. A battery with an emf of 24 V is connected to a 6 Ω resistor. As a result, current of 3 A exists in the resistor. The terminal potential difference of the battery is

    (a) 6 V (c) 18 V

    (b) 12 V (d) 24 V

12. A circuit consists of a battery and two 22 Ω resistors in series. The current through the circuit is 0.55 A. A third resistor is then added to the circuit in parallel with the first two resistors. The current through the new branch of the circuit is 0.35 A. What is the resistance of the third resistor?

    (a) 15 Ω (c) 28 Ω

    (b) 24 Ω (d) 69 Ω

13. A nichrome wire is 1 m long and 1×10^{-6} m^2 in cross-sectional area. When connected to a potential difference of 2 V, a current of 4 A exists in the wire. The resistivity of this nichrome wire is

    (a) 10^{-7} Ω.m (c) 4×10^{-7} Ω.m

    (b) 2×10^{-7} Ω.m

    (d) 5×10^{-7} Ω.m

14. Two copper wires have equal lengths, but the diameter of one is three times that of the other. What is the ratio of the resistance of the thicker wire to that of the thinner wire?

    (a) 1:3 (c) 1:9

    (b) 3:1 (d) 9:1

15. A wire has resistance R. What is the resistance of a wire of the same substance that has the same length but twice the cross-sectional area?

(a) 2R (c) 4R

(b) R/2 (d) R/4

16. Three resistors, 11 Ω, 53 Ω, and R, are connected in series with a 24 V battery. The total current flowing through the battery is 0.16 A. What is the resistance of R?

(a) 150 Ω (c) 66 Ω

(b) 86 Ω (d) 64 Ω

17. A parallel combination consisting of resistors A and B is connected across the terminals of a battery. The resistor A has twice the resistance of resistor B. If the current carried by resistor A is I, then what is the current carried by resistor B?

(a) 2I (c) I/2

(b) I (d) I/4

18. Three identical resistors are connected in series to a 12 V battery. What is the voltage across any one of the resistors?

(a) 36 V (c) 4 V

(b) 12 V (d) 1.2 V

19. The internal resistances of an ideal voltmeter and an ideal ammeter, respectively, are

(a) zero and zero. (c) zero and infinite.

(b) infinite and infinite. (d) infinite and zero.

20. There resistors, R_1, R_2, and R_3, are connected in series in a circuit. If $R_1 < R_2 < R_3$, which resistor dissipates the greatest power?

(a) R_1

(b) R_2

(c) R_3

(d) All are equal in power dissipation

21. When a light bulb is turned on, its resistance increases until it reaches operating temperature. What happens to the current in the bulb as it is warming up?

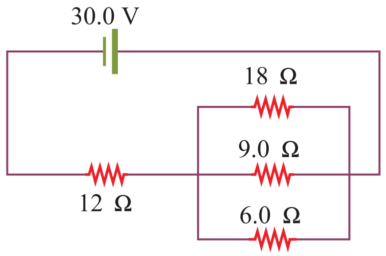
(a) It stays constant.

(b) It increases.

(c) It decreases

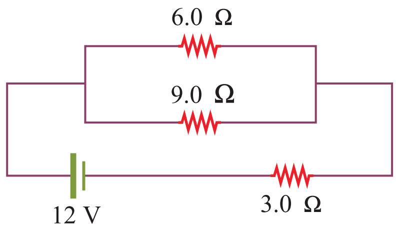
(d) It increases at first and then decreases

22. What is the equivalent resistance of the circuit shown in the figure?



(a) 3 Ω
(b) 12 Ω
(c) 15 Ω
(d) 18 Ω

23. In the circuit shown below, what is the current int the 3 Ω resistor?



(a) 1.8 A
(b) 1.1 A
(c) 0.72 A
(d) 0.45 A

24. A lamp draws 0.50 A from a 120 V generator. How much energy is converted to light and heat in 5.0 min?

(a) 18 kJ
(b) 12 kJ
(c) 9 kJ
(d) 6 kJ

25. A 60 Ω resistor is connected across a 90 V battery. How much energy is used by the resistor in 10 minutes?

(a) 81 kJ
(b) 54 kJ
(c) 5.4 kJ
(d) 1.5 kJ

26. A television set draws 2.0 A when operated on 120 V. If the set is operated for 10 hours, what energy does it consume?

(a) 0.6 kWh
(b) 1.2 kWh
(c) 2.4 kWh
(d) 3.6 kWh

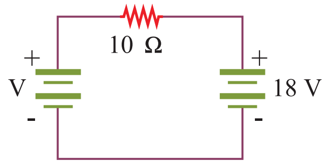
27. A welding machine draws 15 A of current at 220 V. If it operates for a period of 4 hours a day, what is the cost of using th$31.68 e machine for a month if the cost of electricity is $0.08 per kWh?

(a) $31.68
(b) $28.38
(c) $19.68
(d) $16.24

28. Kirchhoff's loop is basically a consequence of the conservation of

(a) charge
(b) energy
(c) momentum
(d) mass

29. In the circuit diagram shown below, what are the magnitude and direction of the current in the 10Ω resistor?



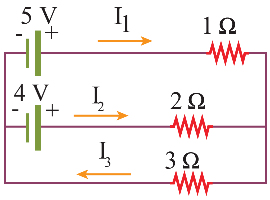
(a) 1.8 A, clockwise

(b) 1.8 A, counterclockwise

(c) 0.9 A, clockwise

(d) 0.9 A, counterclockwise

30. Which of the following equations is not a consequence of Kirchhoff's rules, when applied to the figure shown below?



(a) $9 - I_1 - 2I_2 = 0$

(b) $I_1 + I_2 - I_3 = 0$

(c) $4 - 2I_2 - 3I_3 = 0$

(d) $5 - I_1 - 3I_2 = 0$

P12CH03

CHAPTER 3

MAGNETISM AND ELECTROMAGNETISM

Chapter Contents

Chapter Outcome

Learners will able to:

- recognize that it is magnetic effect that produces electricity.

Chapter Objectives

After completing this chapter, you will be able to:

- describe the origin of magnets;
- describe the behavior of magnets;
- explain why some materials are magnetic;
- relate the behavior of magnets to magnetic fields;
- compare the relationship between electricity and magnetism;
- describe the concept of magnetic flux and how to measure it;
- analyze the relationship between magnetic flux and magnetic flux density;
- explain how moving-coil meters work;
- distinguish between AC and DC motors;
- distinguish motor from generator;
- describe how changing magnetic fields can generate electric potential differences;
- apply this phenomenon to the construction of motors, generators, and transformers;
- elaborate the principle of transformer and its function.

The effects of magnetism have been known since antiquity. For example, a piece of the naturally occurring iron-oxide mineral known as lodestone can behave just like a manufactured magnet. Today, magnets play an increasingly important role in our everyday lives. Electric generators, simple electric motors, television sets, cathode-ray displays, tape recorders, and computer hard drives all depend on the magnetic effects of electric currents. This unit explores the properties of magnets and the magnetic fields they produce. It also reveals the fascinating, and unexpected, connection between magnetic fields and electric currents.

We begin our study of magnetism with a few general observations regarding magnets and the fields they produce. These observations apply over a wide range of scales – from the behavior of small, handheld bar magnets to the global effects associated with the magnetic field of the Earth.

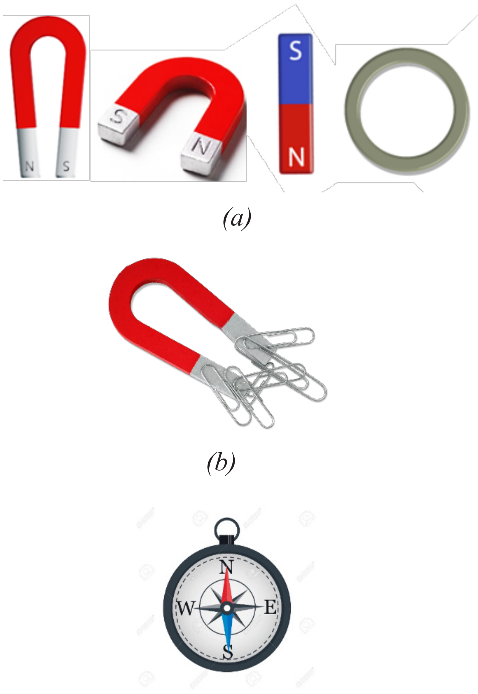
Key Terms

- Magnetic poles
- Magnetic Domains
- Magnetic Fields

Permanent Magnets

A **magnet** is an object which attracts anything containing iron, cobalt, nickel, and some alloys of iron oxide. These materials are called **magnetic materials**. All other materials, which are not attracted by magnets such as paper, plastic, glass, copper and so on, are called **non-magnetic materials**.

Magnets which are found in nature are called **natural magnets**. There are also **artificial magnets** which are produced by different methods. Artificial magnets are manufactured in various shapes for different uses. Horseshoe, U-shaped, bar, and disk magnets are the most common types of artificial magnets (Figure 1 (a)).



(a)

(b)

(c)

Figure 1.

Any magnet, whether it is in the shape of a bar or a horseshoe, has two ends or faces, called **poles**, which is where the magnetic effect is strongest. For example, if a horseshoe magnet is held near some paper clips, they stick to the ends of the magnet Figure 1 (b).

A compass needle is simply a bar magnet which is supported at its center so that it can rotate freely (Figure 1 (c)). When the compass is placed on a horizontal surface the needle rotates until one end points nearly to the north.

Magnetic Poles

If a bar magnet is suspended from its center by a string, it swings until it comes to rest pointing in the north-south direction. The end of the magnet pointing towards the north geographic pole of the Earth we refer to as the "north-seeking pole," or simply the **north pole**. The opposite end pointing towards the south is the "south-seeking pole," or the **south pole**. To identify the poles, the abbreviations N and S are marked on the ends of the magnets.

- Magnets always have two poles

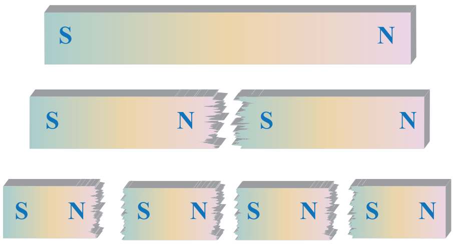


Figure 2.

An interesting aspect of magnets is that they always have two poles. When a bar magnet is cut in half, two new poles appear. Each half has both a north pole and a south pole, just like the original magnet. If the cutting operation is repeated, more magnets are produced, each with a north and a south pole (Figure 2). This behavior is fundamentally different from that of electricity, in that the two types of electric charge (positive and negative) can exist separately.

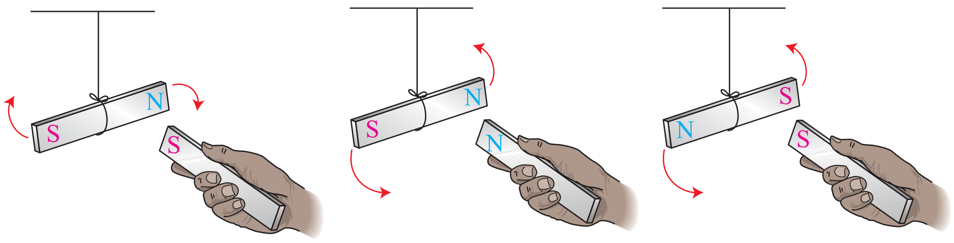
- There is no magnetic monopole.

Magnetic Force

Magnetic poles also exert attractive or repulsive forces on each other similar to the electrical forces between charged objects. In fact, simple experiments with two bar magnets show that

- like poles repel each other and
- unlike poles attract each other.

Thus, the north pole of a magnet is attracted to the south pole of another magnet Figure 3 (a)), and two north poles (or two south poles) brought close together repel each other Figure 3 (b)).



(a) *(b)*

Figure 3.

Activity 1

For this activity, you need two bar magnets, and a piece of string and a stand (support). Use the string to suspend one bar magnet horizontally from the stand (or support). Bring the **north pole** of the other magnet near the **north pole** of the suspended magnet and observe the reaction.

Discuss 1: Do they repel or attract each other? Repeat this with the south pole. What happens?

Perform similar demonstrations with the other types of magnets.

Magnetic Levitation: The repulsive force between the like poles of magnets is used to lift objects. This technique of suspending object in air is called magnetic **levitation**.

Activity 2

For this activity, you need two ring magnets and a stand with wooden or glass rod as shown in Figure 4. Insert the rod through one magnet slowly and keep it at an upright position. Place the other ring magnet on top of the first one so that the poles face each other. Where does the second magnet stay? What happens if the magnet is reversed?

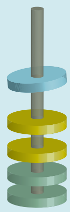


Figure 4.

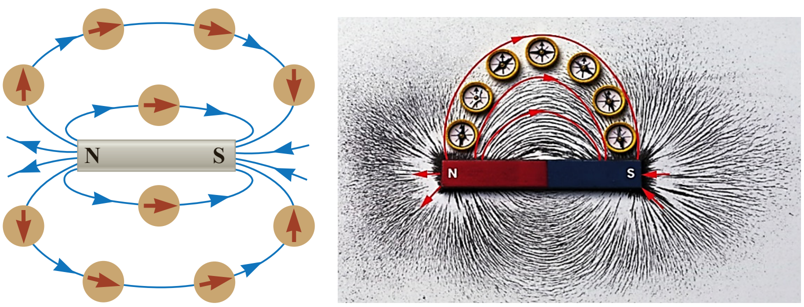
Magnetic fields

In your previous activity with two magnets, you noticed that the forces between magnets, both attraction and repulsion, occur even when the magnets are not touching. In the same way that electric and gravitational forces can be described by electric and gravitational fields, magnetic forces can be described by the existence of fields around magnets.

- A magnet exerts a magnetic force in the space surrounding it. The space around a magnet where the magnetic force is detected is called magnetic field.

The magnetic field of a bar magnet can be explored using a compass, as illustrated in Figure 5 (a). If a small compass is brought near a magnetic field, the compass needle will align with the magnetic field lines. The direction of the magnetic field at any location is defined as the direction that the north pole of a compass needle points at that location.

A magnetic field, which is represented with the symbol $\vec{B}$, can also be visualized using iron filings. In Figure 5 (b), for example, a sheet of paper is placed on top of a bar magnet. When iron filings are sprinkled onto the paper, they align with the magnetic field in their vicinity. The pattern they form gives a good idea of the overall field produced by the magnet.



(a) *(b)*

Figure 5.

Note:

(i) Magnetic field lines appear to begin at the north pole of a magnet and to end at the south pole of a magnet. However, magnetic field lines have no beginning or end. Rather, they always form a closed loop. In a permanent magnet, the field lines actually continue within the magnet.

(ii) The field lines are close together where the field is strong and get farther apart as the field gets weaker.

Activity 3

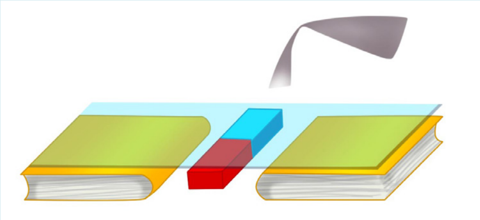
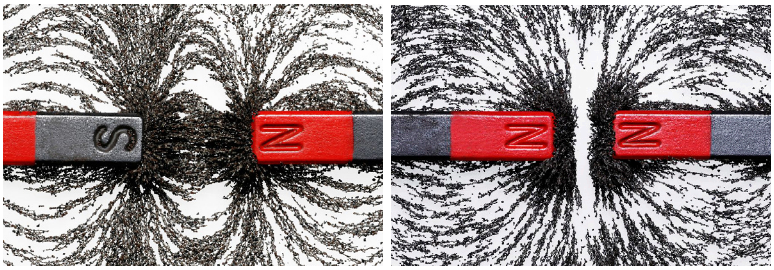


Figure 6.

For this activity you need a bar magnet, a sheet of clear glass, two similar books and and iron filings. Place the bar magnet under the glass. Use the books to support the glass as shown in Figure 6. The iron filings align themselves along the magnetic field lines. Draw this pattern on a sheet of paper.

Discuss 2: Do the magnetic field lines cross each other? Where do the filings concentrate most?

Magnetic field lines between like and unlike poles: When two unlike poles are placed face to face, they attract each other. The magnetic field lines follow the paths from the north pole of one magnet to the south pole of the other as shown in Figure 7 (a).



(a) *(b)*

Figure 7.

However, when two like poles face each other, a repulsive force occurs between the magnets. At the midpoint between the poles, magnetic forces are equal but act in opposite directions, so they cancel each other out (Figure 7 (b)).

Exercises

Two iron bars attract each other no matter which ends are placed together. Are both magnets?

Exercises

Suppose you have three iron rods, two of which are magnetized but the third is not. How would you determine which two are the magnets without using any additional objects?

Explanation: Put one end of one rod close to one end of another rod. The ends will either attract or repel. Continue trying all combinations of rods and ends until two ends repel each other. Then the two rods used in that case are the magnets.

Earth's magnetic field

A compass needle points north because the earth itself is a magnet. The magnetic behavior of the earth is similar to that of a bar magnet with its axis not exactly parallel to the geographic axis as shown in Figure 8. The angular difference between magnetic north and geographic north is called the magnetic **declination**.

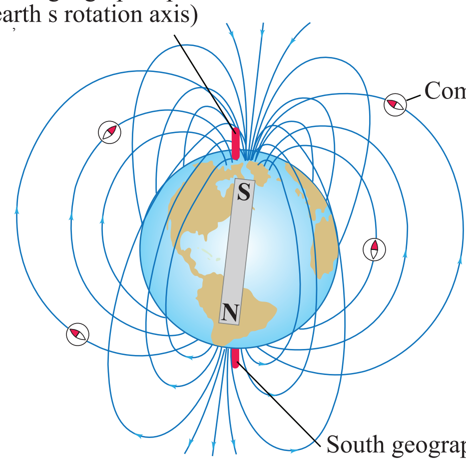


Figure 8.

Exercises

A compass needle is not always balanced parallel to the Earth's surface, but one end may dip downward. Why?

Explanation: The Earth's magnetic field is not always parallel to the surface of the earth, it may have a component perpendicular to the Earth's surface. The compass will tend to line up with the local direction of the magnetic field, and so one end of the compass will dip downward. The angle that the earth's magnetic field makes with the horizontal is called the angle of dip.

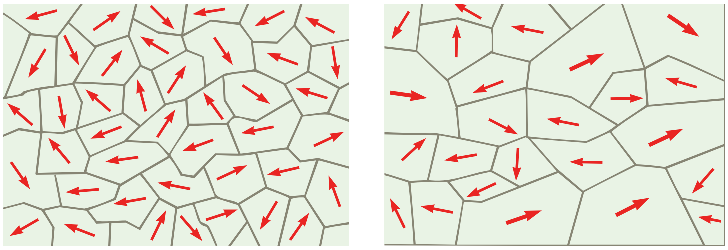
Discuss 3: Is the magnitude of earth's magnetic field at the equator greater than, less than or equal to its magnitude at the north pole?

Magnetic Domains

The magnetic properties of many materials are explained in terms of a model in which an electron is said to spin on its axis. Each electron acts like a small bar magnet. In some materials the magnetic fields of the electrons cancel, leaving zero net magnetic field. In other materials—like iron, nickel, and cobalt—the magnetic fields of the electrons don't cancel, and the electrons in neighboring atoms tend to align with one another, producing a strong magnetic field.

A region within a magnetic material where the electrons are aligned in the same direction is referred to as a **magnetic domain**. Each domain has a strong magnetic field in a given direction. Different domains are oriented differently, however, so that the net effect may be small.

In an **unmagnetized** substance, the domains are randomly oriented, as shown in Figure 9 (a). When an external magnetic field is applied to such a material, the magnetic domains that are pointing in the direction of the applied field tend to grow in size at the expense of the domains with different orientations as shown in Figure 9 (b). The material is now magnetized and becomes a permanent magnet.



(a) *(b)*

Figure 9.

Activity 4

For this activity, prepare two small paper clips, and one bar magnet. Pick up one paper clip with the magnet. Touch the second paper clip to the bottom of the first paper clip so that both are suspended from the magnet as shown in Figure 10. Remove the first paper clip from the bar magnet and the second paper clip remains suspended from the first.

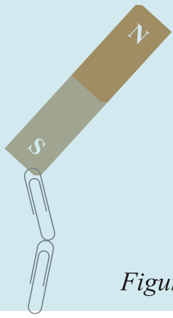


Figure 10.

Discuss 4: Why does the second paper clip remain suspended from the first? Can you tell the polarity of each end of the paper clips?

Now drop both paper clips onto the table ground. Try to pick up one paper clip with the other. Are they still magnetized? If no, why?

Magnetization and demagnetization

An unmagnetized piece of iron can be magnetized by stroking it with a magnet. Another means is that if a piece of unmagnetized iron is placed near a strong permanent magnet, the iron becomes magnetized.

An iron magnet can remain magnetized for some time. However, if you drop a magnet on the floor as in the previous activity or strike it with a hammer, the vibration may cause the domains into randomness. The magnet can thus lose some or all of its magnetism. Heating a magnet too can demagnetize it, for raising the temperature increases the random motion of the atoms which tends to randomize the domains.

Activity 5

Materials you need are – a test tube, a sopper, iron filings, a compass, and a bar magnet.

Fill the test tube with iron filings and close it with a stopper. Stroke the north pole of the bar magnet along the tube many times in one direction only (Figure 11). Now place the compass near the ends of the tube. Is it magnetized? If not, repeat the process until it is magnetized. Which side is the north pole and which side is the south pole?

- Explain why the tube acts as a magnet.

  Now shake the test tube and check with a compass whether it has north and south poles.
- Can we say that the tube has been **demagnetized** by the shaking process?

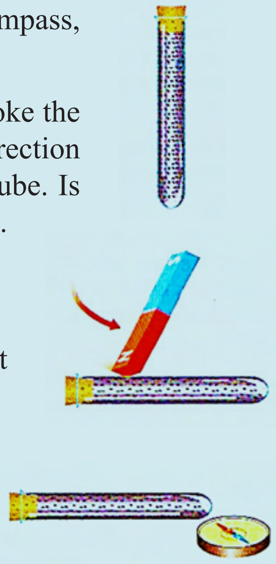


Figure 11.

Discuss 5:

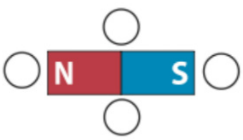


Figure 12.

1. If four compasses are arranged around as shown in Figure 12, in what direction will the north end of each compass needle point? Draw the needles in indicated space.
2. Refer to Figure 13 to answer the following questions. (a) Where is the north pole? (b) Where is the south pole? (c) At what point (s) is the magnetic field the greatest?

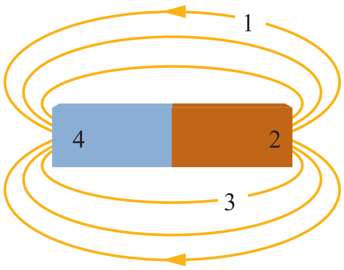


Figure 13.

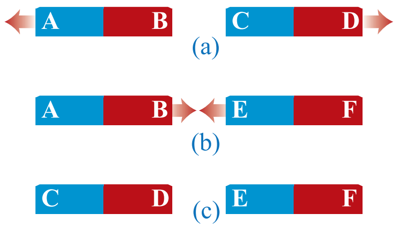


Figure 14.

3. In Figure 14 (a), the ends of the two bar magnets labelled B and C repel each other. If a third bar magnet approaches the first one, it is attracted as in Figure 14 (b). What type force occurs in the arrangement shown in Figure 14 (c)?

Exercises

1. State the rule for magnetic attraction and repulsion.
2. Describe how a temporary magnet differs from a permanent magnet.
3. When you break a bar magnet in half, how many poles does each piece have?

KEY TERMS

- Magnetic flux
- Force
- Magnetic flux density
- Moving-coil meter

Different magnets have different strength of attracting or repelling other materials or magnets. The concept of magnetic flux is related to the measurement of the magnetic field strength these magnets produce. In this section we will consider this field interaction and its application in the construction of moving-coil meters, such as galvanometers.

Concept of magnetic field

Magnetic field is a vector having both magnitude and direction. It is represented by the symbol, $\vec{B}$, (Figure 15).

- The direction of the magnetic field, $\vec{B}$, at a given location is the direction in which the north-seeking arrow of a compass needle points when placed at that location.

Note:

(i) At each point, the magnetic field, $\vec{B}$, is tangent to the field line.

(ii) At each point, the field lines point in the same direction a compass needle would point. Therefore, magnetic field lines point away from N poles and toward S poles.

(iii) The more densely the field lines are packed, the stronger the field is at that point.

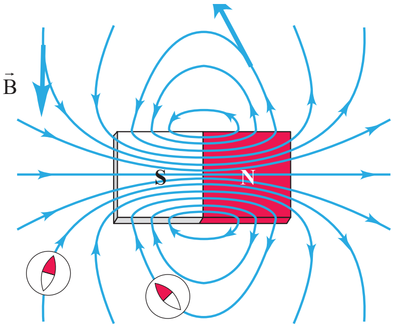


Figure 15.

Uniform Magnetic Field: The simplest magnetic field is one that is uniform—it doesn't change in magnitude or direction from one point to another. A perfectly uniform field over a large area is not easy to produce. But the field between two flat parallel poles of a magnet is nearly uniform if the area of the pole faces is large compared to their separation, as shown in Figure 16 (a).

(a) *(b)*

Figure 16.

The parallel evenly spaced field lines in the central region of the gap indicate that the field is uniform at points not too near the edges. Figure 16 (b) shows how a uniform field can be drawn without the need to draw the poles.

Magnetic Flux

The word flux basically means "flow." For example, the flux, or flow, of air through a window is related to the direction of the wind and the cross-sectional area of the window. If wind blows straight through an open window, the flux is high. The larger the window, the greater the flux. If wind blows parallel to a window, no air passes through it at all. In this case the flux is zero, no matter how large the window.

Similarly, **magnetic flux** is a measure of the number of magnetic field lines that pass through a given area. A magnetic field perpendicular to a surface gives a high flux, and the larger the surface area, the greater the flux. A magnetic field parallel to a surface gives zero flux.

Suppose a magnetic field, $\vec{B}$, crosses a surface area, A, at right angles, as in Figure 17 (a). For a uniform field such as this, the magnetic flux, Φ, is defined as the product of the area A and the magnetic field perpendicular to the surface B.

$\Phi = BA$ (B is perpendicular to area A)

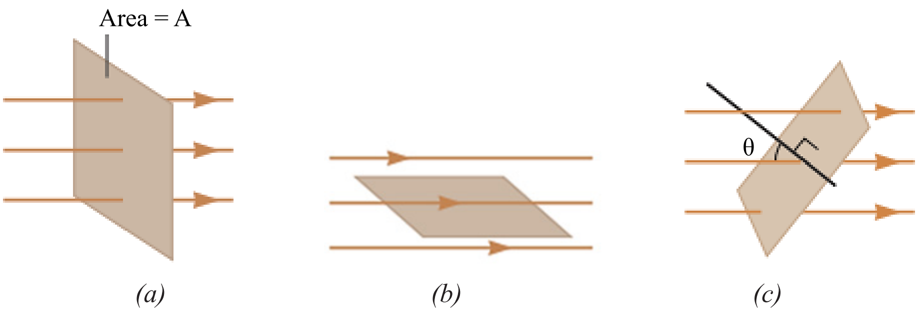


Figure 17.

If, on the other hand, the magnetic field is parallel to the surface, as in Figure 17 (b), we see that no field lines cross the surface. In a case like this the magnetic flux is zero:

$\Phi = 0$

In general, only the component of $\vec{B}$ that is perpendicular to a surface contributes to the magnetic flux. The magnetic field in Figure 17 (c), for example, crosses the surface at an angle θ relative to the normal, and hence its perpendicular component is B cos θ.

The magnetic flux, then, is simply B cos θ times the area A:

$\Phi = B\,A\cos\theta$

Where Φ (Greek-phi) is magnetic flux. The unit of magnetic flux is weber (Wb). If B is measured in tesla (T), and area is measured in m^2, then weber is expressed in $T.m^2$. Thus,

$1\ Wb = 1\ T.m^2$ (Unit of magnetic flux)

Note: Since $B = \Phi/A$, the magnetic field strength, B, is also called the magnetic flux density.

- Magnetic flux depends on the magnitude of the magnetic flux density, B,its orientation with respect to a surface, θ, and the area of the surface, A.

Example

A rectangular coil of wire is situated in a uniform magnetic field whose magnitude is 0.50 T. The coil has an area of 2.0 m^2. Determine the magnetic flux for the three orientations, $\theta = 0°$, $\theta = 60°$, and $\theta = 90°$, shown in Figure 18.

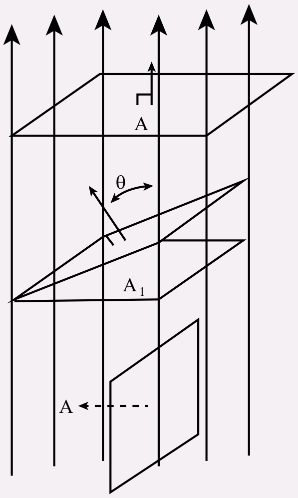


Figure 18.

Solution

The magnetic flux is defined as $\Phi = BA\cos\theta$, where B is the magnitude of the magnetic field, A is the area of the surface through which the magnetic field passes, and θ is the angle between the magnetic field and the normal to the surface.

$\theta = 0°$ $\quad \Phi = (0.50\ T)(2.0\ m^2)\cos 0° = 1.0\ Wb$

$\theta = 60°$ $\quad \Phi = (0.50\ T)(2.0\ m^2)\cos 60° = 0.5\ Wb$

$\theta = 90°$ $\quad \Phi = (0.50\ T)(2.0\ m^2)\cos 90° = 0\ Wb$

Note: Even if a magnetic field is uniform in space and constant in time, the magnetic flux through a given area changes if the orientation of the area changes. This is particularly relevant to electric motors and generators, as we will see later in the next section.

Example

The magnetic flux through a circular loop is $4.5 \times 10^{-4}\ T.m^2$. If the normal to the loop makes an angle of 53° with a magnetic field of 0.50 T, what is the area of the loop?

Solution

$\Phi = 4.5 \times 10\text{-}4\ T.m^2$ $\quad B = 0.50\ T$

$\theta = 53°$ $\quad A = ?$

Solving the equation $\Phi = B A \cos\theta$ for A, gives

$$A = \frac{\Phi}{B\cos\theta} = \frac{4.5 \times 10^{-4}\ T.m^2}{(0.50\ T)(0.6)}$$

$$A = \frac{4.5 \times 10^{-4}\ T.m^2}{0.3\ T} = 1.5 \times 10^{-3}\ m^2$$

Example

A rectangular loop 3.0 cm wide and 5.0 cm long is placed in a magnetic field. The angle between the normal to the loop and the magnetic field is 37°, and the magnetic flux through the loop is $2.0 \times 10^{-3}\ T.m^2$. What is the magnitude of the magnetic field?

Solution

Area of the rectangular loop is: A = 3.0 cm × 5.0 cm = 15 cm2 = 15 × 10^{-4} m^2

θ = 37o

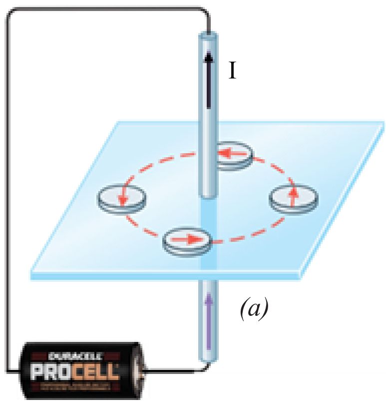
Φ = 2.4 × 10^{-3} T.m^2

The magnetic field intensity, from the equation Φ = BA cos θ, is

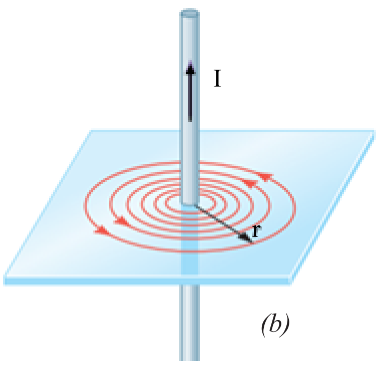
$$B = \frac{\Phi}{A\ \cos\ \theta} = \frac{2.4 \times 10^{-3}\ T.m^2}{(1.5 \times 10^{-3} m^2)(0.8)} = 2.0\ T$$

Electric Currents Produce Magnetic Fields

During the eighteenth century, many scientists sought to find a connection between electricity and magnetism. A stationary electric charge and a magnet were shown to have no influence on each other. But the Danish scientist Hans Christian Oersted discovered that when a compass is placed near a wire, the compass needle deflects if the wire carries an electric current. As we have seen, a compass needle is deflected by a magnetic field. So Oersted's experiment showed that an electric current produces a magnetic field. He had found a connection between electricity and magnetism.



(a)



(b)

Figure 19 (a) illustrates Oersted's discovery with a very long, straight wire. When a current is present, the compass needles point in a circular pattern about the wire. The pattern indicates that the magnetic field lines produced by the current are circles centered on the wire Figure 19 (b)). If the direction of the current is reversed, the needles also reverse their directions, indicating that the direction of the magnetic field has reversed. The direction of the field can be obtained by using the first Right-Hand Rule (RHR- 1), as shown in Figure 19 (c).

Right-Hand Rule (RHR-1) Curl the fingers of the right hand into the shape of a half-circle. Point the thumb in the direction of the conventional current I, and the tips of the fingers will point in the direction of the magnetic field $\vec{B}$.

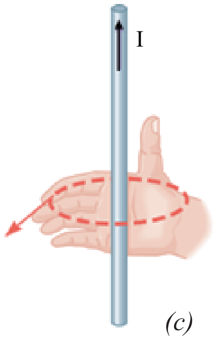


Figure 19.

Direction of $\vec{B}$ In some cases a magnetic field we are considering will point into or out of the page. This can be difficult to draw. Therefore, we establish the convention that the symbol **x** (cross) indicates that the magnetic field points into the page. The way to remember this is to think of a magnetic field vector as an arrow. At the tail end of an arrow are crossed feathers. Therefore, if you view a vector from behind, it looks like an x. Similarly, if an arrow points out of the page, all you see is the point at its tip. Thus, we represent a magnetic field vector pointing out of the page with the symbol • (dot), where the dot represents the tip of the arrow.

Note: This method can also be used for other quantities, such as force and electric current

Exercises

The magnetic field shown in Figure 20 is due to the horizontal, current- carrying wire. Does the current in the wire flow toward the left or the right?

Explanation If you point the thumb of your right hand along the wire toward the left, your fingers curl into the page above the wire and out of the page below the wire, as in the figure. Thus, the current flows toward the left.

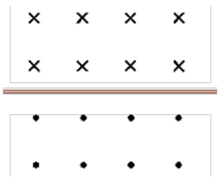


Figure 20.

Magnitude of $\vec{B}$

Experiments show that the magnetic field produced by a current-carrying wire doubles if the current, I, is doubled. In addition, the field doubles if the radial distance from the wire, r, is halved. These observations are easily summarized in one statement: The magnetic field produced by a current in a wire is proportional to the current and inversely proportional to the radial distance from the wire ($B \sim I/r$). The magnitude of the magnetic field, $\vec{B}$, for a long, straight wire is given by the following equation:

$$B = \frac{\mu_o I}{2\pi r}$$

The constant μ_o is known as the **permeability of free space**, and its value is $\mu_o = 4\pi \times 10^{-7}$ T.m/A.

Note: The magnetic field becomes stronger nearer the wire, where r is smaller.

Example

Find the magnetic field strength at a distance of 1 m from a long, straight wire carrying a current of 1 A.

Solution

r = 1 m B =?

I = 1 A

Substituting the given quantities into the equation for B, we get

$$B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi \times 10^{-7})(1A)}{2\pi \times 1\ m} = 2 \times 10^{-7}\ T$$

Note: This is a rather weak magnetic field. It is less than the magnetic field of the Earth, which is approximately 10^{-4} T near the surface of the earth.

In such circumstances, a magnetic field unit called the gauss (G) is sometimes used. Although not an SI unit, the gauss is a convenient size for many applications involving magnetic fields. The relation between the gauss and the tesla is

$$1\ G = 10^{-4}\ T$$

Example

An electric wire in the wall of a building carries a dc current of 25 A vertically upward. What is the magnetic field due to this current at a point P, 10 cm due north of the wire (Figure 21)?

Solution

I = 25 A B =?

r = 10 cm = 0.10 m

The field strength due to the current I is

$$B = \frac{\mu_o I}{2\pi r} = \frac{(4\pi \times 10^{-7})(25\ A)}{2\pi \times 0.10\ m} = 5.0 \times 10^{-5}\ T$$

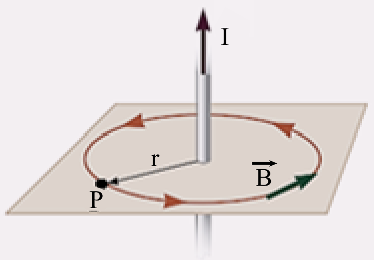


Figure 21.

By the right-hand rule, the magnetic field points due west.

Example

A long, straight wire carries a current of 10 A. How far from this wire is the magnetic field it produces equal to the earth's magnetic field, which is nearly 5×10^{-5} T?

Solution

$I = 10\ A$

$B = 5 \times 10^{-5}\ T$

From the equation for B, we solve for r

$$r = \frac{\mu_o I}{2\pi B}$$

$$r = \frac{\mu_o I}{2\pi B} = \frac{4\pi \times 10^{-7}\ T.m/A \times 10\ A}{2\pi \times 5 \times 10^{-5}\ T} = 4 \times 10^{-2}\ n$$

$r = 4\ cm$

Exercises

How does the strength of a magnetic field, 1 cm from a current carrying wire, compare with each of the following? (a) The strength of the field that is 2 cm from the wire. (b) The strength of the field that is 3 cm from the wire.

Explanation: Because magnetic field strength varies inversely with the distance from the wire, the magnetic field at 1 cm will be (a) twice as strong as the magnetic field at 2 cm, and (b) three times as strong as the magnetic field at 3 cm.

Activity 6

To observe the magnetic effect of electric current you need the following materials: A switch, conducting wires, a compass, and a battery (D-cell).

An electric current – which is defined as the rate of flow of charges – produces magnetic field. This simple demonstration gives an insight of Oersted's experiment.

Set up a circuit including a switch, conducting wires, and a battery as shown in Figure 22. Align the compass in the north-south direction. Place the wire on the compass parallel to the compass needle. Now press the switch for a few seconds. What do you observe when the current is on? Observe carefully the way the needle deflects by reversing the polarity of the battery.

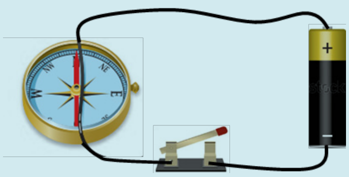


Figure 22.

Force between Two Parallel Wires

We have seen that a wire carrying a current produces a magnetic field. If two such conductors are placed side by side, their magnetic fields exert a repulsive or an attractive force on each other.

If the currents are in the same direction as in Figure 23 (a), the field due to I_1 is into the page and the current I_2 is upward. Using the right-hand rule the force is to the left - which shows that the two wires attract each other.

On the other hand, if the currents are oppositely directed, the field due to I_1 is into the page and the current I_2 is downward. Using the right-hand rule the force is to the right - which is repulsive (Figure 23 (b).

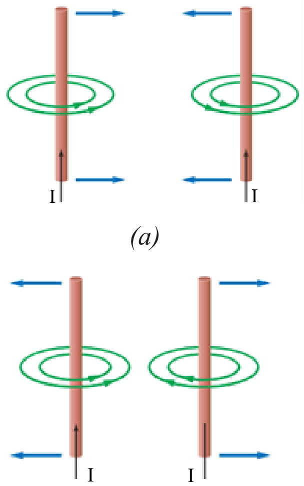


Figure 23.

Magnetic field of a current loop

If a current-carrying wire is bent into a circular loop, the magnetic field lines around the loop have the pattern shown in Figure 24 (a). A comparison of the magnetic field lines around the current loop with that of a bar magnet shows that the two patterns are similar. Not only are the patterns similar, but the loop itself behaves as a bar magnet with a “north pole” on one side and a “south pole” on the other side (Figure 24 (b)).

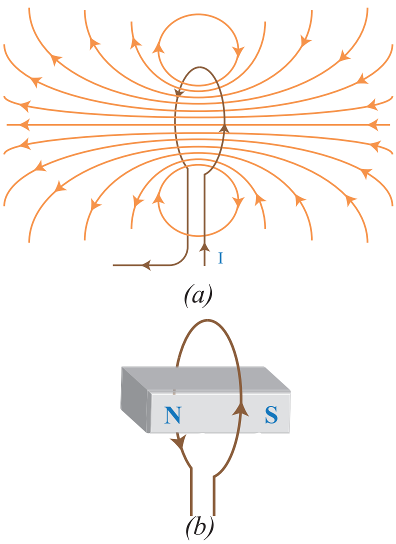


Figure 24.

Exercises

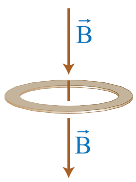


Figure 25.

The current in the circular loop in Figure 25 produces a magnetic field on the axis that points downward as shown. As viewed from above, is the direction of the current in the loop (a) clockwise or (b) counterclockwise?

Explanation: If the thumb points in the direction of the field ($\vec{B}$), the fingers curl in the direction of the current. Thus, the current in the loop is in the clockwise direction as viewed from above.

Force between wire loops

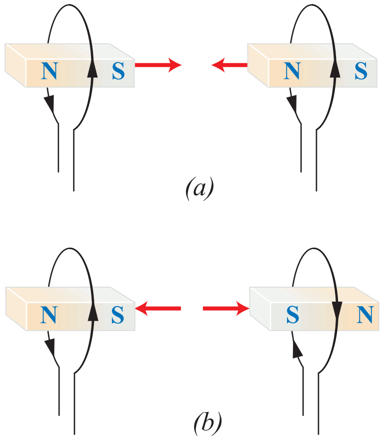


Figure 26.

Because a current-carrying loop acts like a bar magnet, two adjacent loops can be either attracted to or repelled from each other, depending on the relative directions of the currents. Figure 26 includes a "phantom" magnet for each loop and shows that the loops are attracted to each other when the currents are in the same direction (Figure 26 (a)) and repelled from each other when the currents are in opposite directions (Figure 26 (b)).

When a wire is looped several times to form a coil and a current is allowed to flow through the coil, the field around all the loops is always in the same direction. Such a coil of wire consisting of many turns (loops) is called a **solenoid** (Figure 27 (a)).

If a piece of iron is placed inside a solenoid, the magnetic field is increased greatly because the iron becomes a magnet. The resulting magnetic field is the sum of the field due to the current and the field due to the iron, and can be hundreds or thousands of times the field due to the current alone. Such an iron-core solenoid is called an **electromagnet** (Figure 27 (b)).

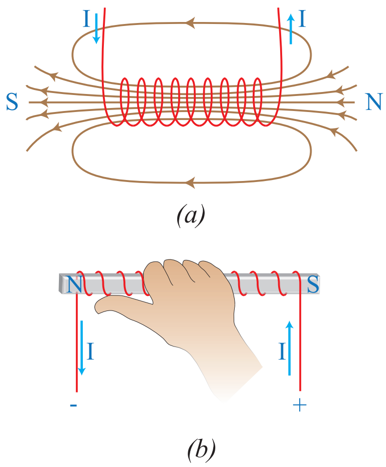


Figure 27.

Magnetic Field of a Solenoid

The magnetic field lines in Figure 27 (a) are tightly packed inside the solenoid but are widely spaced outside. In the ideal case of a very long, tightly packed solenoid, the magnetic field is strong and uniform inside the solenoid. If a solenoid has N loops (turns) and a length L, the magnetic field inside the solenoid is given by the following equation:

$$B = \mu_o \left(\frac{N}{L}\right) I = \mu_o n I$$

In this expression, n is the number of turns per length (n = N/L).

Note:

(i) The magnetic field outside a solenoid is small, and in the ideal case can be considered to be zero.

(ii) The above result is independent of the cross-sectional area of the solenoid. The field depends directly on the number of loops per length and on the current.

Exercises

If you want to increase the strength of the magnetic field inside a solenoid, is it better to (a) double the number of loops, keeping the length the same, or (b) double the length, keeping the number of loops the same?

Explanation: In the expression B = μ0(N/L)I, we see that doubling the number of loops (N→2N) while keeping the length the same results in a doubled magnetic field (B→2B). On the other hand, doubling the length (L → 2L) while keeping the number of loops the same reduces the magnetic field by a factor of 2 (B→B/2). Hence, to increase the field one should pack more loops into the same length.

Example

A solenoid is 15 cm long, has 200 loops, and carries a current of 3 A. Find the strength of the magnetic field inside the solenoid.

Solution

L = 15 cm   I = 3 A

N = 200   B =?

The magnetic field inside the solenoid is:

$$B = \mu_o \left(\frac{N}{L}\right) I = 4\pi \times 10^{-7}\ T\cdot m/A \left(\frac{200}{0.15\ m}\right) 3\ A = 5.0 \times 10^{-3}\ T$$

Discuss 6: Solenoid A has length L and N turns, solenoid B has length 2L and N turns, and solenoid C has length L/2 and 2N turns. If each solenoid carries the same current, in which solenoid is the strength of the magnetic field at the center (a) the largest, and (b) the smallest?

Right-hand rule (RHR-2): To determine the polarity, the second right-hand rule (RHR-2) is used. Hold the electromagnet with your right hand in such a way that the fingers curl in the direction of the conventional current. Then, your thumb will point toward the north pole (Figure 27 (b)).

Exercises

What is the direction of the magnetic field at point P in Figure 28? (P is on the axis of the coil.)

A ↑

B ↓

C ←

D →

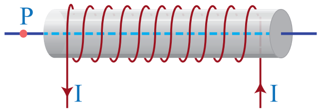


Figure 28.

Explanation Using the right-hand rule (RHR-2), the left end is South Pole.
So, the field lines are into this end. The correct answer is choice (d).

Activity 7

For this Activity you need the following materials: Dry cell, about 1 m length of insulated wire, large nail, compass, and metal paper clips.

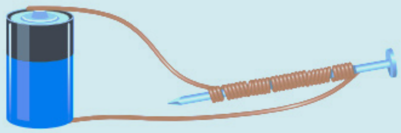
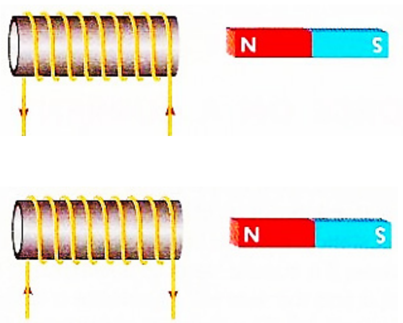


Figure 29.

Wind the wire around the nail, as shown in Figure 29. Remove the insulation from the ends of the wire, and hold these ends against the + and - terminals of the battery. Now, you have an electromagnet. Use the compass to determine whether the nail is magnetized. Next, flip the battery so that the direction of the current is reversed. Again, bring the compass toward the same part of the nail. Explain why the compass needle now points in a different direction?

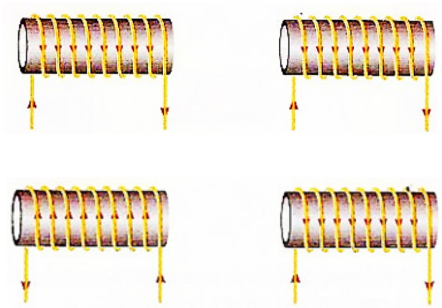
Bring paper clips near the nail while connected to the battery. What happens to the paper clips? How many can you pick up? Try this by adding the D-cell or increasing the number of turns of the coil on the wire or both. Discuss the effects on the strength of the field by picking more paper clips up.

Electromagnets have a very important property: Their magnetism can be controlled by switching the current on or off. Remember that iron is a soft magnetic material and shows temporary magnetism. Therefore, when there is a current through the wire, the iron core becomes a magnet but when the there is no current in the wire, the iron core is not a magnet any more.



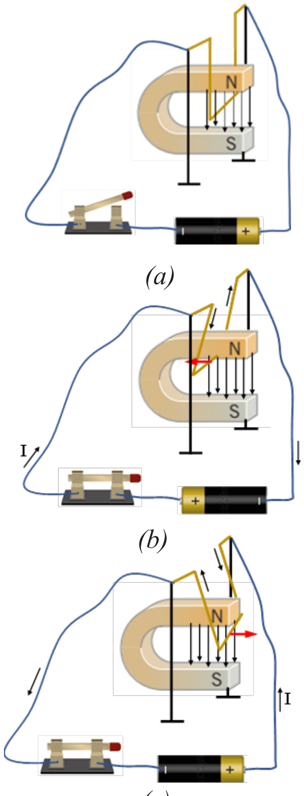
(a)

Discuss 7: (a) For each electromagnet on the left of Figure 30 (a), discuss whether it will be attracted to or repelled from the permanent magnet on the right. (b) For each electromagnet on the left of Figure 30 (b), discuss whether it will be attracted to or repelled from the electromagnet on the right.



(b)

Figure 30.



(a)

(b)

(c)

Figure 31.

Force on an Electric Current in a Magnetic Field: A conducting rod will not produce a magnetic field in the space surrounding it if there is no current through it, as discussed earlier in this section. The conducting rod in Figure 31 (a) will not interact with the magnetic field of the U-shaped magnet, since the switch is open and there is no current in the rod. However, if the switch is closed, the current through the rod produces a magnetic field around it and the wire experiences a **magnetic force** as shown in Figure 31 (b). The direction of the force will be reversed when the current direction is reversed (Figure 31 (c)).

Direction of the magnetic force: Note that the direction of the force is not along the magnetic field direction nor along the current direction. The force is perpendicular to both magnetic field, $\vec{B}$, and current, I, directions. The third right-hand rule (RHR-3) is used to find the direction of the magnetic force.

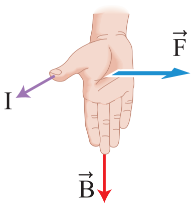


Figure 32.

RHR-3: Point the fingers of your right hand in the direction of $\vec{B}$. Point your thumb in the direction of I. The palm of your hand then faces (pushes) in the direction of the force acting on the wire as shown in Figure (3.32).

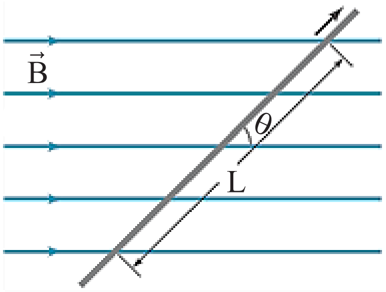


Figure 33.

The magnitude of the force on the wire: It is found experimentally that the magnitude of the force is directly proportional to the current I in the wire, the length L of the wire in the field, and the magnetic field B. The force also depends on the angle θ between the current direction (or wire) and the magnetic field Figure 33). Thus we have the following equation:

$$F = BIL \sin \theta$$

From this the magnetic field strength B is

$$B = \frac{F}{IL \sin \theta}$$

If the field and the current are perpendicular, $\theta = 90^o$ and $\sin 90^o = 1$, which gives the maximum force. The unit for the magnetic field B is the tesla, T.

$$1T = 1\ N/A.m$$

Exercises

A strong current is suddenly switched on in a wire. However, no magnetic force acts on the wire. Can we conclude that there is no magnetic field at the location of the wire?

Explanation: No. A current-carrying conductor experiences no magnetic force if (a) the conductor is parallel to the magnetic field or (b) there is no magnetic field at the location of the conductor.

Example

A straight wire that carries a 5 A current is in a uniform magnetic field oriented at right angles to the wire. When 10 cm of the wire is in the field, the force on the wire is 0.20 N. What is the strength of the magnetic field, B?

Solution

$I = 5\ A$ $F = 0.20\ N$

$L = 0.10\ m$ $B = ?$

Since B and I are perpendicular to each other, $\sin 90° = 1$. Thus,

$$B = \frac{F}{IL} = \frac{0.20\ N}{(5\ A)(0.10\ m)} = 0.40\ N/A.m$$

$$B = 0.40\ T$$

Example

A wire 0.50 m long carrying a current of 8 A is at right angles to a 4×10^3 Gauss magnetic field. How strong a force acts on the wire? (1 Gauss (G) = 10^{-4} T)

Solution

$L = 0.50$ m (the part of the wire inside the field)

$I = 8$ A

$B = 4 \times 10^3 \text{ G} = 4 \times 103 \times 10^{-4} \text{ T} = 0.4 \text{ T}$

$\theta = 90°$ (at right angles)

The magnitude of the magnetic force is:

$F = BIL \sin 90^o = 0.40 \text{ T} \times 8 \text{ A} \times 0.50 \text{ m} \times 1$

$F = 1.6$ N

Example

An electric power line carries a current of 1400 A in a location where the Earth's magnetic field is 5×10^{-5} T. The line makes an angle of 75° with the field. Determine the magnitude of the force on a 120 m length of the line.

Solution

$I = 1400$ A $L = 120$ m $F = ?$

$B = 5 \times 10^{-5}$ T $\theta = 75^o$

Here $\theta = 75°$ is the angle between the magnetic field B and the wire or the current I. The magnetic force on the wire is

$F = BIL \sin \theta$.

$F = 5 \times 10^{-5} \text{ T} \times 1400 \text{ A} \times 120 \text{ m} \times \sin 75°$

$F = 5 \times 10^{-5} \text{ N/A.m} \times 1400 \text{ A} \times 120 \text{ m} \times 0.966$

$F = 8.11$ N

Example

A horizontal straight wire 0.4 m long carries an electric current of 3 A from south to north in a magnetic field of induction 1.0 T directed upward. Find out the magnitude and direction of the force acting on the wire.

Solution

$L = 0.4$ m $B = 1$ T

$I = 3$ A $F = ?$

The magnitude of the force is:

$F = BIL = 1\ T \times 3\ A \times 0.4\ m$

$F = 1.2\ N$

Using the right-hand rule (RHR-3), its direction is east.

Example

When a 2 A current passes through a 3 m length of straight wire that is entirely immersed in a uniform B-field, it experiences a maximum force of 6×10^{-3} N. How strong is the field?

Solution

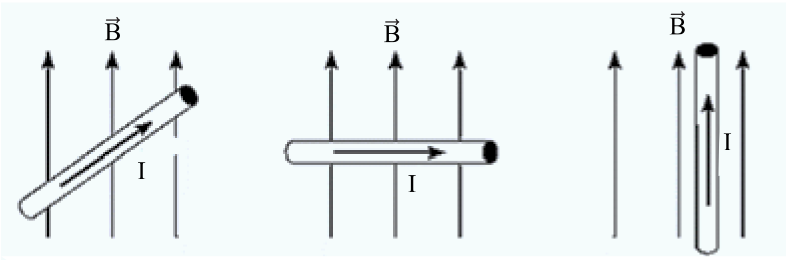
$I = 2\ A$ $\qquad B = ?$

$L = 3\ m$

Notice that maximum force occurs when $\theta = 90°$. The magnitude of the field is

$$B = \frac{F}{IL} = \frac{6 \times 10^{-3}\ N}{2\ A \times 3\ m} = 1 \times 10^{-3}\ T$$

Discuss 8: The same current-carrying wire is placed in the same magnetic field in three different orientations (Figure 34). In which orientation is the magnitude of the magnetic force exerted on the wire (a) the largest, and (b) the smallest?



(a) *(b)* *(c)*

Figure 34.

Example

A thin copper rod of length 1.0 m has a mass of 50 grams. What is the minimum current in the rod that would allow it to "float" in a magnetic field of 0.1 T? (Take $g = 10\ m/s^2$)

Solution

$L = 1.0\ m$ $\qquad B = 0.1\ T$

$m = 50\ g = 0.050\ kg$ $\qquad I = ?$

The weight (or gravitational force, Fg) of the copper rod is balanced by the magnetic force (Fm).

$$F_g = F_m$$

$$mg = BIL$$

$$I = \frac{mg}{BL} = \frac{0.050\ kg \times 10\ m/s^2}{0.1\ T \times 1.0\ m} = 5\ A$$

Moving coil Galvanometer

When an electric current flows in a closed loop of wire placed in an external magnetic field, the magnetic force on the current can produce a torque. This is the principle behind a number of important practical devices, including galvanometers, electric motors, and generators.

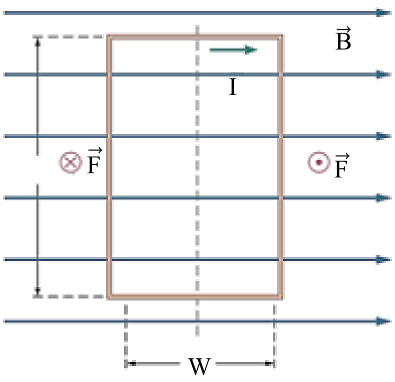


Figure 35.

Figure 35 shows a rectangular loop of height h and width w carrying a current I. The loop is placed in a uniform magnetic field, $\vec{B}$, that is parallel to the plane of the loop. We see that the horizontal segments of the loop experience zero force because they are parallel to the field. The vertical segments, on the other hand, are perpendicular to the field. They experience forces of magnitude F = IhB. One of these forces is directed into the page (left side); the other points out of the page (right side). These forces tend to rotate the loop—that is, they cause a **torque**. This principle is used in a galvanometer.

A galvanometer is a device used to measure very small currents. It can be used as a voltmeter or an ammeter. A small spring in the galvanometer exerts a torque that opposes the torque that results from the current through the wire loop; thus, the amount of rotation is proportional to the current. The meter is calibrated by finding out how much the coil turns when a known current is sent through it, as shown in Figure 36. The galvanometer can then be used to measure unknown currents.

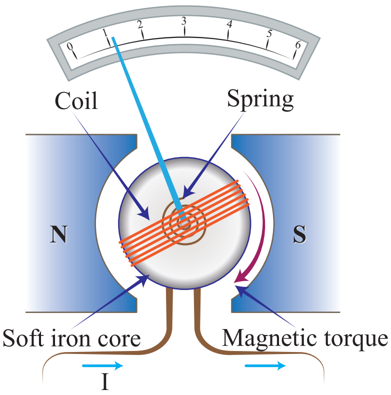


Figure 36.

Exercises

Explain why gavanometers, ammeters and voltmeter are usually called moving coil meters?

Explanation: The basic elements of a galvanometer are a coil, a magnetic field, a spring, and a needle (or pointer) attached to the coil. When a current passes through the coil, it becomes an electromagnet. Attraction and repulsion between the magnetic poles of the electromagnet and the poles of the permanent magnet makes the coil rotate (move). Because of this operation of the meter, it is called moving-coil galvanometer.

Exercises

1. The right-hand rule is used to determine the force on a current-carrying conductor perpendicular to a magnetic field. Identify what part of the hand corresponds to the following physical quantities:
   (a) magnetic force,
   (b) magnetic field,
   (c) current in the conductor.
2. What is the shape of the magnetic field produced by a straight current carrying wire?
3. A wire carries a current of 2.0 A. At what distance from the wire does the magnetic field have a magnitude of 1.0 T? 1 ´ 10−5 T?
4. Why does a current-carrying wire experience a magnetic force?
5. A wire that is 1.50 m long and carrying a current of 10 A is at right angles to a uniform magnetic field. The force acting on the wire is 0.50 N. What is the strength of the magnetic field?
6. A wire carrying a current of 5 A is at an angle of 45° relative to a magnetic field of 0.80 T. What is the force exerted on a 1.5 m length of the wire?
7. A solenoid has 1200 loops per meter and carries a current of 5.0 A. What is the magnetic field inside the solenoid?

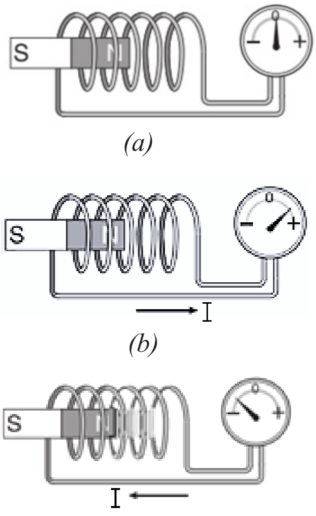
In the last section, we discussed two ways in which electricity and magnetism are related: 1^{st} an electric current produces a magnetic field; and 2^{nd} a magnetic field exerts a force on an electric current. Scientists then began to wonder: if electric currents produce a magnetic field, is it possible that a magnetic field can produce an electric current? A few years later Joseph Henry and Michael Faraday independently

Key Terms

- Induced emf
- Motors
- Transformers
- Faraday's Law
- Generators

found that it was possible. Henry actually made the discovery first. But Faraday published his results earlier and investigated the subject in more detail. We now discuss this phenomenon and some of its world-changing applications including the electric generators, motors and transformers.

The law of electromagnetic induction



(a) *(b)* *(c)*

Figure 37.

Figure 37 shows one of Faraday's experiments, in which a wire loop that is part of a closed circuit is placed in a magnetic field. In fact, there are a number of ways to generate electric current by means of magnetic fields. A bar magnet and a coil of wire, to which a galvanometer is connected is sufficient to obtain current, as shown in Figure 37. When there is no relative motion between the magnet and the coil, as in part Figure 37 (a), the galvanometer reads zero current. When the magnet is moved toward the coil the galvanometer will deflect in one direction (Figure 37 (b)), and when the magnet is taken away the galvanometer needle will deflect in the opposite direction Figure 37 (c).

Faraday found that to generate current, either the conductor can move through a magnetic field or a magnetic field can move past the conductor. It is the relative motion between the wire and the magnetic field that produces the current. The process of generating electric current through a circuit in this way is called **electromagnetic induction.**

The current in the coil is called an **induced current** because it is brought about (or "induced") by a changing magnetic field. Since a source of emf (electromotive force) is always needed to produce a current, the coil itself behaves as if it were a source of emf. This emf is known as an **induced emf**. Thus, a changing magnetic field induces an emf in the coil, and the emf leads to an induced current.

- The magnitudes of the induced current and induced emf are found to be proportional to the rate of change of the magnetic field - the more rapidly the magnetic field changes, the greater the induced emf.

Faraday discovered that the coil in Figure 37 experiences an induced emf only when the magnetic flux through it changes with time. Furthermore, the induced emf for a given loop is found to be proportional to the rate at which the flux changes with time, $\Delta\Phi/\Delta t$. Often the flux passes through a coil of wire containing more than one loop. If the coil consists of N loops, the total induced emf is N times greater than each loop.

Faraday's Law of Electromagnetic Induction

Faraday's law of electromagnetic induction is a basic law of electromagnetism predicting how a magnetic field will interact with an electric circuit to produce an electromotive force (emf).

"Whenever a conductor is placed in a varying magnetic field, an electromotive force is induced. Likewise, if the conductor circuit is closed, a current is induced, which is called induced current."

The change in the magnetic flux or lines of force linked with a circuit produces an induced emf, the strength of which is directly proportional to the flux or lines of force producing it.

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Phi_2 - \Phi_1}{t_2 - t_2}$$

The negative sign in Faraday's law indicates that the induced emf opposes the change in magnetic flux.

Note: Faraday's law gives the emf that is induced in a circuit or a loop of wire. The current that is induced as a result of the emf depends on the characteristics of the circuit itself—for example, how much resistance it contains.

Example

A bar magnet is moved rapidly toward a 45 loop coil of wire. As the magnet moves, the magnetic flux through the coil increases from 1.3 × 10-5 T.m2 to 3.7 × 10-3 T.m2 in 0.25 s. (a) What is the magnitude of the induced emf? (b) If the resistance of the wire in the coil is 3.6 Ω, what is the induced current?

Solution

$N = 45$	$\Phi_1 = 1.3 \times 10^{-5}\ T.m^2$	(a) $\varepsilon = ?$
$t = 0.25\ s$	$\Phi_2 = 3.7 \times 10_{-3}\ T.m^2$	(b) $I = ?$

(a) The magnitude of the induced emf is given by Faraday's law,

$$\varepsilon = N\frac{\Phi_2 - \Phi_1}{t_2 - t_2} = (45)\frac{3.7 \times 10^{-3}\ T.m^2 - 1.3 \times 10^{-5}\ T.m^2}{0.250\ s}$$

$\varepsilon = 0.66\ V$

(b) Use Ohm's law to calculate the current

$$I = \frac{V}{R} = \frac{0.66\ V}{3.6\ \Omega} = 0.18\ A$$

Example

The induced emf in a single loop of wire has a magnitude of 1.2 V when the magnetic flux is changed from 0.9 T.m2 to 0.3 T.m2. How much time is required for this change in flux?

Solution

N = 1   Φ = 0.9 T.m2   Δt =?

ε = 1.2 V   Φ = 0.3 T.m2

A single loop of wire is one that has a single turn; that is, N = 1. Using this value in ε = N(ΔΦ/Δt), and rearranging to solve for the elapsed time, we have

$$\Delta t = -N\frac{\Delta\Phi}{\varepsilon} = -(1)\frac{0.3\ T.m^2 - 0.9\ T.m^2}{1.2\ V} = 0.5\ s$$

Example

A conducting circular loop enclosing an area of $2 \times 10^{-3}\ m^2$ is perpendicular to a uniform magnetic field of 6 T. If the field goes to zero in 0.0048 s, what is the magnitude of the average emf induced in the loop?

Solution

$A = 2 \times 10^{-3}\ m^2$ $B_1 = 6\ T$ N = 1

$\Delta t = 0.0048\ s$ $B_2 = 0$ ε =?

Since the plane of the loop is perpendicular to the field, the flux through the loop is maximum. Thus,

Initial flux: $\Phi_1 = BA = (6\ T)(2 \times 10^{-3}\ Wb) = 12 \times 10^{-3}\ Wb$

Final flux: $\Phi_2 = 0$

Change in flux: $\Delta\Phi = \Phi_2 - \Phi_1 = 0 - 12 \times 10^{-3}\ Wb = -12 \times 10^{-3}\ Wb$

This change occurs in a time interval, $\Delta t = 4.8 \times 10^{-3}\ s$

The induced emf is then

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -(1)\frac{(-12 \times 10^{-3}\ Wb)}{4.8 \times 10^{-3}\ s} = 2.5\ Wb/s$$

ε = 2.5 Volts

Note that Wb/s is the same as Volt.

Lenz's Law The negative sign indicates the direction of the induced emf (or current). Lenz explained it in a law known as Lenz's Law as follows.

"The direction of an induced current is such that its own magnetic field opposes the original change in magnetic flux that induced the current."

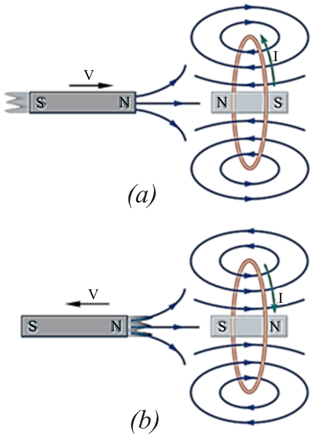


Figure 38.

Figure 38 shows how to apply Lenz's law to a magnet moving toward and away from a current loop. If the north pole of a magnet is moved toward a conducting loop, the induced current produces a north pole pointing toward the magnet's north pole. This creates a repulsive force opposing the change that caused the current (Figure 38 (a)). If the north pole of a magnet is pulled away from a conducting loop, the induced current produces a south magnetic pole near the magnet's north pole as shown in Figure 38 (b). The result is an attractive force opposing the motion of the magnet.

Exercises

A bar magnet is held above the center of a wire loop lying in the horizontal plane, as shown in Figure 39. The south end of the magnet is toward the loop. If the magnet is dropped, what is the direction of the induced current in the loop as viewed from above?

Explanation: As the bar magnet approaches the loop from above, with its south pole downward, magnetic flux through the loop is directed upward and increasing in magnitude. To oppose this increasing upward flux, the induced current in the loop will flow clockwise, as seen from above, producing a flux directed downward through the area enclosed by the loop.

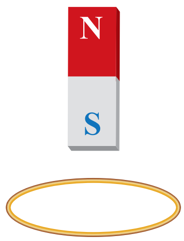


Figure 39.

Activity 8

To observe electromagnetic induction, you need the following materials: A strong bar magnet, a coil of wire with many turns, a coil with fewer turns, a galvanometer (center zero), connecting wire.

Connect the galvanometer to the ends of the coil with less turs. Keep the magnet stationary. Do you see any deflection of the meter. (a) Now, move the magnet in and out and observe the way the pointer deflects. (b) Next, keep the magnet stationary and move the coil back and forth observing the deflection of the pointer.

Now connect the galvanometer to the other coil and repeat the above process.

Discuss 9: Does the number of turns affect the amount of the emf induced? What about if you increase the speed of movement?

Inductance

The induced emf produced by a coil is either due to external magnetic flux changes of permanent magnets and electromagnets, or the magnetic flux changes due to the current that the coil itself produces.

Mutual Inductance: If two coils of wire are near one another, as in Figure 40, a changing current in one will induce an emf in the other. We apply Faraday's law to coil 2: the emf ε_2 induced in coil 2 is proportional to the rate of change of magnetic flux passing through it. A changing flux in coil 2 is produced by a changing current in coil 1. So ε_2 is proportional to the rate of change of the current in coil 1:

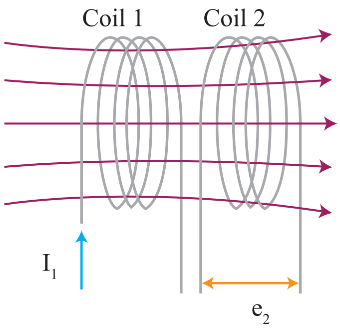


Figure 40.

$$\varepsilon_2 = -M\frac{\Delta I_1}{\Delta t}$$

The constant of proportionality, M, is called the mutual inductance. (The minus sign is because of Lenz's law, the induced emf opposes the changing flux.) Mutual inductance has units of which is called the henry (H), after Joseph Henry:

$1\ \text{H} = 1\ \Omega.\text{s}$

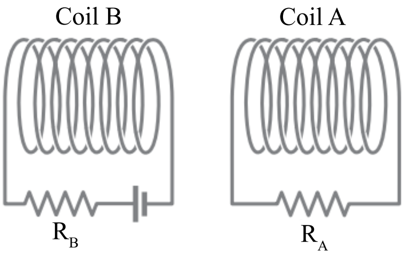
Note: The mutual inductance M is a "constant" that depends on "geometric" factors such as the size, shape, number of turns, and relative positions of the two coils, and also on whether iron (or other ferromagnetic material) is present.

Applications

1. A transformer is one example in which the coupling is maximized so that nearly all magnetic flux lines pass through both coils.
2. Other uses include inductive charging of cell phones, electric cars, and devices with rechargeable batteries.

Exercises

In Figure 41, determine the direction of the induced current in resistor R_A (a) when coil B is moved toward coil A, and (b) when coil B is moved away from A.



(a) *(b)*

Figure 41.

Explanation

(a) The induced current in RA is to the right as coil B is moved toward coil A. As B approaches A, the magnetic flux through coil A increases (there are now more magnetic field lines in coil A pointing to the left). The induced emf in coil A creates a current to produce a magnetic field that opposes this increase in flux, with the field pointing to the right through the center of the coil. A current through RA to the right will produce this opposing field.

(b) The induced current in RA is to the left as coil B is moved away from coil A. As B recedes from A, the magnetic flux through coil A decreases (there are now fewer magnetic field lines in coil A pointing to the left). The induced emf in coil A creates a current to produce a magnetic field that opposes this decrease in flux, pointing to the left through the center of the coil. A current through RA to the left will produce this opposing field.

Self-Inductance: The concept of inductance applies also to an isolated single coil. When a changing current passes through a coil or solenoid, a changing magnetic flux is produced inside the coil, and this in turn induces an emf. This induced emf opposes the change in flux (Lenz's law); it is much like the back emf generated in a motor. (For example, if the current through the coil is increasing, the increasing magnetic flux induces an emf that opposes the original current and tends to retard its increase.) The induced emf is proportional to the rate of change in current (and is in the direction opposed to the change, hence the minus sign):

$$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

The constant of proportionality L is called the self-inductance, or simply the **inductance** of the coil. Like mutual inductance, L is measured in henrys (H). The magnitude of L depends on the size and shape of the coil and on the presence of an iron core.

Because of their inductance, coils are known as **inductors** and are widely used in electronics to prevent the hazards of the sudden current changes. In circuit diagrams, the symbol for an inductor is 

Example

The coil in an electromagnet carries a constant direct current of 4 A. When a switch is suddenly opened, the current drops to zero over a time of 2.50 ms. If the magnitude of the average induced emf during this time is 8.0 V, what is the inductance of the coil?

$I_1 = 4$ A $\Delta t = 2.50$ ms $= 2.50 \times 10^{-3}$ s L =?

$I_2 = 0$ $\varepsilon = 8.0$ V

Solving the equation $\varepsilon = -L(\Delta I/\Delta t)$ for L, we get

$$L = -\varepsilon\frac{\Delta t}{\Delta I} = -(8.0\ \text{V})\frac{(0 - 2.50 \times 10^{-3}\ \text{s})}{4\ \text{A}} = 5 \times 10^{-3}\ \text{H}$$

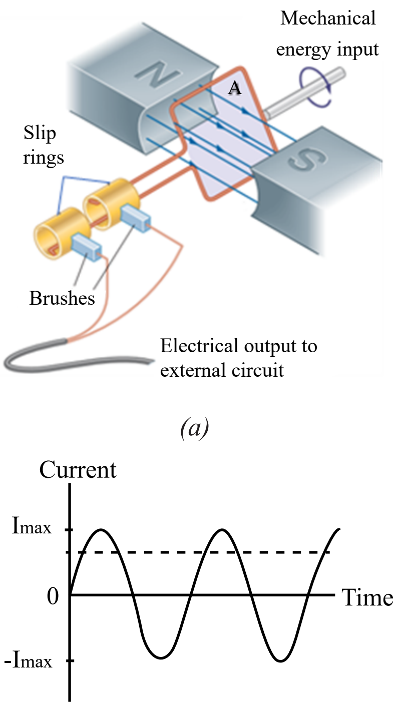
L = 5 mH

Energy stored in an inductor: Work is required to establish a current in an inductor. For example, consider a circuit that consists of just two elements—a battery and an inductor. Even though there are no resistors in the circuit, the battery must still do electrical work to force charge to flow through the inductor, in opposition to its self-induced back emf. What happens to the energy expended to increase the current? We know that it can't be lost; after all, energy is always conserved. We also know that it hasn't been converted to thermal energy (dissipated to heat), because there is no resistance in the circuit. So where did the energy go? It turns out that the energy is stored in the magnetic field, just as the energy in a capacitor is stored in its electric field. The energy stored in an inductor of inductance L is given by

$$\text{Energy} = \frac{1}{2}LI^2$$

Discuss 10: Is more energy stored in an inductor by doubling the inductance or by doubling the current?

Electric Generators: We've just seen that a changing magnetic flux can serve as a means of converting mechanical work to electrical energy. An electric generator is a device designed to convert mechanical energy to electrical energy. The mechanical energy can be supplied by any of a number of sources, such as falling water in a hydroelectric dam, expanding steam in a coal-fired power plant, or the output of a small gasoline motor that powers a portable generator.



(a)

(b)

Figure 42.

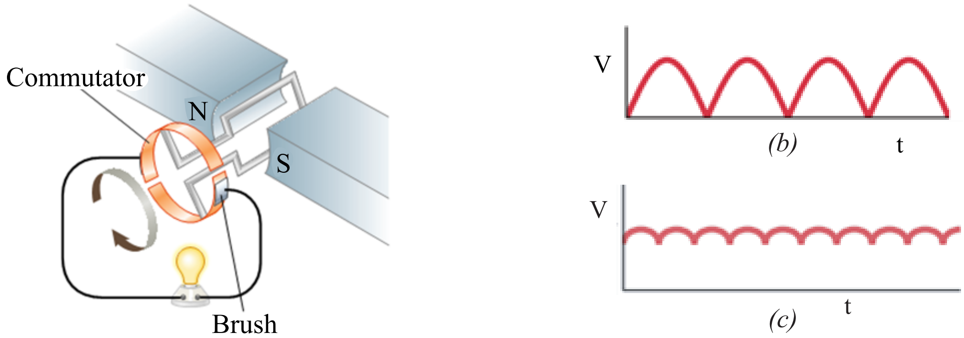
In a simple alternating-current (ac) generator (Figure 42 (a)), a coil of wire is rotated in a uniform magnetic field. The coil of wire (called the armature) is connected to a pair of rotating contacts called **slip rings**, which rub against stationary contacts called **brushes**. As the armature rotates in the magnetic field, the flux through the coil continuously changes. This induces a changing emf, which appears at the brush contacts.

When a generator is connected to an external circuit, the induced emf produces a current whose direction changes continuously (Figure 42 (b)). The graph is similar to a sine curve. The current it produces is a sinusoidally alternating current (ac).

- Generators and motors are almost identical in construction, but they convert energy in opposite directions. A generator converts mechanical energy to electrical energy while a motor converts electrical energy to mechanical energy.

A dc generator is much like an ac generator, except the slip rings are replaced by split-ring commutators (Figure 43 (a)). The output of such a generator is as shown and can be smoothed out by placing a capacitor in parallel with the output. More common is the use of many armature windings which produces a smoother output (Figure 43 (c)).

The symbol for ac source is: 



(a)

(b)

(c)

Figure 43.

Discuss 11: What is the difference between the two circuit diagrams shown in Figure 44?.

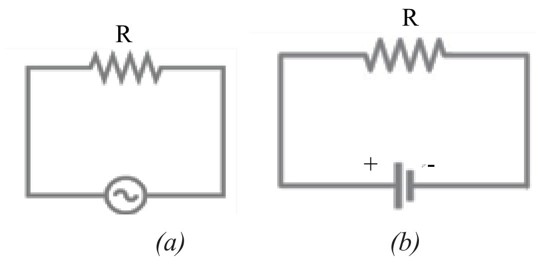


Figure 44.

Motors

An electric motor changes electric energy into mechanical energy. A current-carrying loop in a magnetic field experiences a torque that tends to make it rotate. If such a loop is mounted on an axle, it is possible for the magnetic torque to be applied to the external world.

AC Motors: To see just how this works in practice, notice that the torque exerted on the loop at the moment shown in Figure 45 (a) causes it to rotate clockwise toward the vertical position. Once it reaches this orientation, and continues past it due to its angular momentum, the alternating current from the electrical input reverses direction. This reverses the torque on the loop and causes the loop to rotate away from the vertical - which means it is still rotating in the clockwise sense. The next time the loop becomes vertical, the current from the ac source again reverses, causing the loop to continue rotating clockwise. The result is an axle continually turning in the same direction.

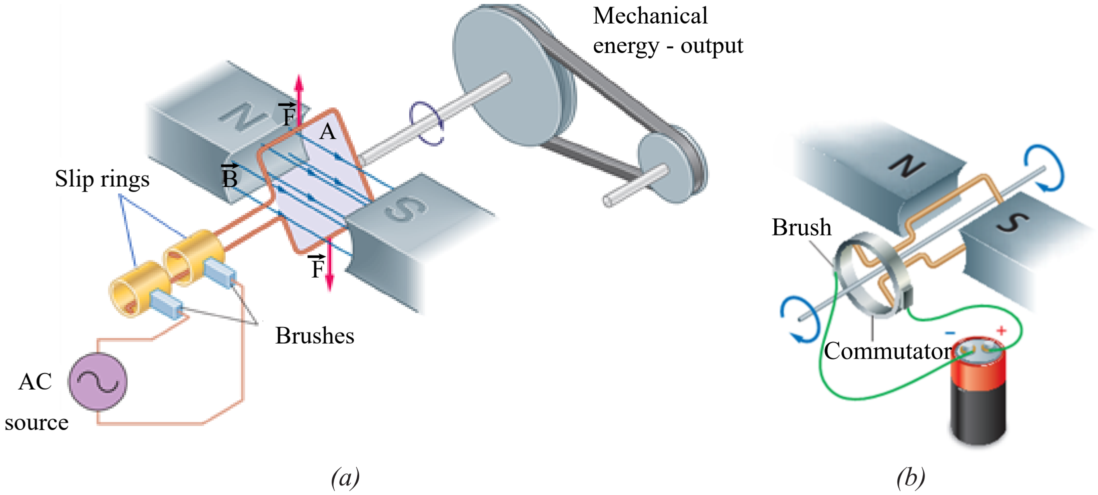


Figure 45.

DC Motors Figure 45 (b) shows a simple type of direct-current (dc) motor. The coil is mounted on an iron cylinder called the armature. The armature is mounted on a shaft or axle. In a dc motor, the wire loop must rotate a full 360° in the magnetic field; thus, the current through the loop must reverse direction just as the loop reaches its vertical position. This can be achieved in a dc motor with the use of commutators and brushes: as shown in Figure 45 (b), input current passes through stationary brushes that rub against the conducting commutators mounted on the motor shaft. At every half revolution, each commutator changes its connection over to the other brush. Thus, the current in the coil reverses every half revolution as required for continuous rotation.

Discuss 12:

(i) The current in the armature of an electric motor changes direction with each half-cycle of rotation of the coil. Which part of the motor produces this change in direction?

(a) The magnet (c) The armature

(b) The commutator (d) The brush

(ii) Which of the following produces alternating current?

(a) Electromagnet (c) Superconductor

(b) Generator (d) motor

Transformers

A transformer is a device for increasing or decreasing an ac voltage. Transformers are found everywhere to reduce the high voltage from the electric company to a usable voltage in houses (120V to 240 V), in chargers for cell phones, laptops, and other electrical devices. A transformer consists of two coils of wire known as the primary and secondary coils both wound on the same iron core. Transformers are designed so that all the magnetic flux produced by the current in the primary coil also passes through the secondary coil. We also assume that energy losses can be ignored.

When an ac voltage is applied to the primary coil, the changing magnetic flux it produces will induce an ac voltage of the same frequency in the secondary coil. However, the voltage will be different according to the number of "turns" or loops in each coil. From Faraday's law, the voltage or emf induced in the secondary coil is

$$V_S = -N_S \frac{\Delta \Phi_B}{\Delta t}$$

Where N_S is the number of turns in the secondary coil, and $\Delta\Phi/\Delta t$ is the rate at which the magnetic flux changes.

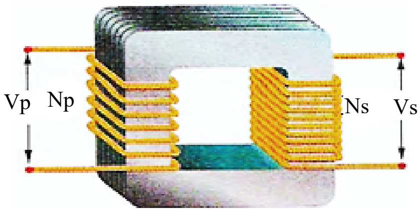
Similarly, the input primary voltage, V_p, is

$$V_P = -N_P \frac{\Delta\Phi_P}{\Delta t}$$

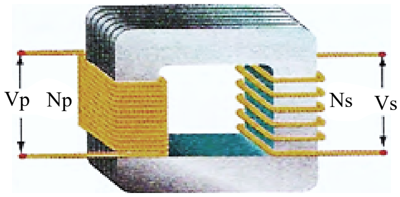
where N_P is the number of turns of the primary coil.

Dividind these two equations, we get

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$



(a)



(b)

Figure 46.

If the secondary coil contains more loops than the primary coil, we have a **step-up transformer** (Figure 46 (a)). In this case, the secondary voltage is greater than the primary voltage. If it is less than we have a **step-down transformer** Figure 46 (b)).

In an ideal transformer, the electric power delivered to the secondary circuit equals the power supplied to the primary circuit. An ideal transformer dissipates no power itself.

$$P_{input} = P_{output} \qquad (P = VI)$$

$$V_p I_p = V_s I_s$$

Rearranging the terms, we get

$$\frac{I_S}{I_P} = \frac{V_P}{V_S} \qquad \text{The transformer equation}$$

Example

A step-up transformer has a primary coil consisting of 200 turns and a secondary coil that has 3000 turns. The primary coil is supplied with an AC voltage of 90 V. (a) What is the voltage in the secondary circuit? (b) The current in the secondary circuit is 2 A. What is the current in the primary circuit? (c) What is the power in the primary circuit?

Solution

$N_P = 200$ $\qquad V_P = 90$ V

$N_S = 3000$ $\qquad I_S = 2$ A

(a) From the transformer equation, we solve for V_S.

$$V_S = \frac{N_S}{N_P} \times V_P = \frac{3000}{200} \times (90\ V) = 1350\ V$$

(b) Assuming that the transformer is perfectly efficient, the power in the primary and secondary circuits is the same. Thus,

$V_P I_P = V_S I_S$

$$I_P = \frac{V_S}{V_P} I_S = \frac{1350\ V}{90\ V}(2\ A) = 30\ A$$

(c) We use the power equation to solve for P_P

$P_P = V_P I_P = (90\ V)\ (30\ A) = 2700\ W$

$P_P = 2.7\ kW$

Example

A transformer has 50 turns on its primary and 100 turns on its secondary. (a) If the primary is connected to a 120 V source, what is the voltage output of the secondary? (b) If the transformer is operated in reverse, and the 120 V input is applied to the coil with 100 turns, what would be the voltage output?

Solution

$N_P = 50$ $V_P = 120\ V$

$N_S = 100$ $V_S = ?$

(a) From the transformer equation, we solve for V_S

$$V_S = \left(\frac{N_S}{N_P}\right) V_P = \left(\frac{100}{50}\right) \times 120\ V = 240\ V$$

The voltage is increased by a factor of 2.

(b) When operated in reverse, the secondary and the primary coils are interchanged.

$N_P = 100$, $N_S = 50$, $V_P = 120\ V$, and $V_s = ?$

$$V_S = \left(\frac{N_S}{N_P}\right) V_P = \left(\frac{50}{100}\right) \times 120\ V = \frac{1}{2} \times 120\ V$$

In this case, the turn ratio is $N_S/N_P = 1/2$ and we have a step-down transformer.

Example

A transformer has 100 turns in the primary and 400 turns in the secondary winding. If the current in the primary is 4A, what will be the current in the secondary winding?

Solution

$N_P = 100$ $I_P = 4$ A

$N_S = 400$ $I_S = ?$

For ideal transformers, we have found the following equation

$$\frac{I_P}{I_S} = \frac{V_S}{V_P} \quad \text{but} \quad \frac{V_S}{V_P} = \frac{N_S}{N_P}$$

Therefore,

$$\frac{I_P}{I_S} = \frac{N_S}{N_P}$$

$$I_S = \left(\frac{N_P}{N_S}\right) I_P = \frac{100}{400} \times 4\ \text{A} = 1\ \text{A}$$

Example

A 150 W transformer has an input voltage of 10 V and an output current of 5 A. (a) Is this a step-up or step-down transformer? (b) What is the ratio of V_{output} to V_{input}?

Solution

$P = 150$ watt $I_s = 5$ A (output current)

$V_p = 10$ V (input voltage)

(a) To decide the type of the transformer, we compare the output voltage to the input voltage. Since

$P_{out} = V_{out} I_{out}$

$150\ \text{W} = V_{out} \times 5\ \text{A}$

$$V_{out} = \frac{150\ \text{W}}{5\ \text{A}} = 30\ \text{V}$$

It is a step-up transformer, because $V_{out} > V_{in}$

(b) $$\frac{V_{output}}{V_{input}} = \frac{30\ \text{V}}{10\ \text{V}} = 3$$

or $V_{output} : V_{input} = 3:1$

Summary

- A magnet is characterized by two poles, referred to as the north pole and the south pole.

- Magnetic field lines point away from north poles and toward south poles, and always form closed loops.
- The direction of a magnetic field at a given location is defined as the direction a compass needle would point if placed at that location.
- Breaking a magnet in half produces two new poles on either side of the break.
- Magnetic fields can be represented with lines in much the same way as electric fields. In particular, the more closely spaced the lines, the more strong the field.
- Like magnetic poles repel; unlike magnetic poles attract.
- Magnetic materials can be magnetized using one of the following methods: By stroking with a permanent magnet, touching with a magnet, and electrical method.
- Heating, hammering, and dropping a magnet destroys its magnetism.
- A galvanometer consists of a loop of wire in a magnetic field, and is used to measure small currents. When current is passed through the loop, a force on the wire loop results in a deflection of the loop.
- The source of magnetic field are moving electrons (or charges).
- In order for a magnetic field to exert a force on an object, the object must have charges moving through it.
- To find the direction of the magnetic field due to a current - carrying wire, point the thumb of your right hand along the wire in the direction of the current, I. Your fingers will then curl around the wire in the direction of the magnetic field.
- Wires with currents in the same direction experience an attractive force; wires with currents in opposite directions experience a repulsive force.
- Magnetic flux is a measure of the number of magnetic field lines that cross a given area.
- Magnetic flux depends on the magnitude of the magnetic field, B, its orientation with respect to a surface, θ, and the area of the surface, A.

  $\Phi = BA\cos\theta$
- A wire of length L carrying a current I at an angle u to a magnetic field B experiences a force given by

  $F = BIL \sin\theta$
- The direction of the magnetic field produced by a current is found by pointing the thumb of the right hand in the direction of the current. Then the fingers of the right hand curl in the direction of the field.
- A long, straight wire carrying a current I produces a magnetic field of magnitude B given by

$$B = \frac{\mu_o I}{2\pi r}$$

- A current loop placed in a magnetic field experiences a torque that depends on the relative orientation of the plane of the loop and the magnetic field.
- The magnetic field inside a solenoid is strong and nearly uniform.
- Faraday observed that the magnitude of the induced emf is proportional to the rate of change of the magnetic field—the more rapidly the magnetic field changes, the greater the induced emf.
- If the magnetic flux in a coil of N loops changes by the amount in the time Δt, the induced emf is

  $$\varepsilon = -N\frac{\Delta \Phi}{\Delta t}$$

- When the current in a coil changes with time, an emf is induced in the coil according to Faraday's law. This self-induced emf is defined by the expression

  $$\varepsilon = -L\frac{\Delta I}{\Delta t}$$

  where L is the inductance of the coil. The SI unit for inductance is the henry (H); 1 H = 1 V.s/A.
- Lenz's law states that an induced current always flows in a direction that opposes the change that caused it.
- All generators use the same basic operating principle - mechanical energy moves a conductor through a magnetic field to produce a motional emf.
- An electric motor transforms energy from electric emf and current into mechanical motion. It follows that an electric motor is basically an electric generator run in reverse.
- A transformer is an electrical device that changes the voltage in an AC circuit.
- The transformer equation shows that if the number of loops in the secondary coil is less than the number of loops in the primary coil, the voltage is stepped down to a lower value. Similarly, if the number of loops in the secondary coil is higher, the voltage is stepped up to a higher value.
- If a transformer increases the voltage by a given factor, it decreases the current by the same factor. Similarly, if it decreases the voltage, it increases the current.

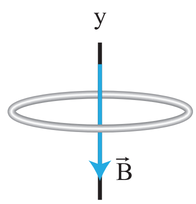
  $$\frac{V_S}{V_P} = \frac{N_P}{N_S} = \frac{I_S}{I_P}$$

Review Exercises

I Conceptual Questions

1. Which one of the following is a magnetic material?
   (a) Copper
   (b) Glass
   (c) Wood
   (d) Cobalt
2. A drawing of the lines of force of a magnetic field provides information about
   (a) the direction of the field only
   (b) the source of the field
   (c) the magnitude of the field only
   (d) both the direction and the magnitude of the field
3. Your teacher tells you to check an unknown sample with a bar magnet. If the sample is attracted by both poles of the bar magnet, it must be
   (a) a magnetic material
   (b) a nonmagnetic material
   (c) an insulator
   (d) a magnet.
4. A magnetic field is produced by
   (a) iron atoms
   (b) non-magnetic substances
   (c) magnetic substances
   (d) moving electric charges
5. Can magnetic field lines cross one another?
6. Explain the difference between a magnetic field and a magnetic flux.
7. Which one of the following is the unit of magnetic flux?
   (a) $\frac{N.A}{m^2}$
   (b) $\frac{N.m}{A}$
   (c) $\frac{T.m}{A}$
   (d) $\frac{T}{m^2}$
8. A uniform magnetic field of flux density 0.20 T makes an angle of 90^o with the plane of a square loop of wire whose side is 10 cm. What is the flux through the loop?
   (a) 2×10^{-2} Wb
   (b) 2×10^{-1} Wb
   (c) 2×10^{-3} Wb
   (d) 2 Wb

9. The magnetic fields of a straight current carrying conductor are
   (a) circular around the conductor
   (b) parallel to the conductor
   (c) perpendicular to the conductor
   (d) A and C are correct
10. Which of the following does not affect the strength of an electromagnet?
    (a) Increasing the number of turns of the wire
    (b) Reversing the current direction
    (c) Inserting an iron core
    (d) Decreasing the current.
11. The magnetic field produced by the current loop in the figure shown is along the y direction. What is the direction of the current in the loop? That is, is it clockwise or is it counterclockwise as viewed from above?



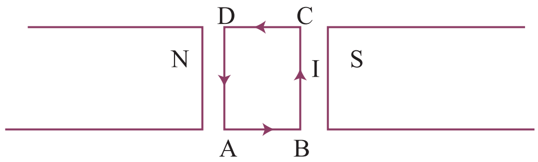
12. A coil of wire lies flat on a table top. You hold a bar magnet vertically above the center of the coil with the south pole down toward the coil. You then move the magnet toward the coil along its axis. What is the direction of the induced current in the coil?
13. A horizontal wire carries a current directly away from you. From your view point, the magnetic field produced by this current
    (a) points directly away from you
    (b) points directly towards you
    (c) circles the wire in a clockwise direction
    (d) circles the wire in a counter clockwise direction
14. Describe how to use the right-hand rule to determine the direction of force on a current-carrying wire placed in a magnetic field.
15. A wire 1.00 m long carries a current of 10 A and makes an angle of 37° with a uniform magnetic field B = 1.50 T. What is the magnetic force on the wire?
    (a) 15 N (c) 6 N
    (b) 9 N (d) 3 N
16. A loop of wire is placed in a uniform magnetic field. (a) For what orientation of the loop is the magnetic flux a maximum? (b) For what orientation is the flux zero?
17. Discuss the different ways of producing an emf in a coil of wire.
18. A current-carrying wire is placed in a uniform magnetic field. The wire experiences zero magnetic force. Why?

19. Which of the following is used to convert electrical energy into mechanical energy?
    (a) Motor
    (b) Transformer
    (c) Generator
    (d) Electromagnet

20. The purpose of a split ring commutator in a simple D.C. motor is to
    (a) give a strong magnetic field
    (b) ensure smooth running
    (c) reverse the current in the coil every half revolution
    (d) keep the current flowing in one direction in the coil

21. A uniform magnetic field of magnitude 0.20 T is perpendicular to a circular loop of wire of area 2×10^{-2} m^2. If the magnetic field decreases to zero in 1ms, what is the emf induced in the loop?
    (a) 1 V
    (b) 2 V
    (c) 4 V
    (d) 2.5 V

22. A rectangular coil of wire carrying a current I is placed between the poles of a permanent magnet as shown in the figure below.



    The magnetic field generates a torque that rotates coil by moving
    (a) side AB out of the page and side CD into the page.
    (b) side AB into the page and side CD out of the page.
    (c) side BC into the page and side DA out of the page.
    (d) side BC out of the page and side DA into the page.

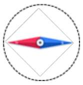
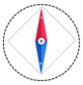
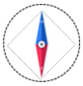
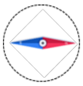
23. The self- inductance of a coil depends on
    (a) the current through the coil
    (b) the flux through the coil
    (c) the geometry of the coil
    (d) the voltage applied to the coil

II Problems

24. A magnetic field of strength 0.30 T is directed perpendicular to a plane circular loop of wire of radius 25 cm. Find the magnetic flux through the area enclosed by this loop.

25. A square coil of wire with 10 turns is in a magnetic field of 0.25 T. The total flux through the coil is 0.50 $T.m^2$. Find the area of the coil if the field is perpendicular to the plane of the coil.
26. How much current will be needed to produce a force of 1.2 N on a 20 cm long piece of wire at right angles to a 0.5 T field?
27. If the current through a coil having a self- inductance of 0.5 H is reduced from 5 A to 2 A in 0.05 s, calculate the average value of the induced emf in the coil.
28. What current is required in the windings of a long solenoid that has 1000 turns uniformly distributed over a length of 0.4 m in order to produce a magnetic field of magnitude 0.1 mT at the center of the solenoid?
29. A uniform magnetic field B = 0.2 T is perpendicular to the plane of a circular loop of wire of radius 10 cm. If the field decreases to zero in 0.50 s, what emf is induced in the loop?
30. Two coils have a mutual inductance of 400 μH. Find the emf induced in one coil when the current in the other coil varies at a rate of 30,000 A/s.
31. A transformer has 90 turns on the primary coil and 18,000 turns on the secondary coil. (a) Is this a step-down or a step-up transformer? (b) If the voltage in the secondary coil is 3.2 kV, what is the voltage being delivered to the primary coil?
32. A transformer has 250 turns on its primary and 500 turns on its secondary. If the primary is connected to a 220 V source, what is the output voltage of the secondary?
33. A transformer on a pole near to a factory steps the voltage down from 3600 V to 200V. What is the current in the primary coil when the transformer delivers 900 kW to the factory?

Sample Test

1. Where is the magnetic force exerted by a magnet strongest?
   (a) At both poles
   (b) Only at the north pole.
   (c) Only at the south pole
   (d) At the center
2. What will the direction of the compass needle be if it is placed between the arms of the U-shaped magnet shown in the figure?
   (a) 
   (b) 
   (c) 
   (d) 

3. If a bar magnet is divided into two equal pieces,
   (a) the north and south poles are separated.
   (b) two magnets are obtained.
   (c) the magnet properties are destroyed.
   (d) an electric field is created.
4. The south pole of a magnet points toward the Earth's
   (a) south Pole.
   (b) north Pole.
   (c) equator.
   (d) center.
5. Which of the following statements best describes the domains in unmagnetized iron?
   (a) There are no domains.
   (b) There are domains, but the domains are smaller than in magnetized iron.
   (c) There are domains, but the domains are oriented randomly.
   (d) There are domains, but the domains are not magnetized.
6. The SI unit of magnetic field, B, is the
   (a) weber
   (b) gauss

(c) watt

(d) tesla

7. A uniform 4.5 T magnetic field passes perpendicularly through the plane of a wire loop 0.10 m^2 in area. What flux passes through the loop?

(a) 5.0 $T.m^2$
(b) 0.45 $T.m^2$
(c) 0.25 $T.m^2$
(d) 0.135 $T.m^2$

8. A magnetic field is generated by a current-carrying wire. Which one of the following statements concerning this situation is false?

(a) The magnitude of this magnetic field decreases with increasing distance away from the wire.

(b) A right-hand rule is useful for determining the direction of the magnetic field at a particular location.

(c) The magnitude of the magnetic field is directly proportional to the magnitude of the current.

(d) The magnetic field is parallel to the direction of the current in the wire.

9. A long wire carries a current of 1 A. What is the magnitude of the magnetic field 20 cm away from the wire?

(a) 1×10^{-3} T
(b) 1×10^{-4} T
(c) 1×10^{-6} T
(d) 1×10^{-7} T

10. A vertical wire carries a current straight down. To the east of this wire, the magnetic field points

(a) north.
(b) east.
(c) west.
(d) south.

11. A 2.0 m wire carrying a current of 0.60 A is oriented parallel to a uniform magnetic field of 0.50T. What is the magnitude of the force it experiences?

(a) zero
(b) 0.15 N
(c) 0.30 N
(d) 0.60 N

12. The force on a current-carrying wire in a magnetic field is the strongest when the current is

(a) parallel to the field lines.

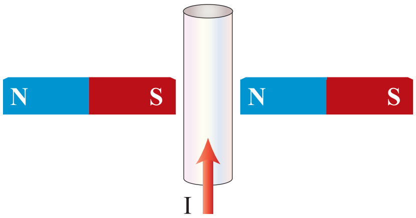
(b) at a 30° angle with respect to the field lines.

(c) at a 60° angle with respect to the field lines.

(d) perpendicular to the field lines.

13. A wire carries a current of 10 A in a direction of 30° with respect to the direction of a 0.30 T magnetic field. The magnitude of the magnetic force on a 0.50 m length of the wire is

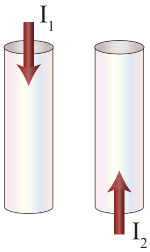
(a) 0.75 N
(b) 1.5 N
(c) 3.0 N
(d) 6.0 N



14. A current-carrying wire is placed between the poles of a permanent magnet as shown in the figure below. What is the direction of the magnetic force on the wire?
    (a) Into the page
    (b) Out of the page
    (c) To the left
    (d) To the right

15. A copper wire of length 25 cm is in a uniform magnetic field of 0.20 T. If it has a mass of 10 g, what is the minimum current through the wire that would cause a magnetic force equal to its weight?
    (a) 1.3 A (c) 2.0 A
    (b) 1.5 A (d) 4.9 A

16. Wire 1 carries current I_1 and creates magnetic field B_1. Wire 2 carries current I_2 and creates magnetic field B_2 (see the figure below). What is the direction of the magnetic field B_1 at the location of wire 2?



    (a) to the left
    (b) to the right
    (c) into the page
    (d) out of the page

17. Which of the following would not change the strength of an electromagnet?
    (a) increasing the amount of current.
    (b) changing the currents direction.
    (c) inserting an iron core inside the coil.
    (d) increasing the number of loops.

18. In the previous question, what is the direction of the force on wire 2 as a result of B_1?
    (a) to the left
    (b) to the right
    (c) into the page
    (d) out of the page

19. What is the magnitude of the magnetic field at the core of a 120-turn solenoid of length 0.50 m carrying a current of 2.0 A?

(a) 2.4×10^{-4} T (c) 6.0×10^{-4} T

(b) 4.8×10^{-5} T (d) 3.6×10^{-5} T

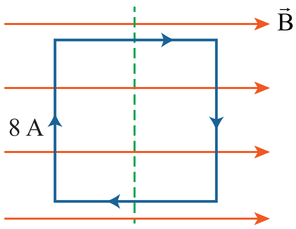
20. A rectangular wire 10 cm by 20 cm carrying an 8 A current is oriented with its plane parallel to a uniform 0.8 T magnetic field as shown in the figure below. What is the net (total) force acting on the rectangular loop?

(a) 5.12 N

(b) 2.56 N

(c) 1.28 N

(d) zero



21. Consider two long, straight parallel wires, each carrying a current I. If the currents are flowing in the same direction

(a) the two wires will attract each other.

(b) the two wires will repel each other.

(c) the two wires will exert a torque on each other.

(d) neither wire will exert a force on the other.

22. Faraday's law of induction states that the emf induced in a loop of wire is proportional to

(a) the magnetic flux.

(b) the magnetic flux density times the loop's area.

(c) the time variation of the magnetic flux.

(d) current divided by time.

23. A square coil of wire with 15 turns and an area of 0.40 m^2 is placed parallel to a magnetic field of 0.75 T. The coil is oriented so that its plane is perpendicular to the magnetic field in 0.050 s. What is the magnitude of the average induced emf?

(a) 6.0 V (c) 45 V

(b) B. 36 V (d) 90 V

24. According to Lenz's law the direction of an induced current in a conductor will be that which tends to

(a) enhance the effect which produces it

(b) produce a greater heating effect

(c) produce the greatest voltage

(d) oppose the effect which produces it.

25. A 10-turn square coil of area 0.036 m^2 and a 20-turn circular coil are both placed perpendicular to the same changing magnetic field. The voltage induced in each of the coils is the same. What is the area of the circular coil?
    (a) 0.072 m^2
    (b) 0.60 m^2
    (c) 0.018 m^2
    (d) 0.036 m^2
26. A flat coil of wire consisting of 20 turns, each with an area of 50 cm^2, is positioned perpendicularly to a uniform magnetic field that increases its magnitude at a constant rate from 2.0 T to 6.0 T in 2.0 s. If the coil has a total resistance of 0.40 Ω, what is the magnitude of the induced current?
    (a) 0.070 A
    (b) 0.140 A
    (c) 0.50 A
    (d) 0.80 A
27. If a bar magnet is falling through a loop of wire, the induced current in the loop of wire sets up a field which exerts a force on the magnet. This force between the magnet and the loop will be attractive when
    (a) the magnet enters the loop.
    (b) the magnet is halfway through.
    (c) the magnet is leaving the loop.
    (d) the magnet stays inside the loop.
28. The current in a coil with a self-inductance of 1.5 mH increases from 0 to 1.0 A in a tenth of a second. What is the induced emf in the coil?
    (a) 15 mV
    (b) 30 mV
    (c) 0.10 V
    (d) 0.30 V
29. What is the self-inductance in a coil that experiences a 3.0-V induced emf when the current is changing at a rate of 110 A/s?
    (a) 83 mH
    (b) 45 mH
    (c) 37 mH
    (d) 27 mH
30. What is the energy stored in a 4.0-mH inductor when the current through it is 4.0 A?
    (a) 16 mJ
    (b) 32 mJ
    (c) 48 mJ
    (d) 64 mJ
31. A step-up transformer is used on a 120 V line to provide a potential difference of 2400 V. If the primary has 75 turns, how many turns must the secondary have?
    (a) 1500
    (b) 1200
    (c) 1000
    (d) 500

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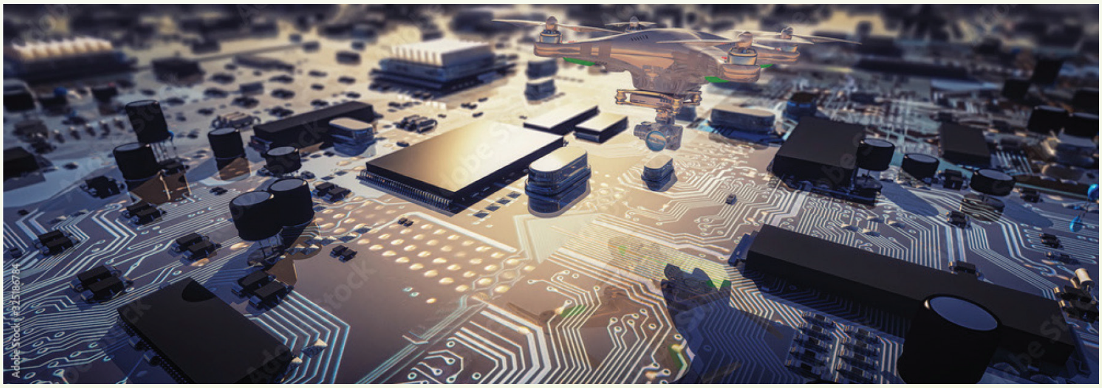
CHAPTER 4

ALTERNATING CURRENT AND ELECTRONICS

Chapter Contents



Chapter Outcome

Learners will able to:

- recognize usage of alternating current and the application of transistors in a circuit.

Chapter Objectives

After completing this chapter, you will be able to:

- identify electrical measuring instruments and state their uses;
- demonstrate the functions of alternating current;
- distinguish between resistance and impedance;
- analyze the R-C-L series circuits and the phase diagram of the (R-C-L) Circuits;
- analyze the principle of the cathode ray tube and applications of transistors and diodes.

Introduction

The electricity supply to our homes is an alternating but a number of appliances need steady potential difference. The conversion of ac to dc is known as rectification. This may be achieved in a number of ways but the study of Semiconductors has led to a great achievement in the study of electronics. In addition, most signals in nature like, sound, temperature, pressure are continuously changing with time, and can be represented as combination of sinusoidal variations. In this unit you will be familiar with basic ac circuit components and with their corresponding working principles.

In previous unit you have learnt that the output e.m.f. of a typical ac generator has a sinusoidal pattern. It is a common wave form in most electronic circuits. But alternating currents and potential differences which are not sine waves can be produced by electronic circuits. Figure 1 shows other possible wave forms. Each of them change the directions and magnitude periodically

Key Terms

- Alternating current.
- Waveform.
- Rectification
- Impedance
- Power factor

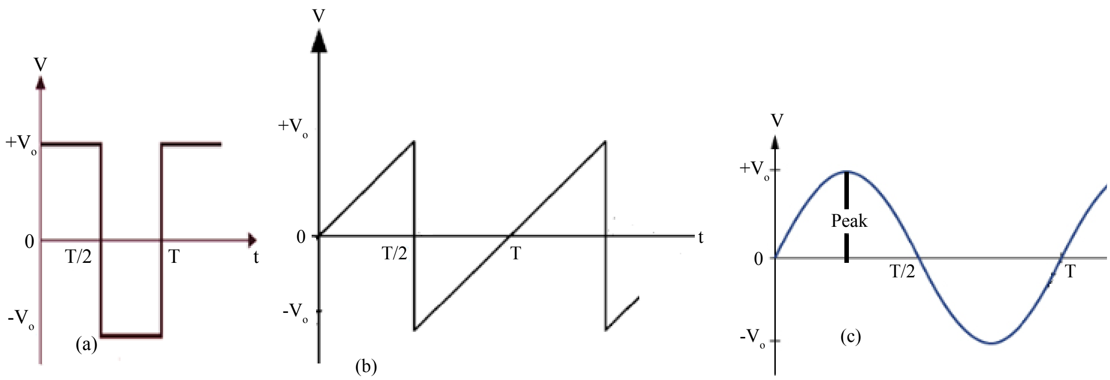


Figure 1. Alternating voltage wave forms. Non-sinusoidal: a) square wave form; b) saw tooth wave form; c) Sinusoidal wave form.

Square wave forms appear in computer circuits, saw tooth wave forms in TV receiver circuits and sinusoidal wave forms in heaters.

Note: Wave form is a line graph which describe the value of some electrical quantity as a function of time.

Activity 1

What is the type of ac voltage wave form from the electrical outlet in your home?

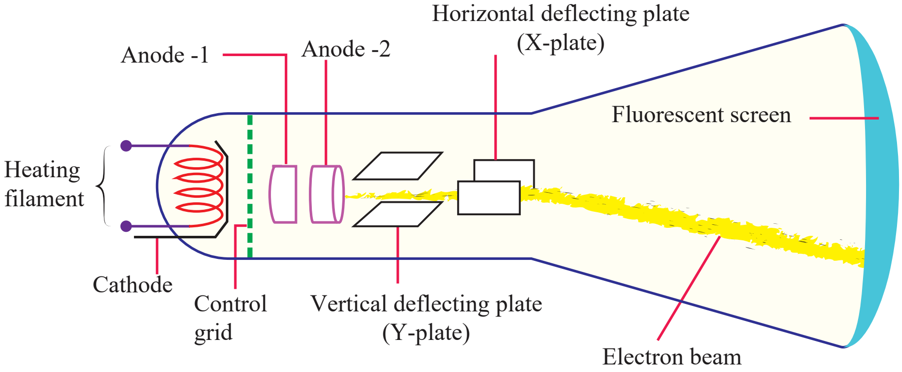


Figure 2. Schematic diagram of cathode ray oscilloscope (C.R.O)

If an alternating voltages is connected across the y-plates a corresponding wave form is displayed on the screen.

ACTIVITY 2

(Demonstration)

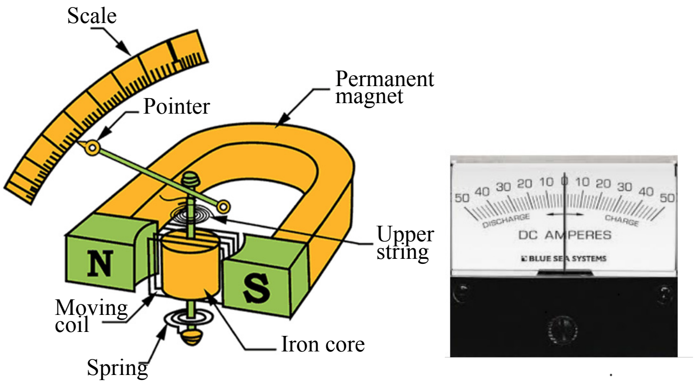
In using an oscilloscope for measurement, to which plate (x or y) does the voltage, the waveform of which is to be analyzed, is applied? What is the direction of the electron beam movement?

1. Demonstrate the electron beam plot when an ac voltage is applied, and the
   (a) time base control is off
   (b) time base control is on
2. Repeat the demonstration with D.C voltage source.

Measurement of Current in ac Circuit

A. Using moving coil meter

The pointer of a center - zero reading meter swings from one side of the scale zero to the other side if it is connected in series with a resistor and a low frequency ac generator.



(a) (b)

Figure 3. Moving coil meter:
a) schematic diagram;
b) center-zero reading Ammeter

Increasing the frequency makes the pointer to oscillate even more rapidly. However, the meter cannot respond quickly enough to the changing current at frequencies above 10Hz. The pointer hardly moves at all even for large peak current.

It is possible to measure alternating current with moving coil meter if the current is first converted into direct current. Converting alternating current to direct current is called rectifying the current. Diodes are used for this purpose. They allow current to pass in one direction

In an ac ammeter where diodes are used with coil meter, the meter is usually calibrated to measure rms (root mean square) values directly from the scale.

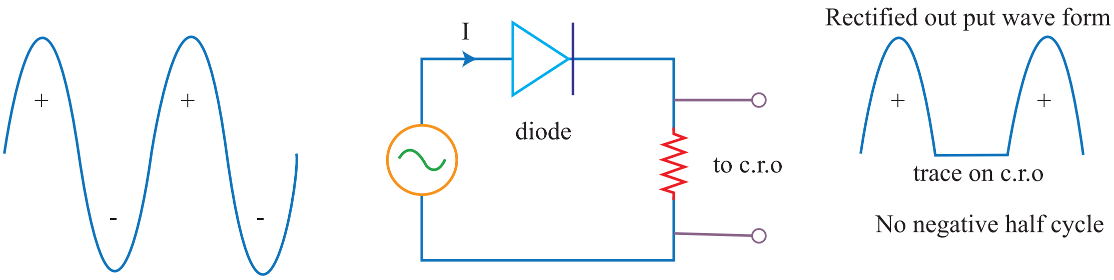


Figure 4. Use of diode in rectifying (Half wave rectification)

Activity 3

Heaters work as well with ac as with d.c. Is the Joule's heating effect similar in the two cases?

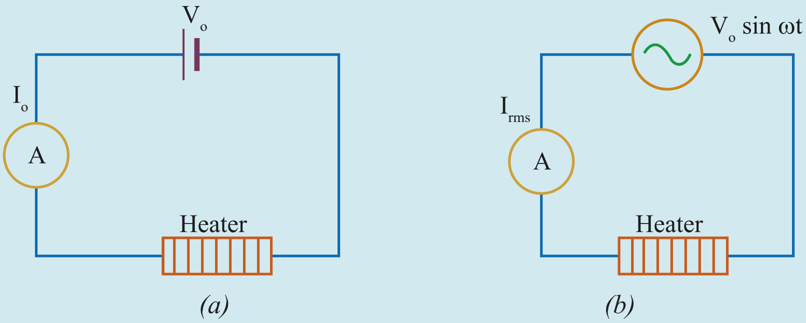


Figure 5. a) Measuring direct current b) Measuring alternating current

Power dissipated across the heater: a) $I_o^2 R$ b) I^2_{rms} R

The current that flows through the resistor connected to an ac source is given by the expression:

$$I(t) = I_o \sin \omega t \qquad 4.1$$

The time rate at which an electrical energy is converted into heat energy by a resistor through which an alternating current of peak value I_o is flowing is not the same as that of a resistor in which a direct current of the same value I_o is flowing through it. It is as if a direct current which is equal to I_{rms} is flowing through the resistor. Therefore, ac ammeters and ac voltmeters in an ac circuit, measure effective magnitude of current and effective magnitude of voltage respectively.

For current with sinusoidal wave form, I_{rms} is given by the expression:

$$I_{rms} = \frac{I_o}{\sqrt{2}} = 0.707\ I_o, \quad \text{where } I_o \text{ is the peak current} \quad 4.2$$

ACTIVITY 4

For how long is the current at its peak value during its cycle?

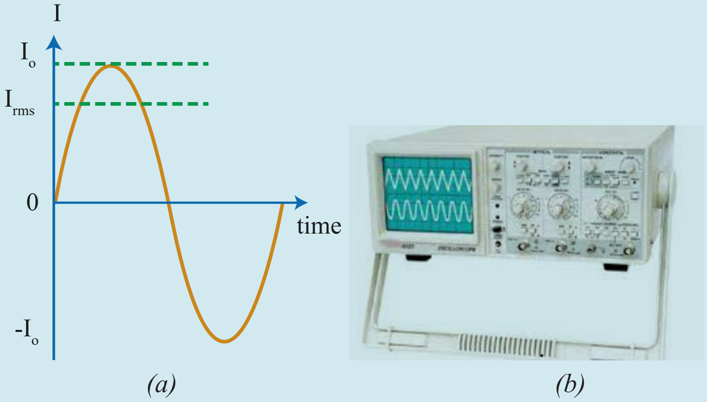


Figure 6. a) rms current compared with peak current, b) Oscilloscope at work

B. Using Oscilloscope

An Oscilloscope gives us the picture of how the current or voltage changes with time. Practically it displays (as shown in Figure 6) the wave form, from which we can calculate the rms values.

ACTIVITY 5

Library research: Describe the working principle of moving - iron Ammeter in measuring alternating current.

Functions of Alternating Current

Unlike batteries, generators can produce either direct or alternating current, depending on their design. However, alternating current has advantages that makes it more practical for use in transferring electrical energy. For this reason, the current supplied to your home is an alternating current than direct current.

ACTIVITY 6

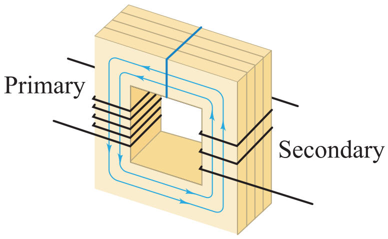
How far is the nearest power plant from your home?

(a)

All of the practical large scale electrical energy supply systems are founded on alternating current. And most countries have what is known as an electric grid which link the generating stations and distribute the electricity (See Figure 7).

When electrical energy is transmitted, some of it is lost as a heat in the line. Thus transmission at low current and low electric resistance is more economical. The current may be kept small by transmitting at high voltages. These high voltages can be achieved by the use of transformers, which may also reduce the voltage to a usable level at homes, schools, hospitals and industries after transmission. And transformers operate on alternating current.

(b)



(c)

Figure 7. (a) High voltage transmission lines,(b) transformer substation; (c) schematic diagram of transformer

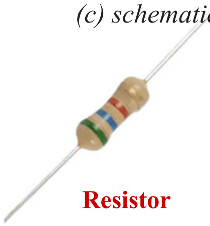
AC Circuits

We often hear about circuit board damage when gadgets are out of work. Circuit boards are studded with resistors, capacitors and other components which are involved with movement and storage of electrical energy. Copper “wires” printed on to the circuit boards conduct current between the components.

Components of ac Circuit

A. Resistor

Resistor is a passive electrical component with two terminals that are used for either limiting or regulating the flow of electric current in electrical circuits. The SI unit of resistor is ohm (Ω).



Resistor

Resistor Symbol

B. Capacitors

(a) Capacitors

Capacitors are used to control current in an ac circuit by storing energy in an electric field. The voltage across them always oppose the source voltage. This opposition is known as capacitive reactance, $x_c = \frac{1}{\omega c}$ where ω is angular velocity and $\omega = 2\pi f$ of the source, and c is capacitance.

C. Inductors, choke or coil

(b) Inductors

Inductance is an electrical property that opposes any change in magnitude of current in a circuit by storing energy in the magnetic field. The opposition is known as inductive reactance, $X_L = \omega L$, where L is self-inductance of the coil.

Both capacitors and inductors provide opposition without changing electrical energy into heat energy. However, it is impossible to build a capacitor or inductor without some resistance. Quality is then a term used to rate capacitors and inductors on their ability with little resistance as possible.

Note: Both capacitive reactance and inductive reactance are measured in units of ohm (Ω).

Impedance

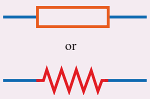
A general term used to describe opposition to current flow is impedance, Z. The opposition to the flow of current where by loss of electrical energy is exhibited is called resistance while the apparent opposition to the flow of current which is not accompanied by energy loss is known reactance.

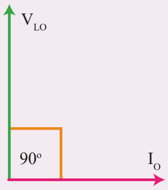
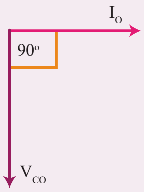
In complex number notation the impedance is written as:

$$Z = R + iX \qquad 4.3$$

Where R is the resistance and, X is the net reactance, $(X_L - X_c)$

Table 1 Voltage and current phase relation in a single circuit element with an ac source of angular velocity, ω. and angular frequency f. (ω = 2πf)

Circuit element	Impedance Z	Peak current I_o	Phase relation	Phase diagram
	R	$\frac{V_0}{R}$	In phase	

	$X_L = \omega L$ or $X_L = 2\pi fL$	$\frac{V_0}{X_L}$	I lags V by p/2	
	$X_c = \frac{1}{\omega c}$ or $X_c = \frac{1}{2\pi fC}$	$\frac{V_0}{X_C}$	I leads V by p/2	

Consider R − L − C in series circuit with an ac source. (Figure 8).

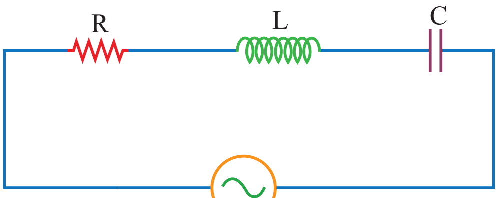
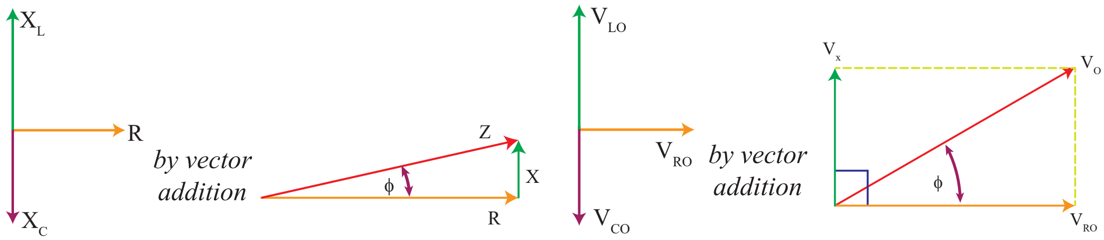


Figure 8. R - L - C in series circuit



Sum of voltages: $V_o^2 = V^2_{xo} + V^2_{Ro}$, $V_{xo} = V_{Lo} - V_{Co}$

Figure 9. (a) Vector addition of impedance *(b) Vector addition of peak voltage*

Sum of resistance and reactance:

$$Z^2 = R^2 + X^2 \qquad 4.5$$

$$Z^2 = R^2\left(1 + \left(\frac{x}{R}\right)^2\right) = R^2\ (1 + \tan^2\phi) \qquad 4.6$$

$$\cos\phi = \frac{R}{Z} \qquad 4.7$$

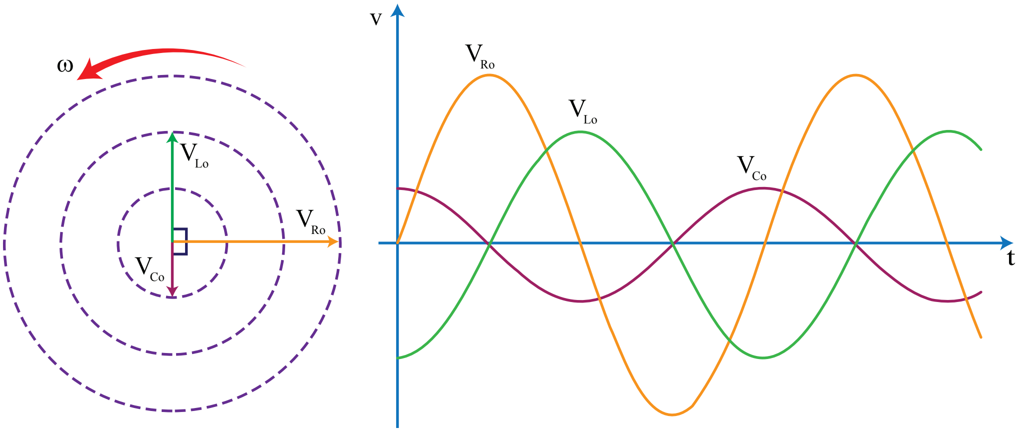


Figure 10. Comparing rotating vector representation with V vs t graph in R − L − C circuit.

Power in an ac circuit

Power loss in an ac circuit is always accounted for electric resistance. The average power delivered by the circuit is equal to the power dissipated across the resistor. The average power over one cycle is given by the expression: 90^o

$$P_{av} = I_{rms} V_{rms} \cos\phi \qquad 4.8$$

The quantity cos φ is called power factor.

Activity 7

(a) Show that $I_{rms} V_{rms} \cos\phi = I^2_{rms} R$

(b) Write an expression for the instantaneous power across a resistor with an ac source.

In measuring a.c power, the ammeter/voltmeter method considers I_{rms} and V_{rms}. The product I_{rms} V_{rms} gives the so called apparent power and it is given in the units (Volt) (Ampere).

The one which is measured by wattmeter measures the true power where the power factor is taken into account.

When $\phi = \pm\dfrac{\pi}{2}$ rad, the net energy conversion is zero, and the current is said to be wattless.

If the circuit is purely resistive or if the circuit is at resonance, the power factor equals one. When R - L - C circuit is resonant condition:

$$X_L = X_C \qquad 4.9$$

$\Rightarrow \omega_o L = \dfrac{1}{\omega_o C}$, which gives the angular velocity at resonant w_o,

$$\omega_o = \frac{1}{\sqrt{LC}} \qquad 4.10$$

$\omega_o = 2\pi f_o$ where f_o is resonant frequency.

$$\text{therefore } f_o = \frac{1}{2\pi\sqrt{LC}} \qquad 4.11$$

Activity 8

1. What is the value of the power factor for which the circuit delivers maximum power?
2. What is the impedance value of R-L-C series circuit at resonance?
3. In an R-L-C series ac circuit, does the voltage reach its peak simultaneously across each element?

Example

R-C-L series circuit

A capacitor of capacitance, C = 6.37μc, inductor with L = 2547mH and resistor with R = 400Ω are connected in series with a sinusoidal voltage source of peak value 250V and frequency 50Hz. Compute:

(a) the rms voltage across the source
(b) the impedance
(c) the power factor
(d) the peak voltage across each element
(e) the average power dissipated in the circuit.
(f) the frequency at which resonance occurs
(g) the average power dissipated in the circuit at resonance.

Solution

(a) The root mean square (rms) value is given by

$$v_{rms} = \frac{v_o}{\sqrt{2}} = \frac{250}{\sqrt{2}}\text{volt} = 177\text{volt}$$

This is a measured value when an ac voltmeter is connected across the source.

(b) Impedance, $Z = \sqrt{R^2 + X^2}$, where $X = X_L - X_c$

$X_L = \omega L = (2\pi f)L$

$= 2(3.14)\,(50\text{Hz})\,(2547 \times 10^{-3}\text{H}) \approx 800\Omega$

verify the unit: (Hertz) (Henry) = Ohm

$$X_c = \frac{1}{\omega C} = \frac{1}{2(3.14)\ (50Hz)\ (6.37 \times 10^{-6} c)} \simeq 500\Omega$$

verify the unit: (Farad) (Hertz) = Ohm

$$X = X_L - X_c$$

$$= 800\Omega - 500\Omega = 300\Omega$$

$$Z = \sqrt{R^2 + X^2}$$

$$= \sqrt{(400|\ )^2 + (300|\ )^2} = 500|$$

(c) The power factor is given by

$$\cos\phi = \frac{R}{Z} = \frac{400\Omega}{500\Omega} = 0.8$$

(d) The peak voltage across each element is given by

(i) across the resistor, where the peak current $I_o = \frac{v_o}{z} = \frac{250volt}{500|} = 0.5A$

$v_o(R) = I_oR = (0.5A)\ (400\Omega) = 200V$

(ii) across the capacitor:

$v_o(c) = I_oX_c = (0.5A)\ (500\Omega) = 250V$

(iii) across the inductor

$v_o(L) = I_oX_L = (0.5A)\ (800\Omega) = 400V$

Note: The peak current across each element is the same

Activity 9

If we take the algebraic sum of the peak values, we obtain quite different value 850V as compared with the source voltage, 250V. (Why?).

(e) The average power dissipated in the circuit is given by

$$P_{av} = I^2_{rms}R,\ I_{rms} = \frac{I_o}{\sqrt{2}} = \frac{0.5A}{\sqrt{2}}$$

$$= (0.125A^2)\ (400\Omega) = 50watt$$

We call this true power. Calculate the apparent power.

You can use different expressions to obtain the true power check it.

(f) Resonance occurs when $x_L = x_c$ and the angular velocity at resonance is given by

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(2547 \times 10^{-3}H)\ (6.37 \times 10^{-6}F)}}$$

$$= \frac{10^4}{\sqrt{1622}} \frac{rad}{sec} = 248\frac{rad}{sec}$$

but, $w_o = 2\pi f_o$, which gives the resonance frequency

$$f_o = \frac{w_o}{2\neq} = \frac{248\frac{rad}{sec}}{2(3.14)} = 39.5Hz$$

(g) The average power dissipated in the circuit can also be given by

$P_{av} = I_{rms} V_{rms} \cos\phi$

but at resonance, $\cos\phi = 1$, which gives,

$P_{av} = I_{rms} V_{rms}$

$$= \frac{I_o}{\sqrt{2}} \frac{V_o}{\sqrt{2}} = \frac{0.5A}{\sqrt{2}} \frac{250v}{\sqrt{2}} = 62.5 \text{ watt}$$

(At resonance the circuit delivers maximum power!)

Let us describe the phase angle relationship between the current and the voltage:

(a) Does the current lead or lag the voltage across the source?

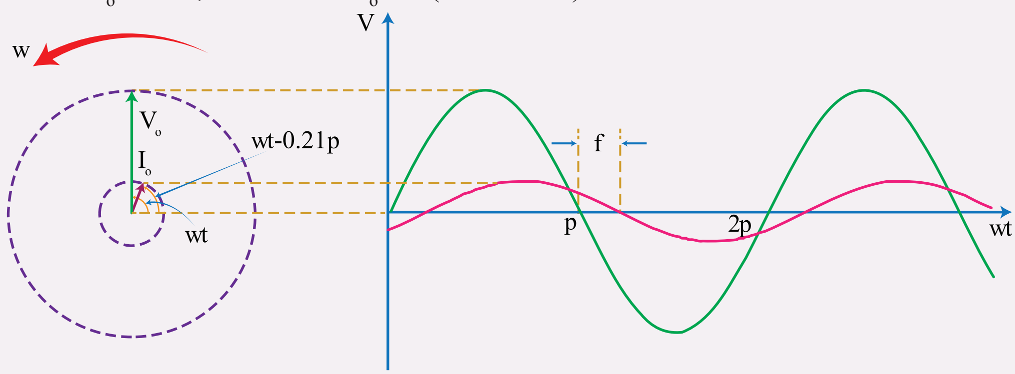
Since $x_L > x_c$, it is more inductive which means the current lags the voltage.

You may use a kind of acronym: CIVL which is to mean in a capacitor, (C) current, I, leads voltage, V as your read it from left. And in inductor (L), voltage, V leads (comes first) current, I as you read it from right.

(b) The phasor diagram can be drawn as shown below.

Since, $\cos \varphi = 0.8$ $\cos^{-1}(0.8) = 37^o = 0.21\pi rad$

If $V = V_o \sin wt$, $I = I_o \sin (wt - 0.21\pi)$



(a) (b)

Figure 11. (a) Phasor diagram of V and I. (b) Graphs of V and I versus wt for the circuit in the example (drawn not to a scale)

Example

R – L circuit

A sinusoidal voltage given by $V(t) = 220 \sin 15\pi t$ is connected to a coil of inductance $L = 1.91H$ and resistance $R = 400\Omega$.

(a) What is the maximum current in the coil?

(b) What is the time lag between the peak voltage and the peak current?

Solution

(a) The maximum current is given by, $I_o = \frac{V_o}{Z}$

$$Z = \sqrt{R^2 + X^2},\ X_L = \omega L = (15\pi)(1.91H) = 90\Omega, \text{ which gives}$$

$$Z = \sqrt{(400|)^2 + (90|)^2} = 410|$$

$$I_o = \frac{220V}{410|} = 0.54A$$

(b) The power factor, which is the cosine of the phase angle difference between current and voltage is given by,

$$\cos\phi = \frac{R}{z} = \frac{400\Omega}{410\Omega} = 0.976$$

Which gives $\phi = \cos^{-1}(0.976) = 12.6^o$

Note: A coil in an Ac circuit is treated as a resistance in series with inductance

ACTIVITY 10

(Experiment on measuring reactance)

Objective: To determine the capacitive reactance of a capacitor.

Theory: Reactance, an opposition to the flow of current due to the property inductance or due to the property capacitance is not a value which could be measured directly. Capacitive reactance, X_c is given by

$X_C = \frac{V_o}{I_o}$, where V_o is peak voltage and I_O is peak current. And the measured current, $I_{rms} = 0.707\ I_O$

Preparation

Prepare a data table with four rows and six columns

Materials

Function generator, Oscilloscope, Capacitor, ac ammeter and connecting wires.

The function generator is to supply a sine wave input. The ac ammeter is to measure the rms current and the oscilloscope to measure the peak voltage across the capacitor

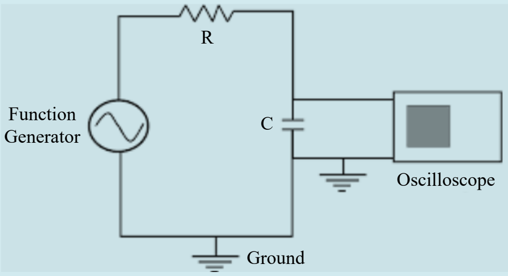


Figure 12.

Procedure

1. Check your equipment's.
2. Set-up the experiment as shown in the circuit diagram.

The function generator has an internal resistance

3. Set a peak voltage. Record the rms voltage and the peak voltage
4. Vary the peak voltage of the supply and record the voltage and the current values for each trial.

Table 2 Data, Data analysis, and Interpretation

Trial	1	2	3	4	5
Peak-voltage					
r.m.s current					
Peak-current					

1. Complete the data table with corresponding values.
2. Graphing data plot the graph of the peak voltage versus the peak current.
3. Determine the gradient of the graph. It gives the capacitive reactance.

Exercises

1. Name a circuit element property which doesn't depend on the frequency of the alternating voltage source.
2. What is the condition in which R-L-C series circuit delivers electrical energy at maximum rate?
3. The peak current of R-L-C series circuit is 1.41A. What is the ammeter reading of the current in the circuit?
4. A sinusoidal voltage given by V(t)=250 sin 2p(159t) is connected to a coil of inductance L = 7mH and resistance R=24Ω
   (a) What is the maximum current in the coil?
   (b) What is the time lag between the peak voltage and the peak current?

Resonance is the response of a system, when forcing frequency is equal to the natural frequency and results in an increase in amplitude of the Oscillation. The frequency at which this occurs is termed as the resonance frequency.

Key Terms

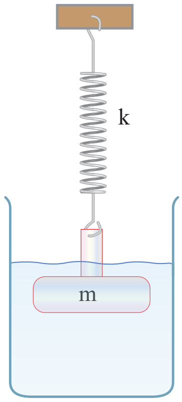
- Resonance
- Oscillator

Oscillation in LC Circuit

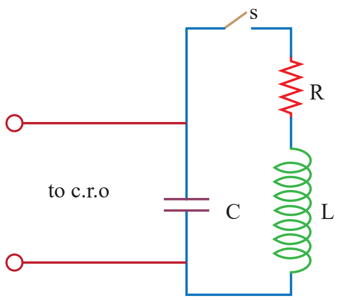
A mass attached to a spring is a familiar example of mechanical Oscillator. The motion will be a sinusoidal function of time for small amplitude and called harmonic Oscillator. The motion will be damped if there is a resistive force proportional to the speed of the mass.

An electric circuit containing capacitance and inductance has essentials of harmonic Oscillator. Ohmic resistance makes it a damped harmonic Oscillator. The Oscillation involves a transfer of energy back and forth from the capacitor to the inductor or from electric field region to magnetic field region. They reach their peak in each half cycle. As the Oscillation goes on, the energy remaining in the field gradually diminishes because of the circuit resistance.

Note: There is no emf source in the circuit. But the capacitor is initially charged.



(a) Mass - spring system in a viscous fluid



(b) R-L-C circuit

Figure 13. comparing damped oscillations: (a) mechanical (b) electrical

In Figure 13 the electrical symbol of the resistor is to represent the circuit resistance. Here, we consider an ideal case where the circuit resistance is zero and no energy is dissipated as Joule's heat. The Oscillation will be undamped.

Let us assume that the capacitor is fully charged with charge, Q_o. The energy stored in the electric field of the capacitor is, $u = \frac{Q_o^2}{2C}$. If the switch is closed at t = 0 the capacitor begins to discharge and the inductor begins to store energy in the magnetic field at the expense of the decrease in the energy stored in the electric field of the capacitor. When the capacitor is fully discharged, it stores no energy. At this time the current through the inductor reaches its maximum value and all of its energy has shifted to be stored in the inductor. This energy is given by $u = \frac{1}{2}LI^2$. The process, then repeats in the reverse direction. The energy continues to transfer between the inductor and the capacitor indefinitely corresponding to the current and the charge respectively.

Suppose the charge on the capacitor at some instant is Q, then the potential difference across the capacitor is, $V_c = \frac{Q}{C}$. The p.d across the inductor, V_L is

$$V_L = L\frac{dI}{dt}$$

but $V_C = V_L$ (why?)

$\frac{Q}{C} = L\frac{dI}{dt}$ $I = -\frac{dQ}{dt}$, current flows as the capacitor discharges.

$$\frac{Q}{C} = L\frac{dI}{dt}\left(-\frac{dQ}{dt}\right)$$

$$\frac{Q}{C} = -L\frac{d^2Q}{dt^2}$$

Which gives

$$\frac{d^2Q}{dt^2} = -\frac{1}{LC}Q \qquad 4.12$$

Compare it with $\frac{d^2x}{dt^2} = -\omega^2 x$ or $\vec{a} = -\omega^2 \vec{x}$ for simple harmonic Oscillation of a mass - spring system.

The solution for the Eq. 4.12 has the same form as, $x(t) = x_o$ coswt

$$Q(t) = Q_o \cos wt \qquad 4.13$$

Similarly the frequency of Oscillation can be derived from, $\omega_o^2 = \frac{1}{LC}$, which gives

$$f_o = \frac{1}{2\neq}\frac{1}{\sqrt{LC}} \qquad 4.14$$

Such circuits have wide practical applications as tuned circuits or resonant circuits, chiefly in radio and television transmitters and receivers.

Note: LC oscillator produce electronic signals from direct current

DID YOU KNOW?

When you tune into your favorite radio station you alter a variable capacitor.

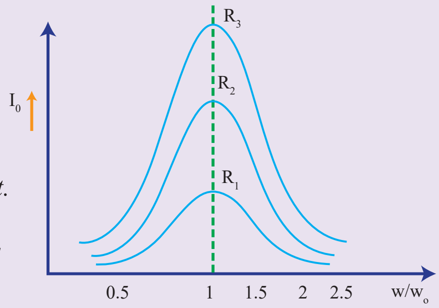


Figure 14. Current Frequency curve for a typical LCR circuit. R3 < R2 < R1. We can say that the LCR circuit is more selective when circuit resistance is low through the current is always maximum at w = wo.

Example

LC Oscillation

An LC circuit contains a 40mH inductor and a 25μF capacitor with an initial charge of 0.01C.

Assume the resistance of the circuit to be negligible. Let the instant the circuit is closed be t = 0.

(a) What is the total energy stored in the capacitor initially? Is it conserved during LC Oscillation?

(b) What is the natural frequency of the Oscillation?

(c) At what time is the stored energy i) completely electrical (i.e. stored in the capacitor) ii) completely magnetic (i.e. stored in the inductor)?

(d) At what time is the total energy stored shared equally between the inductor and capacitor?

(e) If resistance is not negligible, how much energy is dissipated as a heat?

Solution

(a) Initially, just before the circuit is closed, the energy stored in the capacitor is given by

$$u = \frac{Q^2}{2C} = \frac{\left(10^{-2}c\right)^2}{2 \times 25 \times 10^{-6}F} = 2J$$

Yes. In the absence of resistance, sum of energies stored in the inductor and capacitor remains constant. Because reactance is not accompanied by energy loss.

(b) Frequency of Oscillation is given by, $f = \frac{1}{2\neq}\,\frac{1}{\sqrt{LC}}$

$$\frac{1}{\sqrt{LC}} = (40 \times 10^{-3}H \times 25 \times 10^{-6}F)^{\frac{-1}{2}} = 10^3\,\frac{\text{rad}}{\text{sec}}$$

Therefore, $f = \frac{10^3 \text{rad/s}}{2(3.14)} = 159\text{Hz}$

(calculate the time period of Oscillation)

(c) The stored energy is completely electrical at t = 0 (Given by the question).

The amount of charge on the capacitor should be given by Oscillating function: $q_{(t)} = q_o$ coswt, so that $q(t) = q_o$ at t = 0, as it is given in the question.

Hence the stored energy is completely electrical also at $t = \frac{T}{2}$ and T in a cycle.

Where $\cos\omega t = \cos\left(\frac{2\pi}{T}\right)\frac{T}{2} = -1$ and

$\cos\omega t = \cos\left(\frac{2\pi}{T}\right)(T) = 1$ respectively.

Electrical energy stored in the capacitor is zero at $t = \frac{T}{4}$ and at $t = 3\frac{T}{4}$ during a cycle because the charge on the plate is zero at this instants.

And the energy is said to be stored in the inductor (energy associated with magnetic field)

(d) If the energy stored in the capacitor is only half of the total, $u = \frac{1}{2}\frac{q_o^2}{2C}$, the charge on each plate of the capacitor should be $\frac{q_o}{\sqrt{2}}$.

Therefore, $\frac{q_o}{\sqrt{2}} = q_o \cos\left(\frac{2\pi}{T}\right)t$

Which gives, $\frac{2\pi}{T}t = (2n+1)\frac{\pi}{4}$, where $n = 0, 1, 2, 3, \ldots$

The total charge is then shared equally at $t = \frac{T}{8}, 3\frac{T}{8}, 5\frac{T}{8}$ and $7\frac{T}{8}$ during a cycle. Have you noticed that the instants are half way in each consecutive interval where the energy is totally of the capacitor or the inductor?

Compare it with mechanical Oscillation of mass - spring system.

(e) Electric resistance damps out LC Oscillations. The Whole initial energy is eventually dissipated as a heat.

Exercises

1. Which circuit element property in LC oscillation has an effect like that of mechanical oscillator?
2. What is the frequency of oscillation of the charge and the current in LC circuit with inductance of 11mH and capacitance 60nF? Assume negligible resistance.

3. An LC circuit contains a 20 mH inductor and a 50 μF capacitor with initial charge of 0.01C. Assume negligible resistance and let the instant the circuit is closed be t = 0.
   (a) What is the maximum charge stored in the capacitor?
   (b) At what time is all the energy stored in the inductor?

Key Terms

- Energy band
- Intrinsic semiconductor
- Extrinsic semiconductor
- Forward bias
- Reverse bias
- Lattice
- Diode
- Transistor
- Wafer
- Vacuum tube

In this section we discuss some properties of semiconductors necessary for describing devices made out of them. We begin with brief introduction of electronic structure of solids, and then describe the working principles of some devices with their application.

Band Theory

One model that can be used to understand why solids fall in to three categories of conductor, Semiconductor and insulator is called band theory.

Energy levels of atoms become energy bands in solids. When atoms are bound together in a solid, the energy levels of an atom are altered by the influence of the electric field of another atom. The clearly defined energy levels of a single atom widen and blur into energy bands.

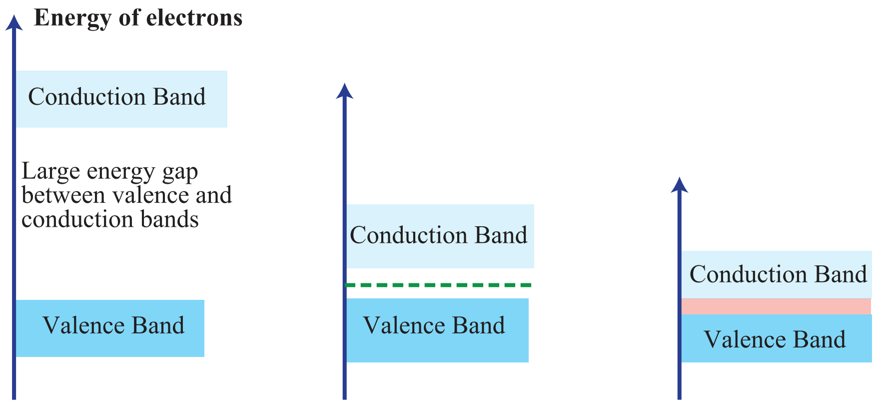
The electrons in a solid normally occupy the lowest energy states one by one until the last electron in the system occupies the state with the highest energy.

Depending on the number of electrons and arrangement of bands a band may be fully occupied or partially occupied. But there are no electrons in higher energy level of an atom unless all of the lower energy levels are completely filled.

There can be more bands of lower energy, but of little importance.

The most important energy band is the highest band containing occupied energy levels, known as valence band. Except in conductors the valence electrons in semiconductors do not conduct electricity.

There are additional unoccupied bands of higher energy besides the valance band. In semiconductors and insulators the band is immediately above the valence band which is known as conduction band as shown in schematic diagram, Figure 15.



(a) Insulator *(b) Semiconductor* *(c) Conductor*

Figure 15. A conductor (c) has a partially filled valence band, and/or partially filled conduction band. Semiconductors (b) and insulators (a) have empty conduction bands and filled valence bands, but the band gap in semiconductor is smaller than that of an insulator.

The range of energies lying between the valence band and conduction band is called the band gap (energy gap). The size of the band gap is different for different materials resulting in different properties.

A Semiconductor has small energy gap with full valence band

The band gap in many Semiconductors is small enough for electrons to be thermally excited into the conduction band leaving an equal number of vacant space (holes) in the valence band, where they are free to move under the influence of even small electric field.

Thermal excitation across the narrow gap is more probable at higher temperatures. Thus, the conductivity of semiconductors improves rapidly with temperature.

Semiconductors

Some materials like Germanium, Silicon, etc., are neither good conductors nor good insulators. The resistivity of these materials lies between conductors and insulators. Such materials are called semiconductors.

Holes are also charge carriers in semiconductors.

As discussed earlier, some of the electrons in a semiconductors move to the conduction band at higher temperatures leaving a vacant space in the valence band. The vacant space is called hole, which is considered to be positive charge carrier. An electron from the neighboring atom can break away to fill the hole completing the covalent bond which in turn creates a hole to be filled by another electron and so on. And we can say the hole movement is random.

At room temperature, about one silicon atom out of every 5×10^{12} is ionized. One carrier for every 10^{12} atoms as compared to one carrier per atom in a metal. Consequently, the room temperature resistivity of silicon is about a factor of 10^{11} higher than most metals.

Intrinsic Semiconductors

Conduction in semiconductors consists of a drift of electrons in one direction and positive holes in the other direction. In earlier discussion we have seen that thermal excitation of electrons into conduction band leaves behind equal number of holes in the valence band. Such a combination of charges are called electron-hole pairs, and a semiconductor that contains, such pairs is called an intrinsic semiconductor.

Another way to change the concentration of charge carriers is to add impurities, atoms that are different from those of an intrinsic semiconductors. The impurity alters the band structure of the semiconductor. New discrete energy levels are formed just below the conduction band or just above the valence band. The number density of charge carriers depends on the concentration of the impurity.

Activity 11

Start up

1. Have you heard of the word ‘doping’?
2. Is doping advantageous or disadvantageous?

Doping a Semiconductor

A crystal is a periodic ordered arrangement of atoms or molecules. However there are several defects in crystals which can affect its property. One of it is when crystals have undesirable impurity. It is difficult to get 100% pure material. Adding impurities at will is called doping. The resulting impure semiconductor is called an extrinsic semiconductor. Generally one impure atom is added for 10^8 atoms of pure semiconductors.

Activity 12

What is the type of bonding which exists between silicon atom?

Doping enhances conduction

The number density of charge carriers is very low in silicon compared with that of metals. Silicon has four valence electrons per atom. In pure silicon crystal at room temperature, almost all of the valence electrons are bound to their respective ion cores, but thermal energy cause some of them to be free.

n - type Semiconductor has electrons as majority charge carriers

If we dope silicon with an atom having five valence electrons (such as phosphorus, Arsenic, or Antimony) called donor, the impurity fits into the silicon lattice, arrangement of atoms in a crystal, without upsetting its electrical neutrality. But the fifth electron which is extra for the lattice structure is free to act as a charge carrier.

The extra electron occupies an energy level, or donor level, that lies just below the conduction band, as shown in Figure 16 (c). Such an impurity, which in effect donates an extra electron to the solid is referred to as a donor atom. And a semiconductor containing donors is known as n - type (n - for negative) semiconductor because the majority charge carriers are negatively charged, electrons.

P - type Semiconductor has hole as majority charge carriers

If we dope silicon with trivalent atoms (such as boron, aluminum, gallium, or indium) called an acceptor the impurity fits into the silicon again without affecting the electrical neutrality, but the absence of a forth electron leaves a vacant space (hole) in the lattice. An electron from the neighboring atom can, breakaway to fill the hole, as shown in Figure 16 (b), completing the covalent bond which in turn creates a hole to be filled by another. Such an impurity, which accepts an electron from the valence band are referred to as acceptors. The energy levels, or acceptor

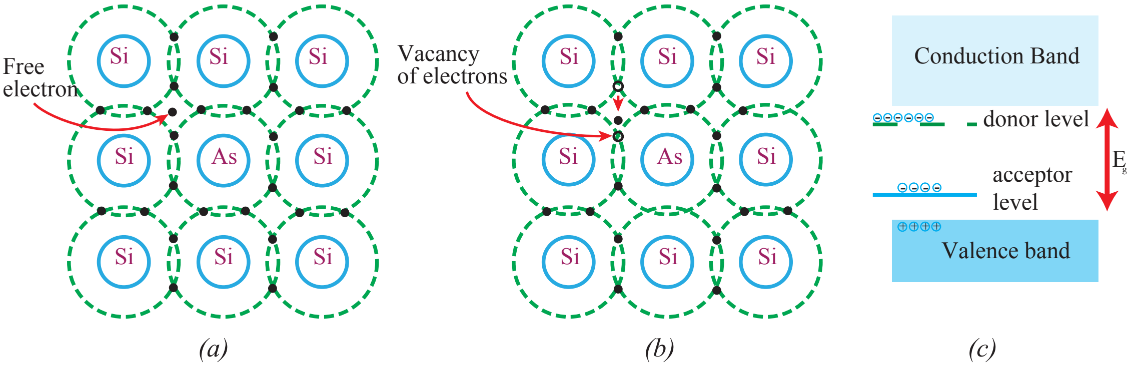


Figure 16. Schematic diagram showing (a) sharing of electrons in covalent bond with group v atoms, (b) with group III atoms (c) band structure of extrinsic semiconductor.

levels, of such impurities lie just above the valence band, as shown in Figure 16 (c) A semiconductor containing acceptors is known as a p - type (p - for positive) semiconductor because the majority charge carriers are holes, which can be considered as positive charge carriers.

Semiconductor devices

In the previous sub-section, it was explained that the addition of impurities to pure semiconductors gives rise to a drastic increase in its conductivity. This fact has been used in the development of many semiconductor devices diodes, transistor, and integrated circuits, which uses n-type and p-type semiconductors.

p - n Junction is a boundary between p - type semiconductor and n - type semiconductor

A p-type or n-type semiconductor, say n - type silicon crystal can be grown by adding appropriate impurity in its melt. This crystals are cut into thin slices called wafer, which used as a base for electronic components. If, in a wafer of n - type silicon, an aluminum film is placed and heated to high temperature, aluminum diffuses into silicon forming a p - region on n - region. This combination is known as p - n junction. Here, the term junction refers to the boundary between p - type semiconductor and n - type semiconductor. Such pn junctions are used in host of semiconductor devices of practical applications.

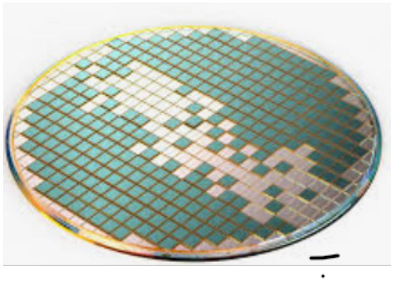


Figure 17. wafer

More than one p - n Junction is Possible

It is possible to make more than one junction on the same wafer which has n - p - n or p - n - p configuration.

The wafer on which such junctions are formed, is cut into small pieces. A piece is encapsulated in casing with electrical connections coming out from p and n regions.

The simplest form is a p - n junction, which is known as diode, having two terminal connections. The other one with n - p - n or p - n - p structure, having three terminal connections is known as transistor.

p - n Junction diode

This semiconductor device allows current to flow preferentially in one direction. Therefore, it can be used to rectify alternating current (i.e convert a.c. to d.c).

ACTIVITY 13

1. Define vacuum tube.
2. Describe the working principle of vacuum tubes and their limitations.

Figure 18. vacuum tube

Diffusion of charge carriers across the junction sets up an electric field.

The diffusion of free electrons to the p - type and free holes to the n - type at the junction leaves a region of no charge carriers called depletion region. In so doing they create a potential barrier which prevents any further migration of charge, as shown in Figure 19 (a). The presence of ions in it creates an internal electric field that opposes the further diffusion of electrons and holes that would cause the region to grow larger.

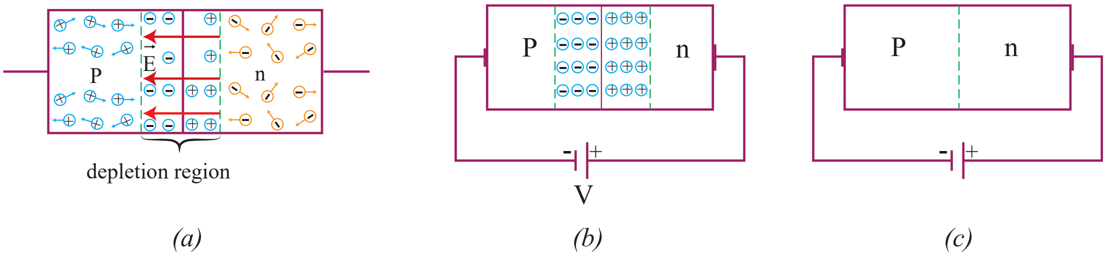


Figure 19. (a) Formation of p - n junction (b) Reverse biased (reinforcing the barrier) and (c) Forward biased (removing the barrier)

The Operation of a p - n junction diode

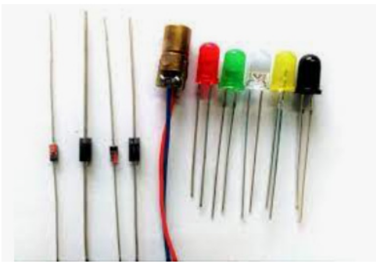


Figure 20. Semiconductor Diodes

The electric field with in the depletion region is directed from right to left, as shown in Figure 19 (a) It means, the right side of the junction is at a higher potential, than the left side.

Positive charge carriers cannot move from left to right and negative charge carriers cannot move from right to left. When a positive external voltage is applied to the n - side (i.e making the n - side of the junction even at a higher potential than the p – side) of the junction, the potential barrier is increased even more, as shown in Figure 19 (b) In such a case the diode is said to be reverse biased. It further limits the current in the junction. A diode in the revers bias has a very large resistance.

When a large enough negative external voltage is applied to the n - side (i.e making the n - side of the junction lower than the p - side) of the junction, the holes flow from left to right and electrons flow from right to left. The electrons can combine with the holes at the junction, mutually eliminating one another (i.e electron - hole pair recombination), which allows the carriers to flow continuously, by removing the barrier, as shown in Figure 19 (c). A diode in the revers bias has a small resistance

A diode doesn't obey Ohm's law, Figure 21 shows a plot of the current in a diode versus the potential difference across the diode. When the diode is reverse biased, the curve is nearly horizontal, which implies the resistance is effectively infinite.

The fact that the current is slightly negative indicates that, a small 'leakage current' still flows in the 'wrong' direction in the reverse biased diode. When the diode is forward biased, the resistance varies, approaching zero for large currents.

Note: Bias is a term used when voltage is applied across a given circuit element

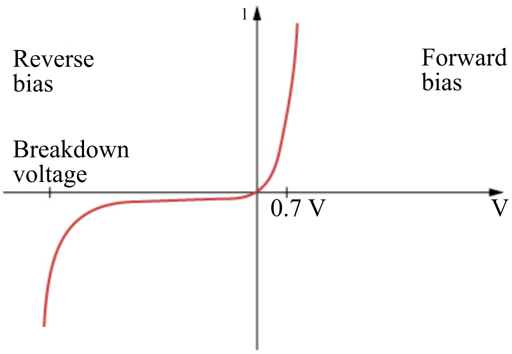


Figure 21. Current versus voltage characteristic curve across a diode.

ACTIVITY 14

1. Plot current - versus voltage graph for Ohmic conductors.
2. What does the slope at a point on the curve shown in Figure 21 describe?

Junction Transistors

Transistors are the other key components in electronic circuits. They use relatively small input signals to control circuits. Semiconductor transistors have two p - n junctions instead of one.

There are many types of transistors, but we consider junction transistor. p - n - p transistor consists of a semi conducting material with a very narrow n region sandwiched between two p regions. The other configuration is n - p - n transistor. The operation of the two transistors is the same.

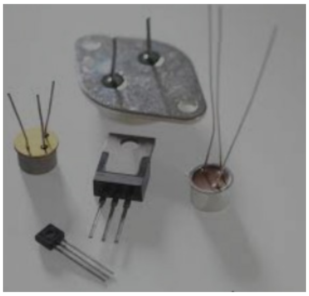


Figure 22. Semiconductor Transistors

The structure of the p - n - p transistor together with its circuit symbol is shown in Figure 23. The central region is the base. Usually the emitter is heavily doped than the base, so there are more charge carriers in the emitter than what would be found in the base. The arrow on the emitter lead is necessary to distinguish between p - n - p transistor and n - p - n transistor. (i.e direction of holes movement).

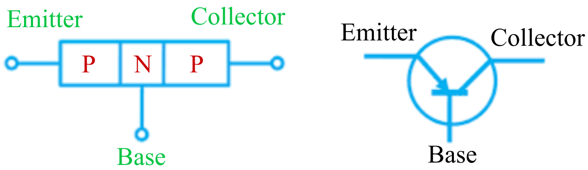


Figure 23. (a) p- n- p junction transistor (b) circuit symbol diagram of p - n - p transistor

Applications of Semiconductor devices

The most remarkable technological advancement has its foundation on semiconductor devices. Portable radios were among the early products of transistor circuits by replacing the vacuum tubes.

A. Semiconductor diodes

A rectifier is a circuit component for producing a unidirectional current from an ac supply. p - n junction diode is the most widely used rectifier. They are cheaper, smaller, less easily damaged and needs no heater as compared with the diode valves which were used in the old days.

A bridge circuit using p - n junction diodes is used as shown in Figure 24 in adaptors to convert a moving coil meter for use in ac circuit.

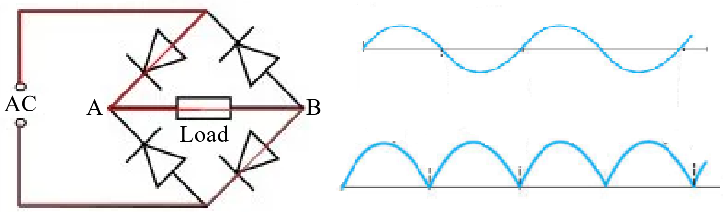


Figure 24. (a) Rectifying bridge circuit, (b) Trace on c.r.o of ac current,
(c) Trace on c.r.o of across AB, full wave rectified current.

Rectifiers can produce a pulsating unidirectional current from an ac supply. To produce a steady direct current the variation must be reduced. This is called smoothing. Capacitors and inductors are used for the smoothening.

Diodes can be used as a protecting equipment if a power supply were connected the wrong way around. They can be also used as a switching for stand by batteries. As the case of computers running with main supply saving your battery and get back to use the battery when the mains supply is cut -off.

Special type Diodes

Light - Emitting Diode (LED) will give out light when they are passing current. The most common type emits red light but it is possible to get yellow or green.

A light emitting diode is an example transducer, it transforms energy from one form to another. LED's are used as indicator lamps. They are small in size, have long life and can operate at low temperatures.

Activity 15

What is the mode of bias at which LED's are working?

Light Dependent resistor (LDR)

When a Semiconductor has a light shining on it, it can conduct electricity more easily. Its resistance depends on the brightness of the light for which it is named Light - Dependent Resistor. They can be used in switch circuits.

Thermistor

It is a thermally sensitive resistor. It has a large variation of resistance with temperature. Thermistors are made from extrinsic semiconductors. They show a decrease in resistance with increase in temperature and can be used as thermometers, electronic thermometer.

Activity 16

Is the variation of resistance with temperature in thermistor linear?

Thermistors can also be used to protect filaments of projector lamps and TV tubes from a current surge as they are switched on and increase the life time of the filament.

B. Semiconductor Transistor

Semiconductor transistors were invented in 1948 in Bell laboratories. By 1960, the transistors had replaced vacuum tube valve circuits which use much more power and need high voltage.

A semiconductor transistor Uses Small Voltage to Operate

A transistor is a three terminal device where small current controls a much larger current. Junction transistors make use of properties of p - n junctions to achieve such control. Its distinct advantage apart from its small size is that it operates at low voltage. It can be used as switch, amplifier and Oscillator.

The Operation of a Junction transistor

A small current which enters the base and leaves via the emitter, called base current is used to control the current which passes through the circuit from the collector to the emitter, called collector current. If the base current is switched off, no collector current can pass through the transistor. Increasing the base current increases the collector current.

Transistor as Switch

Let's look at a simple alarm circuit. A thermistor and a variable resistor connected with a transistor to demonstrate a heat operated switch, as shown in Figure 25

The thermistor and the variable resistor are the two parts of a voltage divider which 'shares' the battery voltage, 5V. As the thermistor gets hotter and its resistance decreases, the voltage across the variable resistor increases. This increases v_{be} and switches on the transistor.

If V_{be} is less than 0.5V the transistor switch is off and if it is more than 0.5V the transistor starts to conduct.

Light - sensor circuit, and burglar arm which are using light dependent resistor (LDR) have the same working principle.

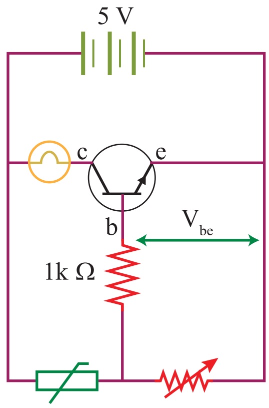


Figure 25. Simple heat operated switch circuit.

ACTIVITY 17

Is the transistor in Figure 25 n- p- n or p- n- p transistor?

Transistor as an amplifier

As electronic signals travel, they become weaker and weaker. Most electronic equipments could not operate without fast and efficient amplification of the weak signals.

The kind of amplifier with which most of us are likely to be familiar is an audio amplifier which is used in sound reproducing system. The amplification involves increasing voltages, currents or both.

Amplifiers are essential in radio transmitters, receivers, television sets, electronic measuring instruments, automatic control equipment's, etc.

Activity 18

(Demonstration)

Objective: To demonstrate "the action of diodes" in a circuit.

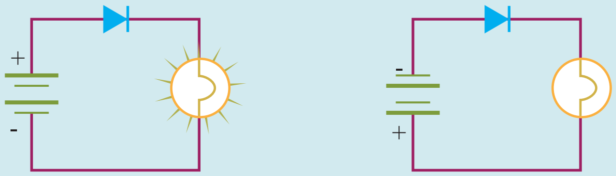


Figure 26. *(a)* *(b)*

Connect a battery, a lamp, and a diode in series, with its arrow end connected to the negative terminal of the battery, as shown in circuit diagram (Figure 26)

(a) What happens to the lamp?

(b) Now turn the diode the opposite way round, Figure4. What happens to the lamp? Does the lamp still light?

Activity 19

(Demonstration)

Objective: To demonstrate "the action of transistors" as an amplifier.

Connect a battery, a transistor, a resistor, and two lamps as shown in the circuit diagram shown below (Figure.27)

(a) Which lamp lights? L_2 in the collector circuit or L_1 in the base circuit?

(b) Now unscrew lamp L_1 so that the base circuit is open. Does lampL_2 still lights? why?

(c) Replace L_1. What do you observe?

Note: L_1 in the base circuit shares a very small current.

A small current in the base circuit causes a larger current in the collector circuit.

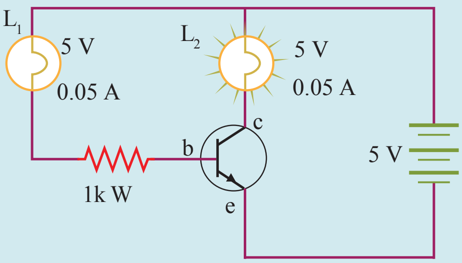


Figure 27.

Exercises

1. Why we add impurities to pure semiconductor?
2. What is the main difference in the band structure of intrinsic semiconductor and insulators?
3. What is the condition which makes a p-n junction diode forward biased?
4. Name three ways in which transistor is used in a circuit?

SUMMARY

- The electricity supply to our homes is an alternating.
- All practical large scale electrical energy supply system is founded on alternating current.
- Ac ammeters, and ac voltmeters measure effective magnitude known as root means square values.
- Ideal capacitors and Ideal inductors as components of ac circuit show opposition without dissipating electrical energy.
- Impedance is a general term given for opposition to current flow in an ac circuit.
- At resonance, an ac circuit delivers maximum power to the load
- Semiconductors have a band structure with empty conduction band, filled valence band, and smaller band gap as compared with insulators
- The era of electronics has its basis on the study of semiconductors
- Electrical conductivity of intrinsic semiconductors depends on their temperature.
- Doping, adding impurities to pure semiconductors, gives rise to dramatic increment in its electrical conductivity.
- The function of electronic devices in a circuit is to control the flow of current. Vacuum tubes are the earliest.
- Diodes are two terminal devices which allow current to flow in one direction.
- P-N junction diodes are widely used in rectification. They can also be used in protecting equipment's, in switching stand by batteries. There are also light-emitting diodes.
- Transistors, three terminal devices, can be used as amplifiers, switches, and oscillators in an ac circuit.
- Transistors have an advantage of working with very small current.
- In R-L-C series circuit,

$$\text{Impedance, } Z = \sqrt{R^2 + X^2},\ X = X_L - X_C,\ X_L = \omega L,\ X_C = \frac{1}{\omega C}$$

$$\text{Voltage, } V = \sqrt{V_R{}^2 + V_X{}^2},\ v_x = v_L - v_C$$

- Average power in an AC circuit is given by $P_{av} = I_{rms} V_{rms} \cos\varphi$

Review Exercises

1. What does the plot of the wave form display on oscilloscope screen represent?
2. What do we mean by saying an alternating voltage? Name instruments used to measure it.
3. Describe the advantage of alternating current over the direct current.
4. Which ac circuit element property decreases with increase in source voltage frequency?
5. Which ac circuit element property give rise to opposition to any change in current with no loss of energy?
6. In which circuit element of an ac circuit does the current attains its peak when the source voltage reaches its peak?
7. In which type of semiconductors does room temperature conductivity is more significant?
8. What is the name given to the impurity atoms in n-type semiconductors?
9. What are the majority charge carriers in p-type semiconductors?
10. Use energy band diagram to explain the extrinsic semiconductors.
11. How does depletion region in p-n junction formed?
12. Which mode of bias in p-n junction corresponds to infinite resistance?
13. What are the earliest electronic devices? Which of these devices are still in use in this era of modern electronics?
14. What is the basic function in the work of electronic devices?
15. What does it mean by saying p-n-p transistor is the complement of n-p-n transistor?
16. The peak voltage of an alternating emf in a circuit is 141volts. What is the voltmeter reading in the circuit?
17. A sine wave voltage is measured using an oscilloscope with y-gain setting of 6 volts per centimeter. The peak-to peak value is founded to be 4cm. What is its rms voltage?
18. How much percent of the maximum voltage of an ac source is the effective value?
19. In R-L-C series circuit the source has a voltage amplitude of 50.0 volts and an angular frequency of 1000 rad per second. If $R=400\Omega$, $L=0.8H$ and $C=2\mu F$, compute:
    (a) the impedance of the circuit
    (b) the power factor
    (c) the peak voltage across each element
    (d) the average power dissipated in the circuit
    (e) the angular frequency at which resonance occurs
    (f) the average power dissipated in the circuit at resonance
20. Use the data given in question number 19 and write an equation for the
    (a) Voltage as a function of time (b) Current as function of time

Sample Test

1. A Cathode-ray oscilloscope can be used to measure,
   (a) AC voltages (c) Frequency
   (b) Dc voltages (d) All of the above
2. The peak voltage of alternating emf is 141 volts. What is the voltmeter reading in the circuit?
   (a) 199volt (c) 100volt
   (b) 141volt (d) 1 volt
3. Which of the following statement about power in L-R-C series circuit is correct?
   (a) The average power delivered by the circuit is maximum at resonance.
   (b) Except at resonance, the true power is smaller than apparent power.
   (c) Apparent power is NOT given in the unit of watt.
   (d) All of the above
4. Which one of the following statements about phase relationship between voltage and current in L-R-C series a circuit is correct?
   (a) In pure capacitive circuit voltage leads the current by $\frac{\neq}{2}$
   (b) In pure resistance circuit current and voltage are in phase.
   (c) In pure inductive circuit current leads the voltage by $\frac{\pi}{2}$
   (d) In any RLC circuit current leads the voltage by $\frac{\pi}{2}$.
5. The average power dissipated by an ac circuit is attributed to
   (a) Resistance (c) Capacitance
   (b) Reactance (d) All of the above
6. Semiconductor Transistors are used as,
   (a) Switchers (c) Oscillators
   (b) Amplifiers (d) All of the above
7. A cathode ray oscilloscope displays peak voltage of 28.2 volts across a 4Ω resistor. What is the rate at which heat is produced across the resistor?
   (a) 199w (c) 100W
   (b) 1.6KW (d) 200W
8. The sensitivity of a CRO is set at 2 V/cm. The spot is seen at a point which is 1.5cm above the time base line when a battery is connected to the y-terminal of an oscilloscope. What is the battery voltage?
   (a) 1.5V (c) 3V
   (b) 2V (d) 1.3V

9. How often does the polarity of the voltage reverse in a 60HZ voltage source?
   (a) 60 times/sec (c) 90 times /sec
   (b) 120 times/sec (d) 30 times/sec
10. Which one of the following circuit elements has no effect on phase relationship between voltage and current in an ac circuit?
   (a) Resistor (c) Inductor
   (b) Capacitor (d) None of the above
11. A circuit element which smoothness a pulsating output of a rectifier is,
   (a) Transistor (c) Diode
   (b) Capacitor (d) Resistor
12. If a silicon crystal is doped with prevalent atom,
   (a) It becomes a better conductor
   (b) It becomes p-type semiconductor
   (c) It will have a donor energy level in the energy band-gap
   (d) All of the above
13. A dopant atom in semiconductor with three valance electron is called,
   (a) Acceptor (c) Moderator
   (b) Donor (d) Extrinsic
14. What are the majority change carriers in p-type semiconductor?
   (a) Electrons (c) Holes
   (b) Protons (d) Ions
15. The depletion region of p-n junction diodes acts as,
   (a) a good insulator (c) a current source
   (b) a voltage source (d) an insulator
16. A p-n junction diode that is forward biased acts as,
   (a) a resistor (c) an open switch
   (b) a good conductor (d) all of the above
17. Which one of the following is not correct about light-emitting diodes?
   (a) They give off light when current flows through them
   (b) They conduct electricity in one direction only
   (c) They consume less energy than a filament type light bulb
   (d) None of the above
18. In which material does electric resistance falls with increase in temperature?
   (a) Copper (c) Silicon
   (b) Aluminum (d) Arsenic

19. A diode is to be used in a circuit that is required to conduct a current of 2.5A. Which one of the following forward-bias diode current rating is the best fit?
    (a) 2A (c) 4A
    (b) 1.5A (d) 1A
20. A unit used to measure inductive resistance is,
    (a) Farad (c) Henry
    (b) Coulomb (d) Ohm
21. Inductors oppose any change in,
    (a) Voltage (c) resistance
    (b) Current (d) electrical energy
22. An ac circuit is said to be at resonant conduction if
    (a) X_L is less than X_C (c) X_L is greater than X_C
    (b) X_L is equal to X_C (d) None of the above
23. The wave form displayed on screen of an oscilloscope represents a plot of,
    (a) Voltage versus current (c) Voltage versus time
    (b) Current versus time (d) All of the above
24. Which one of the following statements describe the working principal of junction transistor?
    (a) Small change in collector current controls a large change in emitter current
    (b) Small change in emitter current controls a larger change in collector current
    (c) Small change in base current controls a large change in collector current
    (d) Small change in collector current controls a large change in base current
25. A p-n junction in a diode consists of,
    (a) two distinct regions (c) one distinct region
    (b) three distinct regions (d) four distinct regions
26. A junction transistor,
    (a) has no depletion layer (c) has two depletion layers
    (b) has three depletion layers (d) has one depletion layer
27. Which section in the transistor structure is heavily doped?
    (a) Base (c) Collector
    (b) Emitter (d) All of the above
28. What are the majority charge carriers in n-p-n transistor?
    (a) Holes (c) Donor ions
    (b) Electrons (d) Acceptor ions

29. The leakage current in semiconductor diodes is due to,
    (a) majority charge carriers
    (b) its forward bias mode
    (c) minority charge carriers
    (d) None of the above
30. What is the mode of bias in diodes for rectification?
    (a) Reverse bias
    (b) Forward bias
    (c) Forward bias and reverse bias
    (d) None of the above

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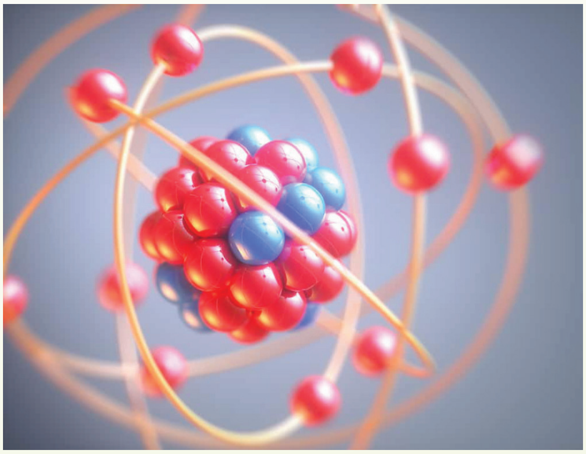
CHAPTER 5

ATOMIC AND NUCLEAR PHYSICS

Chapter Contents



Chapter Outcome

Learners will able to:

- identify substances that can emit harmful particles and take appropriate precautions against the harm- of long term exposure to radioactive substances.

Chapter Objectives

After completing this chapter, you will be able to:

- examine the properties of an electron;
- analyze the effect of radioactive substances on the human body;
- analyze the typical atom and application of nuclear energy;
- distinguish between nuclear fission and nuclear fusion;
- analyze photoelectric emission.

Introduction

The idea of atoms as ultimate indivisible pieces of matter goes back to the time of ancient Greeks. The first evidence for energetic processes going inside atoms had come in 1895 accidentally by Wilhelm Rontgen while he was experimenting with cathode rays. The discovery of x rays stimulated other people to search for other forms of radiation from atoms and in 1896, Heneri Becquerel discovered that atoms of uranium produce another kind of radiation. Two years later Ernest Rutherford showed that there are actually two kinds of radiation which he called alpha rays and beta rays. A third form of radiation, gamma rays was discovered later. The emitted rays revealed the secret of the nucleus.

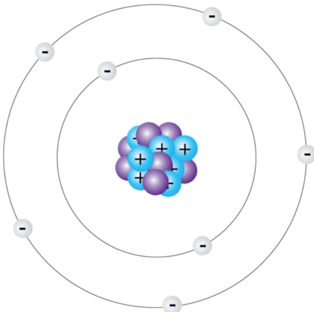
In this unit you are going to learn about the properties of these rays, the secret of the nucleus and ways of harvesting energy from the nucleus.

The accepted model of an atom consists of a central positive nucleus surrounded by a cloud of electrons and the electrons are associated with particular energy levels, Figure 1.

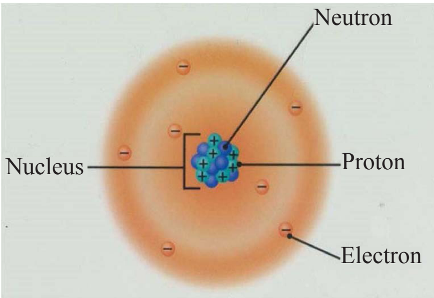
Atom, with diameter of 10^5 times that of its nucleus, is almost empty space. It is the electrons that give an atom its size and it is the nucleus which gives its mass (i.e about 2000 times that of an electron). What goes on in the nucleus of an atom has almost nothing to do with atom's Chemistry and vice versa.

KEY TERMS

- Nucleons
- Strong force
- Energy level



a) planetary model



b) Electron cloud model

Figure 1.

Discussion 1: Suggest a visual model which compares

(a) mass of the nucleus to the mass of the atom

(b) size of an atom to that of the nucleus

The nucleus is a very dynamic place

The nucleus of an atom is made up of protons and neutrons, both of which are known as nucleons. For the nucleus to be stable, the electrostatic force of repulsion between the positively charged protons must be balanced by the force of attraction known as the strong force. The strong force acts at close range (with in the nucleus) to hold all nucleons together, regardless of charge. It is about 10^6 times stronger than the electrostatic force between proton and proton.

Exercises

1. What gives an atom its size?
2. Do atoms exchange protons during chemical reaction?
3. Does strong force act on the electrons?

KEY TERMS

- Binding energy
- Mass defect

When protons and neutrons come together in the nucleus, some of their mass is converted into the energy that binds them together. This is one of the distinctive feature of the strong force. This quantity of energy is called binding energy.

Discussion

1. What determines the atom's chemical identity as an element?
2. What is the most tightly bound of all the nuclei?
3. It takes energy to break chemical bond, does it take energy to separate the nucleus bound together?

How much is the binding energy?

The mass of the nucleus is less than the sum of the masses of the proton and the neutron. The difference in mass is called mass defect. And the energy equivalent of the mass defect equals the binding energy. It is related with Einstein's equation relating mass m to energy E given by:

$$E = mc^2$$

Where c is the speed of light in vacuum. Mass and energy are jointly conserved.

Example

Mass – Energy equivalence.

Compute the binding energy per nucleon of a Lithium atom. Its mass defect is 0.0343u.

Unified mass unit (u) or atomic mass unit and rest energy are used to express the mass of a nucleus.

$$1u = 1.66 \times 10^{-27} \text{ kg}$$

Mass defect, $\Delta m = 0.0343u$

$$= 0.0343\ (1.66 \times 10^{-27} \text{ kg}) = 5.7 \times 10^{-29} \text{ kg}$$

Binding energy = mc^2

$$= 5.7 \times 10^{-29} \text{ kg} \times \left(3 \times 10^8 \frac{m}{s}\right)^2 = 5.13 \times 10^{-12} \text{ J}$$

In units of electron volt, $= \dfrac{5.13 \times 10^{-12} \text{ J}}{1.6 \times 10^{-19} \dfrac{J}{eV}} = 32 \times 10^6 \text{ eV} = 32\text{MeV}$

Example

Binding Energy.

Determine the binding energy of Oxygen $^{16}_{8}O$ if the mass of one oxygen nucleus is 26.557×10^{-27} kg, $m_p = 1.6726 \times 10^{-27}$ kg, $m_n = 1.6749 \times 10^{-27}$ kg

Mass of 8n + 8p = $8(1.6749 \times 10^{-27} \text{ kg}) + 8\ (1.6726 \times 10^{-27} \text{ kg})$

$$= 26.780 \times 10^{-27} \text{ kg}$$

Mass defect = $26.780 \times 10^{-27} \text{ kg} - 26.557 \times 10^{-27} \text{ kg}$

$$= 0.223 \times 10^{-27} \text{ kg}$$

Which gives the binding energy,

$$mc^2 = 0.223 \times 10^{-27} \text{ kg} \times \left[3 \times 10^8 \frac{m}{s}\right]^2$$

$$= 2.007 \times 10^{-11} \text{J} = 125\text{MeV}$$

The mean binding energy per nucleon will be,

$$\frac{1}{16}\left(2.007 \times 10^{-11} \text{ J}\right) = 1.25 \times 10^{-12} \text{ J} = 7.81 \text{ MeV}$$

The mean binding energy per nucleon of a nuclide gives an indication of stability of the nuclide. The larger its value the more stable is the nuclide.

Neutrons help to stabilize the nucleus

The presence of neutrons and the action of the strong force makes it possible for the positively charged protons to stick together in the tiny nucleus. The more protons in a nucleus, the larger number of neutrons are needed to hold it together.

However, for elements with more than 83 protons, even the addition of many neutrons cannot stabilize the nucleus. The force of electrostatic repulsion is too great and the atoms move apart.

Exercises

Compute:

(a) binding energy, and

(b) mean binding energy per nucleon of nitrogen nucleus (14_7N).

Mass of one nitrogen nucleus is 23.253×10^{-27} Kg.

Key Terms

- Radioactivity
- Radioisotope
- Nuclear radiation
- Alpha particles
- Beta particles
- Gamma radiation
- Half- life

Each individual combination of protons and neutrons is known as nuclide. Most of them are unstable with life-time that vary from fraction of a second to several billion years.

Unstable nuclei tend to become more stable by ejecting particles and/or energy from the nucleus of an atom. This spontaneous decay process is known as Radioactivity.

Activity 1

Library research

What was the effect which led the French Scientist Henri Becquerel to discover natural radioactivity?

Did you know?

It was Marie Cure who gave the name radioactivity to the process by which uranium gave off radiation.

Some atoms are naturally radioactive. Many other atoms can be made radioactive by adding particles to the nucleus, forming an unstable nucleus which then radiates particles.

In previous section we have learnt that even the addition of many neutrons cannot stabilize heavy nuclides. They get stable by releasing an alpha (α) particle, which consists of two protons and two neutrons.

In addition, neutrons themselves tend to be unstable when there are no enough protons around. They will decay spontaneously, producing a proton and an electron. This electrons which are released when a neutron changes into proton are called beta (β) particles. An electron from the outer electron shell of an atom is never emitted as a beta particle.

When unstable nuclei undergo changes, they can also emit high energy electromagnetic radiation known as gamma (γ) rays. The emission occurs when a 'rearrangement' of neutrons and proton takes place so that the nucleus reaches a lower energy state where it become more stable.

All the nuclear changes happen in attempt to achieve maximum stability and it is not affected by chemical reactions or the outer electrons

Example

Radioactive decay.

(a) α- Particle emission:

It is exhibited by heavy nucleus

$^{A}_{Z}P \rightarrow {}^{A-4}_{Z-2}D + {}^{4}_{2}\alpha$ where P is for parent nuclide and D is for the Daughter, the nuclei product.

$^{231}_{91}Pa \rightarrow {}^{227}_{89}Ac + {}^{4}_{2}\alpha$ (Protactinum → Actinium + Alpha particle)

Mass number (nucleon number) is balanced: 231 = 227 + 4

Atomi c number(proton number) is balanced: 91 = 89 + 2

(b) β^- particle emission:

Here no nucleon is emitted. It is transmutation of the neutron to proton. Characters of nuclei having large proportion of neutrons.

$^{A}_{Z}P \rightarrow {}^{A}_{Z+1}D + {}^{0}_{-1}e + \overline{\nu}$

$^{12}_{5}B \rightarrow {}^{12}_{6}C + {}^{0}_{-1}e + \overline{\nu}$

Note: Emitted β- particles have different energies. It violates law of conservation of energy and law of conservation of momentum. Hence, another particle, which have zero rest mass and zero charge is suggested to fit the violation. You will learn more about these particles in the coming unit.

(c) β^+ particle emission:

Here again no nucleon is emitted. It is transmutation of the proton to neutron. Characters of nuclei having large proportion of protons.

$$^{A}_{Z}P \rightarrow {}^{A}_{Z-1}D + {}^{0}_{+1}e + v$$

$$^{56}_{27}Co \rightarrow {}^{56}_{26}Fe + {}^{0}_{+1}e + v$$

(d) γ- emission:

Both A and Z remains unaffected.

Excited nucleus → more stable nucleus + γ

Gamma rays are also frequently produced in excited states as a result of decay in α and β emitters. A well known example of such a process is the decay of $^{60}_{27}Co$. First it transmute to $^{60}_{28}Ni$ by β^- particle emission, which in turn reaches its ground state by emitting photons of energies 1.17MeV and 1.33MeV

Many parent radioactive nuclides decay in to daughter products which are also radioactive. Atypical example is seen in the decay series of U-238 by emitting radiation to form Pb-206.

$$^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}\alpha$$

$$^{234}_{90}Th \rightarrow {}^{234}_{91}Pa + {}^{0}_{-1}e \text{ and so on to } {}^{206}_{82}Pb$$

$$^{226}_{88}Ra \rightarrow {}^{222}_{86}Rn + {}^{4}_{2}\pm \text{ (Radium into Radon),}$$

(Again Radon decays by emitting potentially dangerous gamma radiation. They are all in the U-238 decay series.)

HISTORIACAL NOTE:

Curie, Marie (1867-1934) Sklodowska was born in warsaw, Poland. Working with what their daughter called terrible patience, in a small wooden shed with a dirt floor and leaky roof, pierre, her husband and Marie Curie discovered radium and polonium. She was the first person ever to be awarded the Nobel prize twice. She died of overexposure to nuclear radiation.

Summary of the properties of the radiations is given in Table 5.1.

Table 1 Properties of α, β, particles and γ - rays.

Property	α - particle	β - particle	γ - rays
nature	Helium nucleus	Electron	Electromagnetic Radiation
charge	+2	−1	O
Penetrating power	Low	Medium	high
Absorption	Paper 0.01mm Aluminum	~ 2mm Aluminum	~ 5cm lead
Ionization	High	medium	Low
Speed	$\sim 10^7$ m/s	up to ~ $10^8 \frac{m}{s}$	Speed of light
Spread of speed	one or few definite speed	wide spread	the same
Electric field deflection	deflected as positive charge	deflected as negative charge	undeflected
Magnetic field deflection	show small deflection	large deflection show dispersion	Undeflected

Isotopes (sometimes called nuclides) of elements with more neutrons than protons in the nucleus are often naturally radioactive. Hydrogen exist in three isotopes: ^1_1H, ^2_1H (Deuterium), ^3_1H (Tritium). The last two isotopes are unstable because of the additional neutrons.

Carbon atoms with mass number of 14 are also radioactive.

Did you know?

Over 1000 nuclides are known to exist of these, only about one - fourth are stable.
Most of the heavier elements like Uranium - 238, Radium - 226, Thorium - 231, etc, are typical examples of radioactive substances.
Isotopes of some of the common elements can be made artificially. Some are also produced in unclear reactions used for other purposes. Many of these isotopes are radioactive, or radio isotopes. Phosphorus - 32 is an example.

Activity 2

A film badge has three areas: An area open to air, aluminum foil, thin lead. After being exposed to radiation, the film under the open area is black, under the foil the film is slightly exposed ,but the film under the lead is unexposed. Do you think the person is exposed to gamma radiation? Why?

Activity 3

Write the first five equations in the decay series U-238.

Background radiation

Even in the absence of radioactive sources nearby, radiation detectors will still show the presence of radiation. This background radiation comes mainly from natural sources such as the ground beneath our feet and cosmic rays from the sun.

Using radioisotopes

Many uses have been found for radioactive isotopes. Radiation from radioactive isotopes of cobalt and radium is used to kill cancer cells. Radioactive isotope iodine is used to study and treat problems with thyroid gland. A number of different radioactive isotopes are used as tracers of chemical activity in the body.

Plants and Animals both use small amount of phosphorus. Much can be learned about the way the plant or an animal uses by tracing the phosphorus radio isotope.

When plant dies, it stops absorbing carbon dioxide from the atmosphere and when animal dies, it stops absorbing carbon from its food. The carbon - 14 in the dead organic matter slowly changes into nitrogen 14 through radioactive decay. By studying the level of carbon - 14 radioactivity in dead organic matter, scientists can tell how much time has passed since the plant or animal died. This same method can be used to date bones, paper in old manuscripts. Uranium -238 is used to date age of rocks. Uranium-235, Potassium-40 , and Rubidium -27 are also used in determining the age of rocks.

Radioactive isotopes are also used in industry. In paper mills, the thickness of paper can be controlled by measuring how much beta - radiation passes through it. Leaks from a pipeline carrying oil or gas can be traced by injecting or by tagging a radioisotope into it.

Gamma - rays can be used to kill bacteria, moldy and insects in foods, even after the food has been packaged. The treatment prolongs the shelf life of the food, but it sometimes changes the taste. Gamma - rays are also used to sterilize ingredients in cosmetics. Surgical instruments are routinely sterilized in hospitals.

Biological effect of radiation

Radiation can change the nature of any other atom it come across, which in turn changes the structure and behavior of molecules in cells. If a large number of cells are damaged, it affects the processes in our body.

This radiation can cause radiation burns to the skin. These wounds are similar in appearance to heat burn but take much longer time to heal. It can also cause radiation sickness, a disease characterized by nausea, diarrhea, vomiting, internal bleeding, and a feeling of general weakness. Hair loss is a common consequence of exposure to radiation. People who have radiotherapy as part of their medical treatment often surffer hair loss.

The biological damage produced by the radiation is closely related to the amount of energy absorbed. Radiation, damages cells either by ionizing biologically important molecules such as DNA (direct action) or by causing chemical changes in water content of the cell (indirect action). It prefers to interact with water which has small number of molecules rather than DNA molecules.

The biological effect of radiation can be classified either stochastic (i. e random) or non- stochastic. Stochastic, is the one which has no threshold of dose for its severity (if it occurs) e.g cancer.

Non stochastic is the one which occurs only above some threshold level and increase severity with increased dose.

Most of the radiation risks associated with diagnostic imaging are stochastic. Exposure to radioactivity may not have an immediate effect. It is cumulative and long lasting. People often develop leukemia or cancer several years after being exposed to radiation.

Cancer cells are particularly susceptible to the effects of ionizing radiation because they divide more rapidly than normal cells. This is the basis of radiation therapy.
Discussion 2: Link to Biology

1. An exposure to radiation may cause a reduction in lymphocyte count. What is the effect of the reduction in the lymphocyte count?
2. Exposure to radiation can cause mutation. Describe the harmful effect of mutation.

Radioactivity is a random Process

Any sample of radioactive substance, even tiny amount contains large number of atoms. Nuclear decay is a random process. In a given time, a certain number of parent nuclei will disintegrate into their daughter products, but it is impossible to state when one particular nucleus will disintegrate.

Decay rate depends on the amount and type of the sample

The decay rate, or number of nuclei that decay per second, depends only on the type and amount of the sample. Neither chemical combination of nuclide nor changes in physical surrounding, such as temperature or pressure will affect the nuclear decay rate of an isotope. The rate of decay decreases as the amount of the remaining radioactive isotope decreases.

Half - life $\left(t_{1/2}\right)$ refers to the time required for half of the nuclei in the sample of a radioactive isotope to decay.

Half - lives range from microseconds to million years. For example, the half - life of polonium - 215 is 0.0018 second and the half - life of carbon - 14 is 5930 years.

The number of disintegrations of a radioactive substance per second is called activity and it is measured in Becquerel's.

1 becquerel (bq) = 1 disintegration per second (S^{-1})

One gram of radium undergoes 3.7×10^{10} disintegrations per second. Its activity is 3.7×10^{10} Bq, and it is equal to 1 curie, a former unit of activity.

Activity, which is continuously varying as the nuclei decay is given by:

$$\frac{dN}{dt} = N(t)$$ is where the decay constant.

Which gives the number of atoms remaining after time t,

$N(t) = N_o e^{-\lambda t}$, where N_o is the amount nuclides at time t = 0

Table 2 Half - life and activity of radioisotope.

Time from start	0	$t_{1/2}$	$2t_{1/2}$	$3t_{1/2}$
No atoms remaining	N_o	$\frac{N_o}{2}$	$\frac{N_o}{4}$	$\frac{N_o}{8}$
No atoms disintegrate	0	$\frac{N_o}{2}$	$3\frac{N_o}{4}$	$7\frac{N_o}{8}$

Example

Nuclear decay

A freshly prepared sample of Sodium - 24 isotope contains 1020 atoms. The half - life of the isotope is 15 hrs.

Compute:

(a) the initial activity.

(b) the number of radioactive atoms remaining after

(i) one hour

(ii) 30 hours

(c) the number of radioactive atoms disintegrated after 30 hours.

Solution:

(a) Initial activity is given by λN_o. But what is λ?

$$N(t) = N_o e^{-\lambda t}$$

$$\frac{N_o}{2} = N_o e^{-\lambda t_{1/2}} \text{ (by definition of Half - life)}$$

Which gives, $\lambda = \frac{\ln 2}{t_{1/2}} = \frac{0.693}{(15 \times 60 \times 60)\text{sec}} = 1.28 \times 10^{-5}/\text{sec}$

Initial activity, $\lambda N_o = 1.28 \times 10^{-5}/\text{sec} \times 10^{20}$ atoms

$$1.28 \times 10^{15} \text{ atoms/sec}$$

(b) The number of atoms which are still to disintegrate after time t is,

$$N(t) = N_o e^{-\lambda t}$$

$$N(t) = 10^{20} \text{ atoms } e^{-1.28 \times 10^{-5} \frac{\text{atoms}}{\text{sec}} \times (3600 \text{sec})}$$

(i) After an hour,

$$N = 10^{20} \text{ atoms } e^{-1.28 \times 10^{-5} \frac{\text{atoms}}{\text{sec}} \times (3600 \text{sec})}$$

$$= 10^{20} \text{ atoms } (0.955)$$

$$= 9.55 \times 10^{19} \text{ atoms}$$

(ii) After 30 hours,

$$N = 10^{20} \text{ atoms } e^{-1.28 \times 10^{-5} \frac{\text{atoms}}{\text{sec}} \times (30 \times 3600 \text{ sec})}$$

$$= 10^{20} \text{ atoms } (0.955)^{30}$$

$$= 2.51 \times 10^{19} \text{ atoms}$$

$$= 2.51 \times 10^{19} \text{ atoms}$$

(c) The number of radioactive atoms disintegrated after 30 hours out of the 1020 atoms is,

$$= 10 \times 10^{19} \text{ atoms} - 2.51 \times 10^{19} \text{ atoms}$$

$$= (10 - 2.51) \times 10^{19} \text{ atoms}$$

$$= 7.49 \times 10^{19} \text{ atoms}$$

or, 30 hours corresponds to its two half - lives and the number of atoms disintegrated after two half - lives is

$$\frac{3}{4}N_o = \frac{3}{4}(10^{20} \text{ atoms}) = 7.5\ ) = 7.5 \times 10^{19} \text{ atoms}$$

Activity 4

An experiment was performed to determine the half -life of a certain radio isotope. The activity was measured over 24 hours period and the results are recorded as given below. The background counts is 4kBq .

(a) Plot a graph of activity versus time.

(b) Determine the half -life of the sample.

Time(hr)	0	3	6	9	12	15	18	21	24
Activity(KBq.)	23	18.2	14.9	12.5	10.5	9.2	8.3	7.5	7.1

Exercises

1. WRadon -222 is a radioactive gas with half-life of 3.82 days. A gas sample contains 10^8 atoms initially.
   (a) Compute the initial activity
   (b) Estimate the number of radon atoms which will remain after 15 days, and
   (c) Estimate the number of radon atoms which will have decayed after 15 days.
2. The half – life of U– 238 is 4.5×10^9 years. How many disintegrations per second occur in one gram of U – 238?

Key Terms

- Nuclear fission
- Nuclear fusion

In the previous section you have learnt that energy in the nucleus of an atom is much greater than those available among the electrons, and the energy can be released when nucleus of an atom decays.

Efforts to tap this source of energy lead Scientists to finding a way of splitting nucleus, called nuclear fission. Fission reactions are usually caused by shooting a beam of neutrons into a material containing a heavy radioactive isotope.

Activity 5

What makes entering the nucleus fairly easier for the neutrons?

When a neutron enters a heavy nucleus, it upsets the nuclear balance, producing fission reaction.

For example, when a neutron is added to a nuclide of Uranium - 235 nucleus, the nucleus can split into two lighter nuclei such as Krypton - 92 and Barium - 141 including three neutrons and energy.

The energy released in fission is about 100 times greater than the energy released in radioactive decay.

Therefore, nuclear fission can be a very powerful source of energy. The equation for the nuclear reaction of Uranium - 235 can be written as:

$$ {}_{0}^{1}n + {}_{92}^{235}u \rightarrow {}_{36}^{92}kr + {}_{56}^{141}Ba + 3{}_{0}^{1}n + \text{Energy} $$

slow neutron fast neutrons

The energy of nuclear fission can be multiplied in nuclear chain reaction. In nuclear chain reaction, each fission reaction causes another fission reaction. For example, the neutrons released when

Uranium - 235 nucleus fissions can enter other Uranium - 235 nuclei, causing them to fission.

Once a chain reaction is started, it will keep going and can get out of hand very quickly. Nuclear fission occurring under controlled condition provides atomic power which may be converted into electricity. If one mole of U = 235 were used,

$6 \times 10^{23} \times (2 \times 10^{-11} J) = 1.2 \times 10^{13} J$ can be obtained from 235g of Uranium −235.

The principle of atomic bomb is the uncontrolled nuclear fission reaction of certain critical sized piece of Uranium or plutonium.

Safety practices must be followed strictly in the use of nuclear reactors. Excessive usage can led to melt down of the reactor and radioactive substances can escape.

The other problem with nuclear fission is the waste product. It remains dangerously radioactive for centuries to come. No one yet developed, a completely safe way of disposing nuclear wastes.

Did you know?

The atom bomb has put strontium - 90 into atmosphere. A major concern is that it falls to the ground and becomes incorporated in the food chain.

The sun and other stars are sources of even greater amounts of energy. The energy release is the result of the fusion of nuclei. In the nuclear fusion, light nuclei join, or fuse together and release energy.

It is building up of elements from smaller nuclides or protons and neutrons. For example:

$$^{1}_{0}n + ^{1}_{1}p \rightarrow ^{2}_{1}H + \text{Energy } (3.5 \times 10^{-13} J)$$

$$2^{2}_{1}H + ^{3}_{1}H \rightarrow ^{1}_{1}p + \text{Energy } (6.5 \times 10^{-13} J)$$

Starting fusion reaction is more difficult

Although high energies are obtained from nuclear fusion, the two nuclides must collide to initiate the reaction. In many cases, the starting nuclides are both positive and repel each other.

However, if the temperature is considerably increased their mean velocities will increase and more collisions will take place enabling the reaction to proceed. The temperature has to be several million degrees Celsius. Such a high temperatures exist deep in the center of stars. the sun, a star can be thought of as a giant nuclear furnace that is producing energy in nuclear fusion.

At such high temperatures matter consists of free electrons and bare nuclei. This form of matter, a hot, ionized gas is referred to as plasma state. In this form of matter the nuclei can be close enough to fuse in the interiors of stars.

Scientists are working on ways to produce useful energy from nuclear fusion. Isotopes of hydrogen, deuterium can be used as the fuel in a nuclear fusion reaction. The oceans contain large supply of deuterium. The fusion of deuterium produces no radioactive wastes and it releases a large amount of energy.

DID YOU KNOW?

Fusion reactions in stars cause them to shine.

The condition for fusion which exists naturally in the mother of energy, the sun is difficult to produce and control here on Earth. The use of nuclear fusion to supply energy may not be possible for quite some time. Its advantages are great enough to encourage researchers in their efforts to use as a source of energy.

Discussion 3:

1. Do you favor the use of nuclear power?

2. What advantages do the use of nuclear fusion have over nuclear fission as a source of energy?

Did you know?

The hydrogen bomb, first detonated in 1952, is an example of an uncontrolled thermonuclear fusion reaction.

Scientists continue to explore ways to obtain and use fission and fusion energy. So far, working on ways to produce useful energy from nuclear fission have been more successful than those involving fusion. In addition to using nuclear energy as a source of electric power for cities, industries, submarines, aircraft carriers and space probes.

Exercises

1. What is nuclear fission?
2. What is nuclear fusion?
3. Are fission fragments from nuclear reactor light? Medium or heavy?

Key Terms

- Photoelectric effect.
- Work function.
- Stopping potential.
- Threshold frequency.

Among the observations made by early experimenters using electrical discharges in gases is that the electrode connected to the negative side of the electrical source emits what are called cathode rays. These particles were always the same, regardless of the cathode material or nature of the gas through which the electrical discharge was passed.

The English Physicist J.J Thomson, showed that particles with essentially the same properties as those in cathode rays were produced in other ways. Such particles which are known to be electrons can also be emitted by heating a filament of metal or when ultraviolet light falls on certain metals.

When an electric light bulb is turned on, the wires inside give off both heat and light.

Are Electrons and Light Particles or waves?

The electron has been treated as a particle obeying the laws of mechanics, but it may also show wave properties such as diffraction. Similarly light may not be treated as a wave all the time.

In cathode ray tube the electron beam can be deflected and observed by the spot the electrons produce on luminescent screen. The motion of the electrons is found to obey the equations of classical mechanics. This is one of the pieces of evidence for the particle aspect of the electrons.

Wave aspect of electrons

When a beam of electrons is fired at a thin gold film, a diffraction pattern, similar to that of x - rays diffraction in crystals as shown in Figure 2 below are observed.

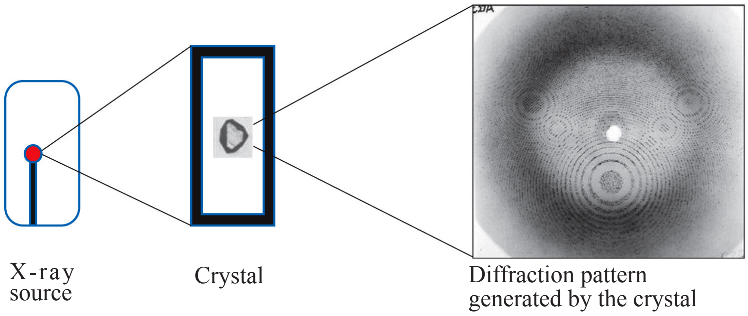


Figure 2. Typical x- ray diffraction pattern. The experiment reveals wave nature of electrons.

The wavelength, λ associated with matter waves as proposed by De Broglie often called the de Broglie wavelength is given by the equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

Where $h = 6.63 \times 10^{-34}$Js is Planck's constant and P is object's linear momentum.

Particle Aspect of Light

Emission of electrons from a heated cathode is known as thermionic emission. Emission of electrons from surface of a metal under the action of light, is referred as photo electric effect. It is one of the experimental evidence for the particle nature of light.

Presence of threshold frequency

The emission of electrons does not occur for light of all frequencies. If below a certain value, called threshold frequency, f_o, then emission will not occur no matter how the light is intense.

For a given frequency f where $f > f_o$ the kinetic energy, K .E of the emitted electrons has spread of values from zero to a maximum kinetic energy, $K.E_{max}$ which is proportional to $(f - f_o)$. Moreover, the maximum kinetic energy is the same for any chosen emitter no matter how weak or intense the light is, but the threshold frequency depends on the type of the metal.

If emission occurs it does so as soon as the light reaches the metal, no matter how weak the light is.

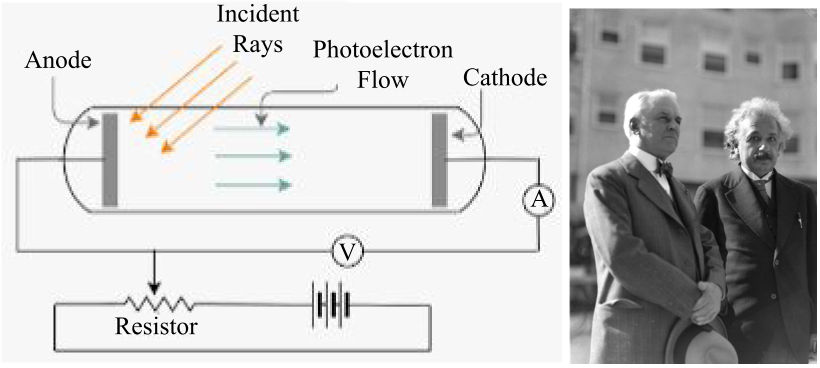


Figure 3. Schematic diagram of Photoelectric emission

Effect of the intensity of light

If emission occurs, the electron current (i.e. the number of electrons emitted per second) is proportional to the light intensity. Intense light produce more photoelectrons whenever there is an emission.

Classical prediction versus experimental evidence

The classical wave theory associated the energy of light with intensity. Thus, it predicts that an electron would break free when it absorbs enough energy (i.e it may take time) from light of any frequency.

But none of these classical prediction are actually observed. Albert Einstein resolved this conflict in his paper on the photoelectric effect, by extending Planck's concept of quantization to electromagnetic waves. Einstein assumed that an electromagnetic wave can be viewed as stream of particles called photons. Each photon has an energy, E given, by plank's equation ($E = hf$). In this theory each photon can be absorbed as a unit by an electron .When a photon's energy is transferred to an electron in a metal, the energy acquired by the electron is equal to, hf.

The metal's work function

In order, to be ejected from a metal, an electron must overcome the force that binds it to the metal. The minimum amount of energy the electron must have to escape from the metal is known as the work function, ϕ.

So the maximum kinetic energy of a photoelectron, the electron emitted by light beam shining on the metal, is equal to the difference between the energy gained (*hf*) by absorbing a photon of frequency (*f*) and the work function (ϕ) of the metal surface.

$K.E_{max} = hf - \phi$, where *h* is plank's constant.

The equation is known as Einstein's equation for photoelectric effect. It shows that the maximum kinetic energy depends only on the frequency of light.

The work function is equal to hf_o, where f_o *is the threshold frequency.* Photons with energy less than, hf_o lack enough energy to eject an electron from the metal surface.

The Stopping Potential

So far, we have considered photo electric emission from metals at zero potential. But if the potential is negative, the photo electrons escape more easily and if the potential is positive, extra work is needed to emit electrons from the metal surface. The extra work needed is equal to the product of charge and potential, eV. The maximum K.E is given by the equation:

$K.E_{max} = hf - (\phi + eV)$

The equation indicates that there is a situation where no photo electrons are emitted. The positive potential which must be applied to the metal surface to stop photo electric emission at a particular light frequency is known as stopping potential, V_s.

$O = hf - (\phi + eV_s)$, where ϕ is work function and V_s is stopping potential.

Experimental confirmation of Einstein's theory was obtained in 1916 by Robert Millikan. Millikan measured the stopping potential for three different metals using different frequencies for each metal in turn. Typical results for two of the metals is shown in Figure 4.

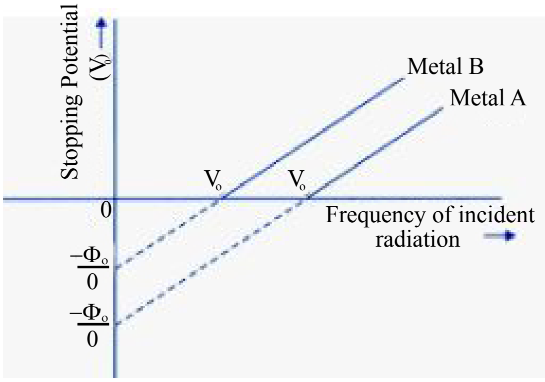


Figure 4.

Did you know?

Einstein published his paper on photoelectric effect in 1905 while working 1905 in patent office in Bern, Switzerland.

Typical result of Millikan photo electricity investigation.

The results obtained by Millikan fitted Einstein's predictions exactly, which is stated in his words: If the formula is correct, then V_s must be a straight - line function of the frequency of the incident light.

Example

Stopping potential and threshold frequency.

A mercury lamp is a convenient source for studying frequency dependence of photoelectric emission since it gives a number of spectral lines ranging from uv light to the red end of the visible spectrum. If the spectrum of lines from the mercury source with wavelength i) 227.1nm and ii) 404.7nm were used to irradiate a photocell made of silver ($\varphi = 4.73$ eV),

(a) what is the threshold frequency?

(b) which radiation gives photoelectric emission?

(c) estimate the stopping potential.

Solution

(a) From the work function of silver, $\varphi = 4.73$ eV

$$hf_o = 4.73 \text{ eV}$$

$$6.63 \times 10^{-34}\text{Js}\, f_o = 4.73\,(1.6 \times 10^{-19}\text{J})$$

which gives, $f_o = 1.14 \times 10^{15}$ Hz

(b) First, let's calculate the energy of the photon which falls on the metal. we are given with wavelength.

$\lambda = 227.1$nm, $c = \lambda f$, where c is speed of light

$$3 \times 10^{8}\ \frac{\text{m}}{\text{s}} = 2271 \times 10^{-10}\,\text{m}\, f$$

$$f = 1.32 \times 10^{15}\ \text{Hz}$$

which gives the photon energy hf

$hf = 6.63 \times 10^{-34}$ Js $(1.32 \times 10^{15}$ Hz$) = 8.75 \times 10^{-19}$ J and in electronvolts,

$$\frac{8.75 \times 10^{-19}\,\text{J}}{1.6 \times 10^{-9}\,\text{J/ev}} = 5.47\text{eV}$$

Similarly, for the wavelength $\lambda = 404.7$nm, the photon energy is 3.07 eV

Comparing with the work function of silver, 4.73 eV

(a) 3.07eV < 4.73 eV, photo emission is not possible with light of wavelength 404.7nm.

(b) 5.47 eV > 4.73 eV, photo emission is possible with light of wavelength 227.1nm.

(c) In Einstein photoelectric equation,

$K.E_{max} = hf - (\varphi + eV)$, the stopping potential is a positive potential for which no photo electrons are emitted.

$0 = hf - (\varphi + eV_s)$ which gives, $eV_s = hf - \varphi$

$eV_s = 5.47\text{eV} - 4.73\text{ eV} = 0.74\text{eV}$

$1.6 \times 10^{-19}\,\text{C}\ V_s = 0.74\,(1.6 \times 10^{-19}\,\text{J})$

$$\Rightarrow V_s = 0.74\ \frac{\text{J}}{\text{c}} = 0.74 \text{ volt}$$

X rays

In 1895 the German physicist Wilhelm Röntegen was experimenting in darkened laboratory with a gas discharge tube enclosed in black cardboard, when he noticed the flourescence painted with barium plantinocyanide. He realized that part of glass wall of the discharge tube being struck by high speed electrons, cathode rays, cathode rays were causing the glass to emit an invisible radiations which was sufficiently penetrating to pass through the cardboard round the tube. In view of their unknown, mysterious nature he called them X-rays. They can penetrate more or less all matter.

The most penetrating ones are absorbed by 1mm thick sheet of lead. They can affect the photographic plates and films in much the same way as light, making X-ray photographic possible. In radiography, low energy X-rays which would are absorbed by body tissue are removed from the beam by placing suitable metal plate in the path of the beam.

As a scientific probe, X-rays have been used to determine the structure of crystals and molecules of many compounds. (Every element has its own characteristic X-ray spectrum which reveals much about its atomic structure

Did you know?

Discovery of X-rays preceded discovery of radioactivity by two months.

Exercises

1. Monochromatic light of wavelength 365 nm irradiates photo cell made of sodium metal ($\varphi = 2.28$ eV) which is placed 1m away from the source.
   (a) What is the threshold frequency for the metal?
   (b) What is the stopping potential?
   (c) What happens if the source is brought nearer and placed 50cm away from the source?
2. Which light has more energy per photon? Red light or Green light?

Summary

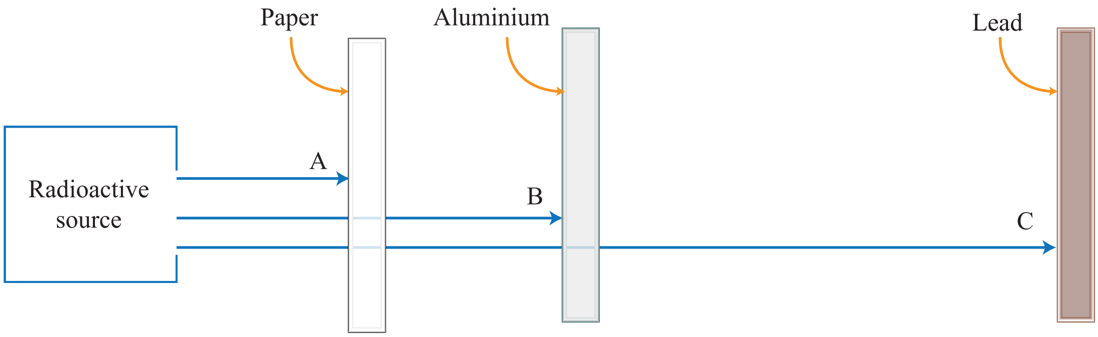
- It is the electrons that give an atom its size and the nucleus its mass.
- The nucleus is the very dynamic place. Neutrons help to stabilize the nucleus.
- The binding energy of a nucleus is the difference in energy between its nucleons when bound and its nucleons when unbound.

- Radioactivity refers to spontaneous disintegration of the nucleus if an atom while radiation refers to what ever comes out from spontaneous decay of a nuclei (alpha particles, beta particles and gamma radiation)
- Radioactive isotopes are used in tracing techniques, radioactive dating, radiotherapy and in nuclear power, yet, you have to compromise.
- Half -life ($t_{1/2}$) of an isotope is the time it takes for half of its nuclei in a same undergo radioactive decay.
- Nuclear reaction involves change in the nucleus of an atom. Enormous energy can be obtained from the reaction.
- In nuclear fission, heavy nucleus splits into two lighter nuclei. In nuclear fission two light nuclei combine to form a heavier nucleus. And its start up is difficult.
- Light has a particle aspect. Photo electric effect is one of the experimental evidence for particle nature of light. Wave theory which favors intensity , of light fails to explain it.
- X-rays are emitted when the innermost electrons of heavy elements are excited.
- The maximum kinetic energy of a photoelectrons, is given by
  $K.E_{max} = hf - \varphi$, where h is plank's constant, *f* is photon of frequency and (φ) the work function of the metal surface.
- The positive potential which must be applied to a metal surface to stop photo electric emission at a particular light frequency is known as stopping potential, V_s, which is given by the equation,
  $O = hf - (\varphi + eV_s)$, where φ is work function and V_s is stopping potential.

Review Exercises

1. Describe the acceptable model of an atom.
2. What makes nucleus a very dynamic place?
3. What is the difference between living in a concrete or brick house than living in wooden house in terms of radioactive material exposure?
4. Do you think human bodies tolerate radiations?
5. Which radiation is the most dangerous?
6. Do you think keeping distance from radioactive source is one way of protection?
7. An element radon comes 86^{th} in a list of elements in periodic table. What information does this fact alone give you about atoms of radon?

8. If you have equal amount of radioactivity material one that has short half -life and another long half -life, which one will give a higher reading?
9. How was the splitting nucleus first achieved?
10. What is meant by Uranium fission? Why does it yield energy?
11. Identify the type of radiation labeled as A, B and C?



12. Radioactivity sources can be used to control thickness of paper and metal sheeting in rolling mills. Explain why gamma radiation is not suitable to use in paper rolling mill.
13. If a certain radio isotope has a half- life of two days, how long will it take for 100g of the sample to decay to 25g?
14. Radiation from a point source follows inverse square law. If a detector that is 1m away from a small source counts 100 counts per minute, what will be its reading 2m from the source?
15. An X-ray tube produces a continuous spectrum of radiation with its short wavelength end at $0.45A^0$. What is the maximum energy of a proton in a radiation?
16. An isotope of zinc $^{71}_{30}Zn$ is unstable with half-life of 2.4 minutes. It decays with emission. Suppose the sample initially contains 1.88×10^{22} atoms,
    (a) how many atoms exist after 2.4 minutes?
    (b) how many zinc-71 nuclides remain after 2.4 minutes?
    (c) write the nuclear equation for the decay process.
    Hint: use periodic table to name the daughter nuclei.
    (d) how many zinc nuclides are disintegrated in 12 minutes?
    (e) how many zinc nuclides are disintegrated in 12 minutes?
17. Determine the unknowns to complete the following nuclear reaction equations.
    (a) $^{238}_{92}U \rightarrow {}^{a}_{b}Th + {}^{4}_{2}He + \gamma$ (photon)
    (b) $^{227}_{89}Ac \rightarrow {}^{227}_{90}Th + {}^{c}_{d}X + \overline{\nu}$ (anti-neutrino)

(c) $^{18}_{9}F \rightarrow {}^{A}_{Z}X + {}^{0}_{+1}e + \gamma$ (neutrino)

Note: You will learn more about this particles in the next unit.

18. Which one of the following metals exhibit photoelectric emission for incident radiation of wavelength 428 nm? The work functions of the metals are:

    Silver: 4.7eV, Sodium: 1.92eV and, lithium: 2.3eV

19. The threshold frequency for a certain metal is 3.3×10^{14} Hz. What is the cut - off voltage (stopping potential) for a light from mercury source, 3050 A^0 ?

20. A typical alpha particle could have energies in the range, 4MeV to 6MeV. As it passes through air it loses, on average, 30eV each time it ionizes an atom. How many ion pairs can it create?

Sample Test

1. Most of an atom's volume is:
   (a) protons
   (b) peutrons
   (c) electrons
   (d) empty space
2. When nuclide undergoes a decay process by beta emission its:
   (a) mass number changes
   (b) orbital electron number changes
   (c) proton number changes
   (d) nucleon number changes
3. Which particle is involved to fit the violation in conservation of energy and momentum in decay process of nuclides with large proportion of protons?
   (a) positron
   (b) photon
   (c) anti-neutrino
   (d) neutrino
4. Radon is a radioactive element which emits alpha particles with energy 6.3MeV. Each particle loses on average 30eV of energy for each pair of ions it creates in collision with air molecules. How many ion-pairs would you expect an alpha-particle from radon to create?
   (a) 2.1×10^5
   (b) 1.89×10^8
   (c) 2.1×10^{-8}
   (d) 4.76×10^5
5. Which element would possibly yield energy by fission?
   (a) Gold
   (b) Carbon
   (c) Helium
   (d) All the above
6. When Uranium -235 captures a slow neutron, it may split into two smaller nuclides with net gain in energy. What is the value x in the following nuclear reaction?

   $${}^{1}_{0}n + {}^{235}_{92}U \rightarrow {}^{92}_{36}Kr + {}^{x}_{56}Ba + 3{}^{1}_{0}n + \text{energy}$$

   (a) 144
   (b) 140
   (c) 141
   (d) 143

7. The activity of a radioactive source falls to one-fourth of its original value in 112 years. What is the half-life of the decay process?
   (a) 14 yr
   (b) 28 yr
   (c) 56 yr
   (d) None of the above
8. When light of wavelength 400 nm is incident on a particular surface, the stopping potential is 0.5 Volt. What is the work function of the material?
   (a) 4 eV
   (b) 3 eV
   (c) 3.5 eV
   (d) 2.5 eV
9. What type of radioactive transformation does the following nuclear reaction describe?

   $${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H$$

   (a) Nuclear fusion
   (b) Nuclear fission nor nuclear fusion
   (c) Nuclear fission
   (d) None of the above
10. Which one of the following statements is correct about emission of electrons from a metal surface illuminated by a short wavelength radiation?
   (a) The emission can take place the moment light falls on it.
   (b) It supports particle nature of light.
   (c) Maximum KE of the photoelectron depends on the type of metal.
   (d) All the above.
11. Which radiation has the highest ionization for the same amount of energy?
   (a) α -particle
   (b) β^- -particle
   (c) β^+ -particle
   (d) gamma radiation
12. In alpha decay, the number of protons in the nucleus:
   (a) increase by 1
   (b) decrease by 2
   (c) remains constant
   (d) decrease by 1

13. An isotope of ${}^{238}_{92}U$, most likely emits:
    (a) beta particle
    (b) alpha particle
    (c) both beta and alpha particle
    (d) gamma ray
14. What percentage of radioactive nuclides remains after its two half-lives?
    (a) 50%
    (b) 75%
    (c) 25%
    (d) 12.5%
15. Photoelectrons with maximum energy of 1.86 eV are produced when a metal is irradiated. With light of wavelength 300 nm. What is the work function of the metal? (Take $h = 6.4 \times 10^{-34}$ Js)
    (a) 2.30 eV
    (b) 2.14 eV
    (c) 0.8 eV
    (d) 4.14 eV
16. The half-life of Strontium-90 is 28 years. How many years will it take for its activity to be reduced to 6.25% of its original?
    (a) 7 years
    (b) 14 years
    (c) 112 years
    (d) 56 years
17. Which one of the following statements about β-particles is correct?
    (a) They can be positively charged or positively charged.
    (b) Their creation is accompanied by transmutation of proton to neutron and vice versa.
    (c) They can pass through paper.
    (d) All of the above.
18. Which combination of Scientist-discovery is NOT correct?
    (a) Rutherford-Nucleus
    (b) Fermi-neutrino.
    (c) J.J Thomson-Electron
    (d) Chadwick-Neutron

19. Which one of the statements about an isotope with high binding energy per nucleon is correct?
    (a) It is unstable.
    (b) It has more neutrons than protons.
    (c) It is more stable.
    (d) It has more protons than neutrons.
20. A radioactive source, astatine-198 has a half–life of 4.2sec. How long will it take such that only one eighth of it is to remain?
    (a) 4.2 s
    (b) 12.6 s
    (c) 8.4 s
    (d) 16.8 s
21. Which one of the following statements about the wavelength of electrons and protons accelerated from rest through the same potential difference is correct?
    (a) They will have same wavelength.
    (b) Protons will have longer wavelength.
    (c) Electrons will have longer wavelength.
    (d) None of the above.
22. Lots of energy is released during nuclear fusion because the product nuclide:
    (a) have higher binding energy per nucleon.
    (b) have less mass.
    (c) have lower binding energy per nucleon.
    (d) none of the above.
23. What aspect of wave–particle duality is shown by photoelectric effect?
    (a) waves can behave like particles.
    (b) particles can behave like waves.
    (c) light is made up of particles.
    (d) particles are also waves.
24. Which particles are considered in computing binding energy?
    (a) electrons
    (b) protons
    (c) neutrons and protons
    (d) neutrons

25. The band gap for silicon at 300 K is about 1.14eV. What is the lowest photon frequency that will make an electron to jump from the valance band to the conduction band?
    (a) 1.14×10^{14} Hz
    (b) 2.75×10^{14} Hz
    (c) 6.63×10^{14} Hz
    (d) 1.7×10^{14} Hz
26. An X-ray photon which is scattered by an electron will have:
    (a) a shorter wavelength.
    (b) a longer wavelength.
    (c) the same wavelength.
    (d) none of the above.
27. Which one of the following isotopes is used in radioactive dating?
    (a) Potassium-40
    (b) Uranium-238
    (c) Carbon-14
    (d) All of the above
28. Which one of the following is an outcome of nuclear fission?
    (a) energy
    (b) unstable fragments
    (c) neutrons
    (d) all of the above
29. A Geiger counter registers a count rate of 4000 counts per minute from a sample of a radioisotope. Twelve minutes later, the count rate is 1000 counts per minute. What is the half-life of the radioisotope?
    (a) 6 min
    (b) 3 min
    (c) 4 min
    (d) 2 min
30. A nucleus of Strontium-87 undergoes a decay process given by the equation, ${}^{*87}_{38}\mathrm{Sr} \rightarrow {}^{87}_{38}\mathrm{Sr} + ?$ Identify the missing radiation?
    The asterisk (*)indicates that the nucleus is in an excited state.
    (a) α -particle
    (b) β^- -particle
    (c) β^+ particle
    (d) gamma radiation

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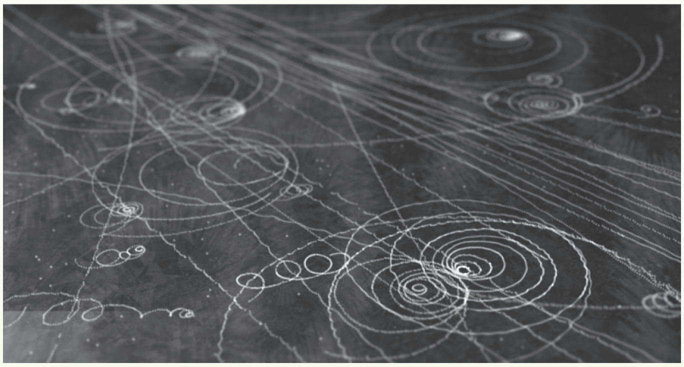
CHAPTER 6

HIGH ENERGY PHYSICS

Chapter Contents



Chapter Outcome

Learners will able to:

- describe the disintegration of atoms that can produce energetic particles and that can be accelerated and captured.

Chapter Objectives

After completing this chapter, you will be able to:

- examine the uncertainty principle;
- analyze the principal quantum number and describe the motion of an electron;
- discuss various types of particle accelerators;
- identify and discuss the four basic interactions between particles of matter;
- analyze the conservation laws of particle physics.

Introduction

The modern Scientists picture the large-scale world as put together from smaller and smaller units, down to elementary particles, the indivisible building blocks. And pictures phenomena as arising from the laws which ultimately have their simplest expression in the micro world.

In addition to changing the perspective on what constitutes elementary particles, this chapter will allow you to glimpse the way Scientists are studying the elementary particles and also the interaction involved between these particles.

Classical mechanics involves the study of familiar things such as motions, energy, momenta of massive objects that behave in a predictable manner obeying Newton's laws.

In the micro world, where everything is a bit and where values of energy, momentum, position and perhaps time are all too small to be measured, there are fundamental uncertainties in the measurements. The uncertainties are comparable to the magnitudes of the quantities to be measured themselves. They are all governed by probabilities rather than certainties.

The fundamental laws of quantum mechanics are then laws of probability rather than laws of certainty. For example, in contrast to the Bohr's theory, which confines the position of the electron in the ground state of hydrogen to points of fixed distance in a plane, quantum mechanics predicts that the electron can be found in spherical region surrounding the nucleus.

Quantum mechanics has extended man's power of prediction and quantitative description of the subatomic world. However, instead of two sets of physical principles, one for macro world and the other for micro world, there is only a single set included in quantum mechanics. Classical mechanics turns out to be just an approximate version of Quantum mechanics.

Key Terms

- Quantum mechanics
- Uncertainty principle
- Wave function
- de Broglie wavelength
- Quantization
- Principal quantum number

Wave - particle duality

In micro-world, waves and particles appear to be different aspects of one and the something. This remarkable fact was first implied by Einstein's theory of the photon and came to be fully appreciated after the work of de Broglie, Erwin Schrödinger, and others on the quantum theory.

According to de Broglie, the wavelength, λ of a particle with mass, m moving with non- relativistic speed, v is given by:

$$\lambda = \frac{h}{mv}, \text{ where h is Planck's constant.} \qquad (6.1)$$

The answer to the wave particle duality conflict is obtained in the principle of complementarity, the wave descriptions and the particle descriptions are complementary. It was first stated by Niels Bohr in 1928, we need both descriptions to complete our model of nature.

The wave - particle paradox

In 1906, J.J Thomson had received the Nobel prize for discovering an electron and proving that it is a particle. In 1927, George Thomson (Son of J.J) carried out an experiment which unambiguously showed electrons behaving like waves when bounced off the atoms in a crystal. Both off them were right, and both prizes were merited. Similarly, light behaves as a particle up on interacting with matter as we have seen in photoelectric effect. And it exhibits its wave property in passing through slits. The best way to understand this wave-particle paradox is to say that quantum entity such as a photon or an electron travel as a wave and arrives as a particle. However, it is a feature of waves that they are spread-out things, and they never concentrate to what the mathematician calls a point, a dot with zero dimension. When you measure their location, they instantaneously concentrate at definite location.

Example

Use de Broglie equation to calculate the wavelength associated with an electron of energy 10keV.

Solution

$K.E = \frac{1}{2}mv^2$ and can be written as $K.E = \frac{p^2}{2m}$ which gives,

$$p^2 = 2(9.11 \times 10^{-31}\,kg)(10 \times 10^3 \times 1.6 \times 10^{-19} J)$$

$p = 5.4 \times 10^{-23}\ kg\frac{m}{s}$. Therefore, $\lambda = \frac{h}{p} = \frac{6.63\times10^{-34}}{5.4\times10^{-23}}\frac{Js}{kg\frac{m}{s}} = 0.123$ Å

Particles as a wave to understand the subatomic world

A surprising number of facts about subatomic world can be understood in terms of the wave nature of particles. The most essential fact about a wave as far as the world of particles is concerned is its unlocalized nature.

A wave can't be said to be exactly at this point or exactly at that point, at best it is said to be in this region or in that region.

The relevance of this unlocalized nature in the sub microscopic world is simply that the position of a particle can never be known, even in principle to an accuracy much greater than the wavelength of the particle.

The uncertainty principle

In classical mechanics, there is no limitation to the accuracy of our measurements in experiments. But in the micro world, nature imposes a fundamental limitation on the precision of our measurements.

Limitations on the precision of our measurements.

Limitation inherent in nature due to the wave nature of particles apart from limitations on human ingenuity.

One of the most important insights into nature revealed by quantum mechanics is Heisenberg Uncertainty Principle, a general principle that can take many forms. It will be sufficient to consider one of its forms, which can be written.

$$\Delta x\ \Delta p \approx \hbar \qquad (6.2)$$

Where Δx is uncertainty in position, ΔP is uncertainty in momentum and $\hbar = h/2\pi$, h is Planck constant.

Suppose we wish to measure the position and momentum of an electron as accurately as possible. In order to see an electron, at least a photon of light must bounce off the electron and re-emerge to be observed. When the photon strikes the electron it transfers some of its energy and momentum to the electron. So, in the process of attempting to locate the electron we have caused much uncertainty in its momentum. The measurement procedure limits the accuracy to which we can determine position and momentum simultaneously.

Example

An electron is confined with in a region of width 1.0×10^{-10} m. Estimate the minimum uncertainty in the x-component of the electron's momentum?

Solution

Based on uncertainty principle, $\Delta p_x \Delta x = \hbar$, where $\hbar = 1.055 \times 10^{-34}\ Js^{-1}$

We don't know the exact position of the electron with in the given region, so the width of the region is the position uncertainty.

$$\Delta p_x = \frac{1.055 \times 10^{-34}\ Js^{-1}}{1.0 \times 10^{-10}\ m} = 1.1 \times 10^{-24}\ kg.ms^{-1}$$

Electron's location

The wave nature of particles is intimately connected with the fundamental role of probability in nature. The simplest aspect is its probability of position.

If you consider the hydrogen atom at its ground state, the electron must be visualized in terms of a spread-out wave instead of being at any particular point. In any particular state of motion, the electron is described by the wave function. Wave function describes the distribution of a particle in space, just as the wave function for the electromagnetic wave describes the distribution of electric fields and magnetic fields. The square of the wave function is proportional to the probability of finding the electron at a given position.

The physical meaning of the square of a wave function is probability per unit volume.

Discussion 1: Do you think particle waves are the same as mechanical waves?

Did you know?

In 1926 Max Born gave the probability interpretation to the Schrödinger's wave function that remains as a key stone of quantum mechanics.

The probability of finding the electron at various distances from the nucleus in the ground state of hydrogen is shown in Figure 1. Note that there is zero probability of finding the electron in the nucleus at this state.

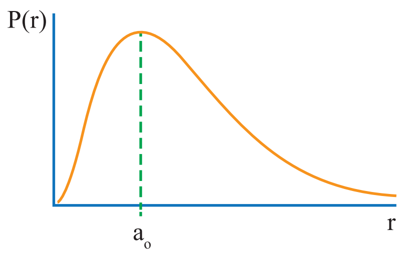


Figure 1. The probability of finding the electron in the ground state of hydrogen.

The peak of the curve represents the distance from the nucleus at which the electron is most likely to be found in the ground state.

The electron may be found at various distances from the nucleus, but the probability of finding it at a distance corresponding to the first Bohr orbit (5.3×10^{-11}m) is greater than that of finding it at any other distance.

Quantum mechanics also predicts that the wave function for the hydrogen atom in the ground state is spherically symmetric. Hence, the electron can be found in spherical region surrounding the nucleus.

Analysis of each of the energy levels of hydrogen reveals that the most probable electron location in each case is in agreement with each of the radii predicated by Bohr theory.

Bohr model of atom

In 1913 Niels Bohr (1885 - 1962) modified the Rutherford's planetary model for hydrogen. This modified model represented the beginning of a quantum theory of matter. Bohr postulated the existence of certain stationary orbits around the nucleus, each corresponding to a certain fixed amount of energy but no other in between and it is stationary in the sense that no radiation is emitted by an electron orbiting in one of these circular orbits.

The wave nature of particles has taught us that this picture of sharply defined orbits is wrong. Bohr's allowed orbits yield the correct order of magnitude for the size of an atom and speed of the electron. They also suggest quantization of angular momentum in nature, a principle that has proved to be central in quantum mechanics. The quantization condition for the magnitude of angular momentum in circular orbits can be written as

$$L = \frac{nh}{2\pi} \quad (n = 1, 2, 3, \ldots) \qquad (6.3)$$

Where h is Planck's constant and n is a positive integer. Thus, Bohr model of hydrogen atom allows the electrons to have only quantized values of angular momentum. The integer n is called the principal quantum number, and each integral value of the quantum number corresponds to a stationary state or quantum state.

The principal quantum number, n gives an information on size of an atom and energy corresponding to each orbit.

Energy levels and the principal quantum number, n

Applying Bohr's quantization condition of angular momentum to an electron in circular orbit, we have:

$$mvr = n\frac{h}{2\neq} \qquad (6.4)$$

and the magnitude of the centripetal force which is responsible to keep the electron in its circular orbit is given by:

$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_o r^2} \qquad (6.5)$$

where electron of charge, −e is orbiting a nucleus of charge, +e if hydrogen atom is considered. The total energy associated with the system can also be derived as follows:

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{e^2}{4\pi\varepsilon_o r}\right) \qquad (6.6)$$

$$P.E. = \frac{-e^2}{4\pi\varepsilon_o r} \qquad (6.7)$$

Which gives the electron's energy.

$$E = -\frac{e^2}{8\pi\varepsilon_o}\frac{1}{r} \qquad (6.8)$$

The energy turns out to be one - half of the potential energy. The zero energy refers to the condition of the atom when the electron has been completely removed, hence negative values for the energy levels.

Multiplying both sides of Eq. (6.5) by r^2 and dividing it by Eq. (6.4) we obtain quantization condition for the electron's orbital speed:

$$V_n = -\frac{e^2}{2h\varepsilon_o}\frac{1}{n} \quad (n = 1, 2, 3, \ldots) \qquad (6.9)$$

The speed is quantized, where n is the principal quantum number.

If we substitute V_n in Eq. (6.4) for the speed, we obtain an expression for the quantized orbit radius:

$$r_n = \frac{\varepsilon_o h^2}{\pi m e^2} n^2 \quad (n = 1, 2, 3, \ldots) \qquad (6.10)$$

Which can be written as:

$$r_n = a_o n^2 \qquad (6.11)$$

The smallest radius corresponds to n = 1; it is called the Bohr radius which is denoted by a_o.

Activity 1

Substitute the constant values and calculate the Bohr radius, a_o = 0.53Å

The Bohr radius is an important quantity because it characterizes the size of atoms. An expression for the quantized energy can be written as

$$E_n = -\frac{me^4}{8\varepsilon_o^2 h^2}\frac{1}{n^2} \quad (n = 1, 2, 3, \ldots) \qquad (6.\ 12)$$

The lowest energy state, which is called the ground state, corresponds to n = 1 having a value which is equal to − 13.6eV.

The stationary states of a quantum system are usually shown through energy level diagrams. Figure 2. shows the energy level diagram for stationary state of hydrogen atom. The principal quantum number, n labels the stationary state in ascending order of energy.

Bohr's model of the atom explained both the appearance of bright lines in the emission spectrum of an element, and the presence of dark lines in the absorption spectrum at the same wavelengths.

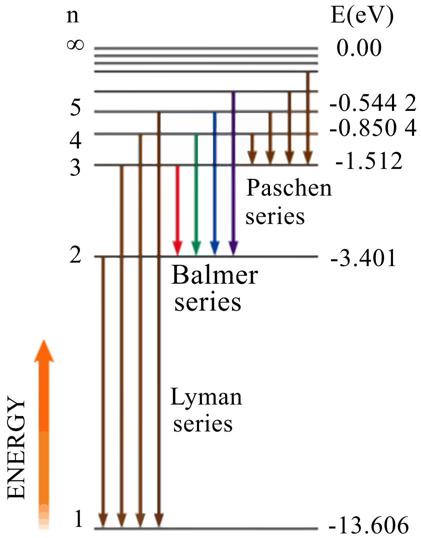


Figure 2. Energy level diagram for the hydrogen atom. Some transitions in the Lyman, Balmer and Paschen series are also shown.

The spectrum of light from hot object is a series of bright lines at sharply defined wavelengths, corresponding to quantum energy radiated when an electron made a particular energy jump down the energy level. The pattern of these lines in the spectrum is different for each element and acts as a unique fingerprint which shows the presence of a particular element in the hot object. One of the most familiar everyday examples of this kind of process is the distinctive orange-yellow color of many street-lights.

Example

Photons are emitted when electrons moves down from one energy level to another. Consider the energy levels for hydrogen atoms. What is the frequency of the emitted photons when it jumps from energy level n = 2 to energy level n = 1?

Solution

$E_2 - E_1 = hf$

$E_2 = -3.41$ eV and $E_1 = -13.6$ eV (use equation 6.11 to check the values.)

$-3.41\text{eV} - (-13.6\text{eV}) = 10.19$ eV

$10.19(1.6 \times 10^{-19}\text{J}) = 6.63 \times 10^{-34}\text{Js.f}$

which gives $f = 2.46 \times 10^{15}$hz

Example

Energy of an electron in the *n*th orbit of hydrogen atom is given by $E_n = -\frac{13.6 \text{ eV}}{n^2}$. How much energy is required to take an electron from the ground state to the first exited state?

Solution:

Here $E_n = -\frac{13.6 \text{ eV}}{n^2}$

When n = 1 (ground state), $E_1 = -13.6$ eV

When n = 2 (first exited state), $E_2 = -\frac{13.6\text{eV}}{4} = -3.4\text{eV}$

Therefore, energy required to take an electron from the ground state to the first exited state:

$= -3.4$ eV $- (-13.6$ eV$)$

$= 10.2$ eV

Example

The ground state energy of hydrogen atom is −13.6 eV. If an electron makes a transition from an energy level −0.85 eV to −3.4 eV, calculate the wavelength of spectral line emitted. To which series of hydrogen spectrum does their wavelength belong to?

Solution:

Energy emitted $= \Delta E = E_2 - E_1$

$= -3.4$ eV $- (-0.85$ eV$) = -2.55$ eV

$= -2.55 \times 1.6 \times 10^{-19}$ J

$$\Delta E = \frac{hc}{\lambda}$$

Therefore, wavelength of emitted spectral line

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{(6.63 \times 10^{-34} \times 3 \times 10^{8})}{(2.25 \times 1.6 \times 10^{-19})}$$

$= 4.875 \times 10^{-7}$ m = 4875 Å

Because, $E_2 = -3.4 = \frac{13.6}{n_i^2}$

Therefore, $n_i = 2$

This wavelength belongs to Balmer series of hydrogen spectrum.

Bohr model and de Broglie waves

In the Bohr model, we pictured the energy levels of the hydrogen atom in terms of definite electron orbits. The most important idea in Bohr's theory was the existence of discrete energy level and their relationships to the frequencies of the emitted photons.

In quantum mechanics, where the motion of electrons is described by wavefunction, the energy levels predicated by quantum mechanics turns out to be the same as those given by Bohr's theory. In more complicated atoms for which Bohr's theory does not work, the quantum mechanical picture is in an excellent agreement with observation.

The de Broglie wave hypothesis has an interesting relationship to the Bohr's model. The method is analogous to determining the normal mode of frequencies of standing waves. For an orbit with radius r and circumference $2\pi r$ we have,

$$2\pi r = n\lambda, \text{ where } n = 1,2,3 \ldots \quad (6.13)$$

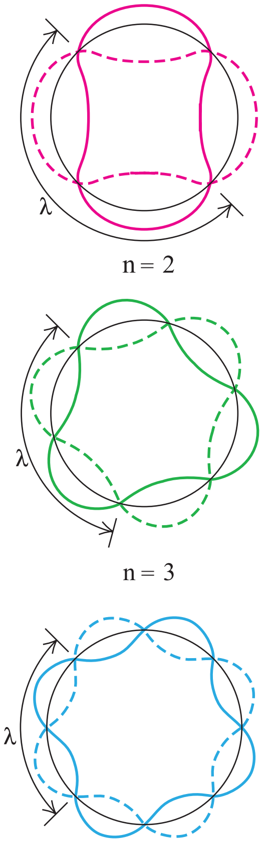


Figure 3. The idea of fitting a standing wave around a circular orbit.

(For the wave to join onto itself smoothly, a circumference of the orbit must be an integral number *n* of wave lengths, (Figure 2).

Substituting λ from de Broglie equation $\lambda = \frac{h}{mv}$, gives $mvr = \frac{nh}{2\neq}$ an equation identical to Bohr's result.

A standing wave on a string transmits no energy .And electrons in Bohr's orbits radiates no energy, so think of an electron as a standing wave lifted around a circle in one of Bohr orbits.

The most important idea in Bohr's theory was the existence of discrete energy levels and their relationships to the frequencies of the emitted photons. In more complicated atoms, for which the Bohr's theory does not work, the quantum mechanical picture is an excellent agreement with observation.

The principal quantum number and Bohr's energy equation survived in the full theory of quantum mechanics although the physical picture of atom underwent revision. And two more quantum numbers were also added to the description of stationary states.

A more recent and useful model of atomic structure is the electron cloud model. In this model, each electron is represented by a cloud in the space around the nucleus. According to this model, the electrons are not in a fixed orbit around the nucleus, but they do have fixed amounts of energy. Each electron is in an energy level, which corresponds to one of these fixed amounts of energy.

Quantization of energy

Although we encounter it frequently, the term quantization seems to be quite unfamiliar. For example, money is quantized. The Banknotes are integral multiples of the coins. As you have learnt in grade 11 in the topic of waves, a stretched string oscillates only in definite stationary states whose frequencies are quantized. In Bohr model of an atom, electrons orbit a nucleus at a specific distance from the nucleus. The electron orbits are direct result of the quantization of energy. Hence, the bright lines of different colors at sharply defined wavelengths in the emission spectrum of hot objects we have mentioned above corresponds to a package of energy or quanta of energy called photons. An electron can radiate or absorb energy as radiation in quantized amounts.

Max Planck postulated that the energy of a particular quantum of radiant energy could be described explicitly by equation:

E = hf, where the proportionality constant, h is called Planck's constant

$h = 6.626 \times 10^{-34}$ Js

The observed manifestations of quantization are much easier to examine than the origins of quantization. Existence of quantized energy levels in Bohr's hydrogen atom and the particle nature of light in photoelectric effect are attributed to quantization.

Bohr had postulated that when a hydrogen atom makes a transition from the state with quantum number n_i to a state with quantum number n_f, the frequency f radiation is given by:

$$hf = E_f - E_i$$

$$= -\frac{me^4}{8\varepsilon_o^2 h^2}\left(\frac{1}{n_f^{\,2}} - \frac{1}{n_i^{\,2}}\right)$$

Which can be written as,

$f = cR_H\left(\frac{1}{n_i^{\,2}} - \frac{1}{n_f^{\,2}}\right)$, where $R_H = \frac{me^4}{8\varepsilon_o^2 ch^2}$ is Rydberg constant, an experimentally measured quantity $R_H = 1.097 \times 10^7 \, m^{-1}$.

The spacing between the energy levels depends on the size of Planck's constant and, this in turn determines the spacing between the Balmer series. The lines in the spectrum that provide the unique fingerprint of hydrogen are named after Johann Balmer, a Swiss school teacher who worked out a mathematical formula describing the pattern in 1884.Using the spacing between the lines, Bohr was able to calculate the spacing of the energy levels, and rewrite Balmer's formula to include Planck's constant.

Thus, Bohr's theory is found to be in perfect agreement with the spectroscopic data. Balmer series was one of the keys to unlock the mystery in describing the structure of the hydrogen atom.

Activity 2

Draw a diagram which shows some of the energy levels (n =1 to n = 8, where n is the principal quantum number)

Exercises

1. What is meant by a certain quantity is quantized?
2. What does Heisenberg's uncertainty principle claim?
3. An electron moves with a speed 2×10^6 ms^{-1}. What is its de Broglie wavelength ?
4. In the Bohr model of an hydrogen atom, what is the de Broglie wavelength of an electron when it is in its (a) n = 1 energy level (b) n = 3 energy level.

The discovery of neutron, techniques used to isolate neutrons and artificial transmutation, a process by which one element is changed into another provided Physicists a new tool to probe the nucleus. They also discovered that something of greater value, energy can be obtained besides from discovering a new element.

Key Terms

- Cosmic rays
- Particle accelerator
- High energy particles
- Particle detector

Did you know?

James Chadwick, an English Physicist won the Noble Prize for Physics in 1935 for his discovery of the neutron.

The first artificial transmutation occurred when Rutherford and his colleagues investigated the effect of 'firing' alpha particles into various gases. Many more studies were made using alpha particle sources, but their scope was limited by the low energy of the alpha particles which could be repelled when they encounter a nucleus and the inability to control the particles adequately.

Activity 3

Protons and alpha particles can be accelerated to have huge kinetic energy and can be guided using magnetic field and electric field. What property of this particles is responsible?

Natural sources of high energy particles

Cosmic rays are natural sources of high energy particles produced by the sun and other stars. Particles such as protons enter the earth's atmosphere from space with huge energies. They are called 'primary' cosmic rays with typical energies of the order 10^4 MeV. A primary particle entering the atmosphere creates a shower of 'secondary' cosmic rays by smashing into nuclei of atoms in the atmosphere.

DID YOU KNOW?

Cosmic rays continually create Carbon-14 from nitrogen in the atmosphere.

Cosmic rays supplied concentrated energy needed in early studies of elementary particles. By sending photographic plates high above the earth's surface in balloons and rockets, cosmic ray collisions have been recorded and studied in detail. In 1937 Carl Anderson discovered the positron (the positive electron B^+) from the plates. Particles called mesons with a mass about 300 times the mass of the electron were also discovered.

Artificial sources of high energy particles

In studying elementary particles and seeking a pattern of nature in the subatomic domain man uses a remarkable array of technological aids to observation.

In most contemporary research, accelerators or high energy machines are used to obtain energetic particles. The energetic particles, concentrated in a narrow beam, strike atomic nuclei in a target, and there undergo interactions or create other particles which fly on to interact with another target or a detector.

The van der Graaf accelerator

This accelerator gives particle energies up to 20 MeV. A series of electrodes in a tube are supplied with high voltage in steps from the dome of the machine, charged particles such as positive ions enter the accelerator near the dome positive, the ions are accelerated down the tube. A positive potential of the order 10MV gives the positive ions enough kinetic energy to smash target nuclei.

The linear accelerator

This accelerator gives particle energies of the order of MeV. It has a series of electrodes with determined length supplied by a high frequency alternating voltage.

Alternate electrodes are connected to the same terminal voltage supply Figure 3. Charged particles accelerate as they are attracted by alternately charged drift tubes.

The Stanford linear accelerator can accelerate electrons to energies of the order of 20 GeV. The accelerator is about 3 km long and is like a giant TV picture tube, but different targets are placed in the path of the beam instead of a screen.

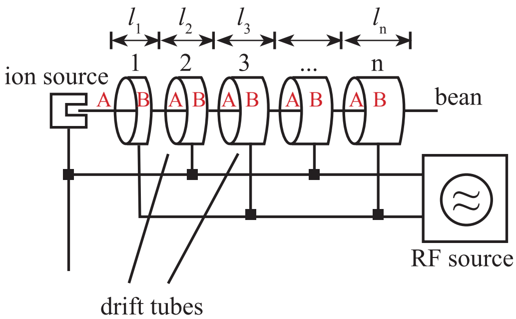
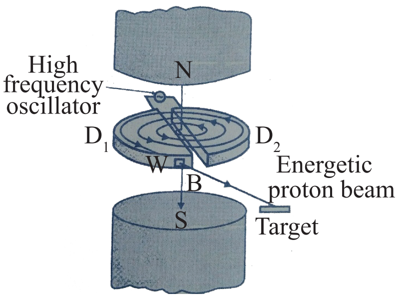


Figure 4. Schematic diagram of linear accelerator

Cyclotron

Two D-shaped electrodes, called '*Dees*', enclose an evacuated chamber containing the circling particles. A high frequency alternating potential difference is applied between the Dees and a uniform magnetic field provided by an electromagnet is directed perpendicular to the Dees. The frequency of the applied voltage should be adjusted so that the polarity of the Dees is reversed each time the ion completes half of its revolution. The motion continues under the action of the magnetic field with a gain of kinetic energy equal to (qV) in each half of its revolution.



(a)

(b)

Figure 5. cyclotron a) schematic diagram to show the principle b) cyclotron at work

ACTIVITY 4

Does the ions gain kinetic energy when moving in the magnetic field? If not, why?

When the radius of the orbit is nearly that of the Dees, the energetic ions leave the system through an exit slit as shown in Figure 4.

We can obtain the maximum kinetic energy of the ion when it exits from the cyclotron in terms or radius R of the Dees. The magnetic force provides the necessary centripetal force for the ion to describe the circular path:

$$Bqv = m\frac{v^2}{r} \qquad (6.11)$$

Where B is the magnitude of the magnetic field applied, v is the speed of the ion r is the radius of the circular path it describes and m is mass of the ion.

The maximum K.E. up on leaving the cyclotron is given by:

$$KE_{max} = \frac{1}{2}m\,V_{max}^2 = \frac{1}{2}m\left(\frac{qBR}{m}\right)^2$$

$$KE_{max} = \frac{q^2B^2R^2}{2m}$$

Example

1. What is the energy transfer in eV when:
   (a) proton passes through a potential difference of 2.0 V?
   (b) a doubly charged ion passes through potential difference of 1.0 V?

Solution

The amount energy gained by a charged particle (q) when it passes trough a potential difference (V) is given by: U= qV

(a) $U = 1.6 \times 10^{-19}$ C (2.0 V) = 2.0 eV

(b) $U = 2(1.6 \times 10^{-19}$ C) (1.0 V) = 2.0 eV

2. What is the maximum KE of protons in cyclotron of radius 0.5 m if the magnitude filed is 0.3T?

$$K.E = \frac{(qBR)^2}{2m} = \frac{\left(1.6\times10^{-19}C\times0.3T\times0.5m\right)^2}{2\left(1.67\times10^{-27}kg\right)} = 1.7 \times 10^{-13}\,J = 1.06\ MeV$$

Note: The limit for a cyclotron is about 20 Mev. This is because the mass increases due to relativistic effect. For this reason the time period of the orbit increases and the rotating ions do not remain in phase with the applied voltage. Accelerators which solve this problem have been built.

Activity 5

Draw a schematic diagram of a cyclotron and demonstrate its working principle.

Synchrotrons

A ring of electromagnets is used to keep the ions on a circular path. The field strength of the magnets is increased to compensate for the gain of the mass as the particles are accelerated.

The energetic particles that emerge from the cyclotron are used to bombard other nuclei. The bombardment in turn produces nuclear reaction of interest to researchers. Today we have about 300,000 accelerators all over the world. A number of hospitals use Cyclotron facilities to produce radioactive substances that can be used in diagnosis and treatment.

Activity 6

Draw a labeled diagram of a linear particle accelerator and describe its working principle.

Exercises

1. Derive an expression for the time period of a rotating ion in a cyclotron.
2. What is the speed of a proton with kinetic energy 5 eV?
3. Describe the effect of the magnetic field in the cyclotron.

The devices used in elementary particle research are basically of two kinds, those which are used to create particles and those used to detect particles.

Thomson detected electrons by observing a fluorescent glow where the beam of electrons struck the glass wall of his cathode-ray tube. Rutherford detected alpha particles by noticing that they ionize the air in passing through it and rapidly discharging an electroscope which is oppositely charged.

Methods to detect single particle rather than the cumulative effect of many particles were also discovered. Rutherford observed through microscope the tiny flashes of light at points on a zinc sulphide screen where alpha particles struck.

Most detectors indicate the arrival of energy which may produce:

- ionization as in Geiger-Müller tube, spark counter, cloud chamber and the bubble chamber.

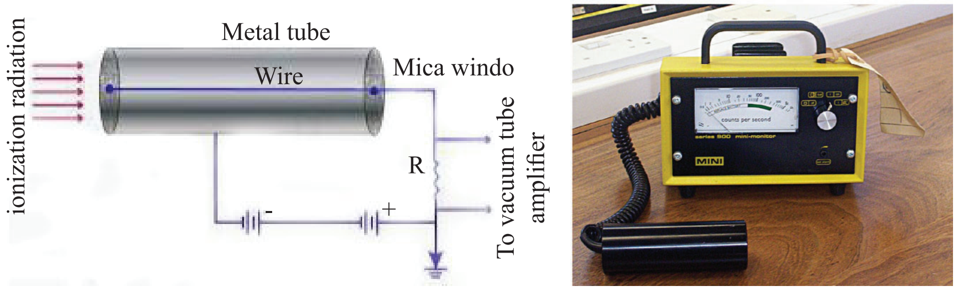
- fluorescence on phosphorus, or
- effect on photographic plate.

Geiger-Müller tube

Geiger-Müller tube consists of a large cylindrical cathode, with a wire down its center serving as an anode. The tube is filled with argon or neon gas at low pressure and a trace of bromine. When a high energy particle enters the tube through a thin mica window on one end, it ionizes some of the gas molecules in the tube. The positive ions move toward the cathode and the negative ions towards the anode, colliding with other molecules on the way to make more ions and create a pulse of current to a meter connected to the output. One pulse is recorded for each radioactive event regardless of the ionizing particle.

The size of the pulse is independent to the original amount of ionization, for example an alpha particle that produces 10^5 ion pair directly creates a pulse of the same size as a gamma ray that has produced only a single ion pair. The output of Geiger-Müller tube is therefore proportional to the number of ionizing 'particles' rather than to the amount of ionization they produce. A rate meter records the average number of pulses per second, the count rate.

Geiger-Müller tubes are primarily used to detect alpha- and beta- particles with counting rate limits of few hundred particles per second. Gamma rays are not easily detected by this instrument.



(a) (b)

Figure 6. Geiger Müller tube a) Schematic diagram b) the device

ACTIVITY 7

Collision between a charged particle and an atom can result in forming an 'ion pair'. Describe the event of forming an ion pair.

Example

A certain alpha particle track in a cloud chamber has a length of 37 mm. Given that the average energy required to produce an ion pair in air is 5.2 × 10^{-18} J and that alpha particle in air produce on average 5 × 10^3 such pairs per mm of track. Find the initial energy of the alpha particle?

Solution

$$5\times10^{3}\,\frac{\text{pairs}}{\text{mm}}\times 37\text{mm}\times 5.2\times10^{-18}\,\text{J} = 96.2\times10^{-15}\,\text{J}$$

which gives,

$$\frac{96.2\times10^{-15}\,\text{J}}{1.6\times10^{-19}\,\frac{\text{J}}{\text{eV}}} = 0.6\text{MeV}$$

Ionization produced in air is used as the basis for monitoring X-rays and gamma rays. Air is used because its "effective" atomic number is very close to that of soft tissue.

Did you know?

Ernest Rutherford, a physicist from New Zealand studied as a research student at Cambridge university under J.J Thomson. And Hans Geiger, an assistant of Rutherford invented Geiger counter.

The cloud chamber

The cloud chamber is a container filled with super-saturated alcohol vapor or water vapor. As the charged particles travel, they ionize the air molecules they happen to hit along their path. These ions attract nearby vapor molecules, which are about to condense or about to boil, forming tiny drops of liquids that are seen as a small vapor track in the chamber. The path, which varies with type of the particle, can be seen and photographed.

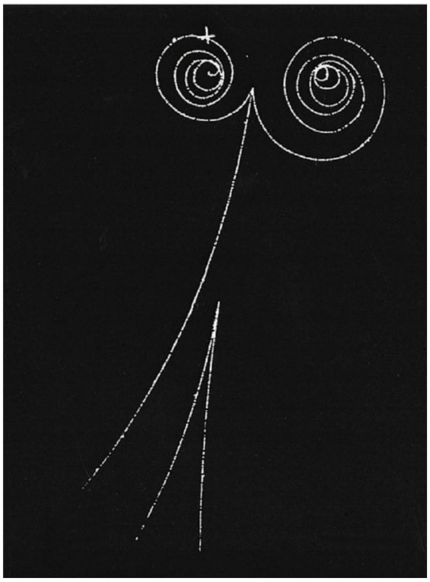


Figure 7. Particle tracks in bubble chamber

In bubble chamber, liquid hydrogen at its boiling point fills a chamber. When a charged particle passes through the liquid, bubbles of vapor form along the path of the particle. Usually, a strong magnetic field is applied across the bubble chamber, causing the particles to move in curved paths as shown in Figure 6. By considering the direction of the curvature, the type of charge can be determined.

By measuring the radius of curvature, the mass and speed of the particle can be calculated. This type of detectors are used in high energy particle research where physicists investigate the basic structure of matter.

Activity 8

Identify the negatively charged particle from the track in the bubble chamber shown in the Figure 6. Assume what is seen in the picture is along the direction of the magnetic field which is perpendicular to the plane of the paper.

Photographic emulsion

The photographic emulsion is the oldest of all particle detectors having figured in the discovery of radioactivity by Becquerel in 1896. At the time of the emulsion, a paper that contains chemicals which are sensitive to light, reacted by fogging the plate. Later individual tracks of alpha particles and protons were observed and in 1940's technical development converted it to be used as a basic research tool.

The specific features of the track such as its grain density and its deviation from straight line could be used to have detailed information about the mass, speed and charge of the particle that made the track.

Scintillation detectors

It was William coolers who noted, a flash of light, or scintillation when an alpha particle strikes a thin layer of zinc sulphide. Crooker alpha particle detector was very useful for the purpose of counting alpha particles when the atom was being investigated.

Modern scintillation counters use electronic means to convert weak flashes of light in to measurable electrical currents. By suitable choice of phosphor most radiations may be detected by this method. This modern scintillation counters, can detect all the three types of radioactive emissions.

Solid-state detectors

A solid-state detector is a p-n junction semiconductor, which produces a current pulse whenever it is exposed to ionizing radiation. The amplitude of this pulse depends on the extra electron-hole pairs produced. Alpha particles produce the largest pulses but still require amplification before they can be counted by scalar or rate meter. By suitable choice of solid-state materials, α and β particles or γ-rays may be detected.

The energy required to produce an ion-pair in conventional chambers is 10 times larger than the energy needed to produce an electron-hole pair in silicon.

Background radiation

Cosmic rays and radioactive materials provide a general background radiation, which can be detected by a suitable device. It is necessary therefore to take a count or reading before and after an experiment involving a radioactive source. The average of these reading is called the background count. This count should be measured over a suitable period of time (say 8 minutes) and must be subtracted from all experimental readings to obtain results due to the activity of the source alone. The background count rate recorded shows random variations.

Exercises

1. A Geiger-counter placed 20 cm from point source of gamma radiation registers a count rate of 6000 per second. What would the count rate be 80 cm from the source?
2. How many ion-pairs are formed if the detector were solid state instead of cloud chamber?
3. A student uses a Geiger-counter and measures background radiation as 50 counts per minute. She then measures the count rate from the sample, and records 1650 counts per minute. If the half-life of the sample is 56 seconds. How long will it take for the count rate to be similar to that of the background radiation?

Before the discovery of neutron, scientists thought protons and electrons were elementary particles because they were stable. But experiments with particle accelerators have demonstrated that new particles are often discovered in high energy collision between known particles. The particles are unstable. They have a very short half-life. So far more than 300 new particles are known to exist. Mesons are one of them. It is mesons that hold atomic nuclei together, constantly being exchanged among the protons and neutrons that make up the nucleus.

Key Terms

- Weak interactions
- Strong interactions
- Sub-atomic particles
- Anti-particles
- Interaction mediators
- Strange particles
- Hadrons
- Leptons

Fundamental interactions between particles

The key to understanding the properties of elementary particles is to be able to describe the interaction between them. All particles in nature are subject to four fundamental interactions: gravitational, electromagnetic, strong, and weak nuclear interactions.

Gravitational interactions

You are familiar with gravitational interactions at least in lifting your bags and in football fields or basketball courts. The same interaction is responsible to keep planets in their orbits. It is this interaction that holds, the planets, stars, and galaxies together. But its effect on elementary particles is negligible.

Electromagnetic interactions

As the name implies it includes both electric and magnetic interactions, both of which are results of the same property of matter, the electric charge. This interaction is responsible for binding atoms and molecules. The underlying cause of the contact forces we experience at microscopic scale such as friction, the electric shock we feel in quite different situations are all electromagnetic interactions.

Strong nuclear interactions

It is this interaction that binds protons and neutrons together to form an atomic nucleus. If it were not for the dominance of the strong nuclear interaction, the electromagnetic interaction between the protons, which is manifested as repulsion, would make the protons fly apart. Hence, the nucleus couldn't exist.

Weak nuclear interactions

It is an interaction which is responsible for some nuclear reactions, beta decay. It is also responsible in controlling the rate of some of the nuclear reactions that occur in stars.

Comparison of interactions

The gravitational interactions and the electromagnetic interactions are long-range interaction and obey inverse-square law. The magnitudes of these forces decrease with distance but never becomes zero. The range of these two forces is infinite.

The strong interaction is short-ranged and negligible for separations greater than the size of a nucleus, which is about 10^{-15} m. The range of the weak interaction is less than 10^{-16} m.

The gravitational interaction is the weakest of all fundamental interactions. On relative strength scale where the strong interaction has a value of 1 the electromagnetic interaction has a value of 10^{-2}, the weak interaction has a value of 10^{-5}, and the gravitational force has a value 10^{-39}. These relative strengths are merely estimates and they depend on the considered range of interaction.

The weakest is dominant

Objects in the macroscopic world of direct human experience are beyond the range of the strong and weak interactions. The direct effect of these forces is seen in experiments using high energy accelerators that probe into the nucleus.

In the case of electromagnetic interaction, the electric charge on macroscopic objects under normal condition is often too small to produce noticeable effect. The amount of negative charge is almost exactly equal to the amount of positive charge which makes the object neutral. Nevertheless, it is the basis of a large part of our modern technology.

The gravitational force is often the dominant force on objects we regularly encounter, but it is by far the weakest of the fundamental interactions.

Activity 9

(Library research)
Describe the so-called electro weak force.

Conservation laws in particle physics

We have seen that the largest scientific instruments ever built are used to study the microscopic things known to exist, the elementary particles. These particles are created by accelerating α-particles, protons, neutrons and others nearly to speed of light, then making them collide with other particles. The objects of investigation at the colliding particles and 'by products' from the collision.

Conservation laws are the key to examine the properties of these elementary particles and the forces that exist between them. A conservation law demands that the total amount of a physical quantity such as energy, linear momentum or angular momentum remains the same regardless of what goes on during a collision.

Classical conservation laws, (Revisited)

We use the laws of conservation of mass, energy, linear momentum, angular momentum, and charge to derive information about physical systems on a 'just before and just after' basis.

These laws of classical physics were developed from the observed behavior of macroscopic systems. The same laws are found to apply equally well to the interaction of the elementary particles.

When two particles collide, linear momentum of the system must be conserved. Likewise, mass-energy must be conserved. The implication of these laws for particle physics is that no single particle can spontaneously decay into particles whose mass exceeds its own. One might imagine, for example, that the high-speed particle could suddenly self-annihilate, converting some of its kinetic energy into mass to form another particle of greater mass moving more slowly. A careful analysis of this situation shows if these were to happen either i) Conservation of mass-energy would be violated, or ii) Conservation of linear momentum will be violated.

Conservation laws and the photon decay

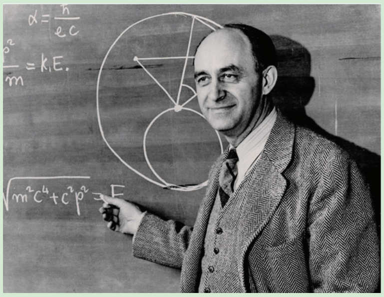
In spontaneous decay of a photon to produce an electron and positron we see that the net charge after the decay is the same as the charge before the decay, which is zero. This is obeying law of conservation of charge. Besides, we can apply the law of conservation of angular momentum that we used to analyze spinning ballet dancer and orbiting planet. Obeying law of conservation of angular momentum, the spin of the departing electron and positron must lie along the same direction so that the sum is equal to one, which is the spin of the photon.

Note: Spin, like mass and charge, is an intrinsic property of all elementary particles. It measures the angular momentum carried by the particle. The spins of all known particles are either integral or half-integral multiples of the basic unit $\frac{h}{2\neq}$, where h is Planck's constant.

Conservation laws and the neutrino

The β-particles emitted during β-decay have a range of velocities, which implies that they have different kinetic energies. Hence, another particle emitted with different kinetic energies must be involved so that the total energy remains constant.

Historical Note



Fermi Enrico (1901-1954) Italian physicist was both a theorist and an experimentalist. He was a brilliant student and earned his doctorate at the age of 20. He decided to use the uncharged neutrons instead of alpha particles to induce artificial radioactivity. Fermi and his group produced about 40 radioisotopes, for which he wins the 1938 Nobel prize. Fermi discovered the quantum statistical laws governing the particles called the fermions.

In 1930, W. Pauli proposed that the missing energy may have been carried off by undetectable neutral and light particles. Shortly, there after E. Fermi named this particle the neutrino (little neutral one). The decay process can be written as:

$$\text{(neutron)}\ {}^{0}_{1}\text{n} \longrightarrow {}^{+1}_{1}\text{p} + {}^{-1}_{0}\text{e} + \overline{\upsilon}\ \text{(anti neturino)} \qquad 6.1$$

$$\text{(Proton)}\ {}^{+1}_{1}\text{p} \longrightarrow {}^{0}_{1}\text{n} + {}^{+1}_{0}\text{e} + \upsilon\ \text{(neutrino)} \qquad 6.2$$

Did you know?

The decay of the proton into a neutron and a positron is sometimes called inverse β-decay.

Furthermore, if the β-particle has an angular spin in one direction there must be another particle with an equal spin in the complementary direction for the angular momentum to be conserved. The neutrino (υ, read as nu) and antineutrino $\left(\overline{\upsilon}\right)$ are involved in β^+ particle $\left(\text{positron, } {}^{+1}_{0}\text{e}\right)$ and β^- particle (electron or negatron, $\left({}^{-1}_{0}\text{e}\right)$ emission respectively. It is to ensure the processes obey conservation of energy and conservation of angular momentum.

Note: The different name 'negatron' is to give an emphasis that outer most electrons are not emitted in decay process.

Absence of the expected has led to the new

The fact that a certain outcome is never seen regardless of the number of times a particular collision happens has led physicists to suspect that there exist additional conservation laws that operate only at subatomic scale.

The recognition and the use of these additional conservation laws have led to a remarkable progress in particle physics.

Before we consider the new conservation laws let us get acquainted with the groups of the elementary particles.

Classification of elementary particles

By mid-1930, the proton, neutron, electron, positron, photon and the neutrino (a charge less, massless particle that interacts very weakly with matter) were considered to be elementary particles. Since that time hundreds of additional elementary particles have been discovered.

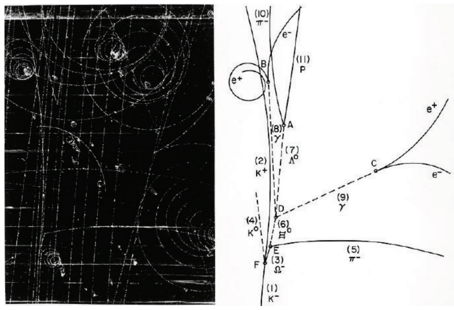


Figure 8. Subatomic particles track

ACTIVITY 10

(Web-search)

How many subatomic particles have been discovered as of today?

As more subatomic particles were discovered it was found that they could be classified just as we have classified matter into solid, liquid, gas and plasma state or as element, compound and mixture.

Elementary particles can be classified on the basis of spin, on the basis of mass or on the basis of interaction. A particle and its interactions are inseparable; the best approach to understand the properties and behavior of the particles is through their interaction.

The interaction has the mediator

In the same way that we come to understand the details of a supernova explosion by the particles emitted during the interaction, particle physicists come to know the characteristics of the four basic interactions by the particles involved in these interactions.

In particle physics the interaction of matter is usually described in terms of the exchange of special particles. In the case of electromagnetic interaction, the particles are photons. Thus, the electromagnetic force is said to mediated by photons.

Did you know?

The interaction of charged particles by the exchange of photons is described by a theory called quantum electrodynamics.

Similarly, the strong interaction is mediated by particles called gluons, the weak interaction is mediated by particles called the W and Z bosons, and the gravitational interaction is mediated by gravitons. All of these except gravitons have been detected.

Leptons and Hadrons are not mediators

All particles other than the mediating field particles can be classified into two broad categories: Leptons and Hadrons. The difference is based on whether they interact through the strong interaction or not. Leptons are particles that interact through the weak, electromagnetic and gravitational interactions. Hadrons are group of particles that participate in the four basic interactions, including the strong interaction.

Activity 11

(Library search)

On what basis does elementary particles are classified as Fermions and Bosons.

Leptons are considered as elementary particles

Electrons and neutrinos are both leptons. They have no measurable size or internal structure and do not seem to break down into smaller units.

Currently, Scientists believe there are only six leptons: the electron (e^-), muon (μ^-) and tau lepton (ι^-) and their corresponding neutrinos, with a symbol υ_e, υ_μ and $\upsilon\iota$ respectively. Each of these six leptons has an anti-particle.

Hadrons are thought to be made of quarks

Hadrons, the strongly interacting particles, can be further divided in to two classes: mesons and baryons. Baryons and mesons are distinguished by their internal structure. The most common examples of baryons are protons and neutrons. All mesons are unstable. Thus, they are not considered as constituents of matter.

Did you know?

Earlier classification schemes of particles were based on their masses. Leptons means ‘light one’ in Greek and hadron has its Greek origin ‘adros’ to mean ‘thick’ or ‘strong’.

Particle-collision experiments involving hadrons seem to involve many short-lived particles, implying that hadrons are made up of more fundamental particles.

In 1963, Murray Gell-Mann and George Zweig independently proposed that hadrons have a more elementary structure, which came to be called quarks.

The six quarks which seem to fit together in pairs are: up and down, charm and strange, and top and bottom.

All quarks have a charge associated with them. The charge of hadron is equal to the sum of the charges of its constituent quarks, and it is either zero or a multiple of fundamental unit of charge, e. This implies that quarks have a very unusual property having fractional electric charge. In other words, the familiar elementary charge, e is not the smallest possible charge that a particle can have.

The difference between mesons and baryons is due to the number of quarks that compose them. Mesons contain a quark and an anti-quark. Baryons contain three quarks as shown in schematic diagram, Figure 9.

Figure 9. Schematic diagram to show composition of mesons (pion, π^+ and Kaon k^-) and baryons (proton, p^+ and neutron, n)

The 'new' conservation laws

In previous section we have seen that the same laws which are familiar to us in classical physics are found to apply equally well in elementary particle interaction. However, some reactions that are not forbidden by the classical conservation laws are never observed. Thus, in particle physics, it is widely assumed that any reaction that is not strictly forbidden will occur although it is not frequently observed.

Conservation of Baryon and Lepton numbers

Consider a collision between two protons. Out of many particles that can be produced in such an interaction, mesons are never seen as a single product, even in cases like shown below, which does not violate any of the four classical conservation laws.

$$ {}_{1}^{+1}p + {}_{1}^{+1}p \xrightarrow{\times} \pi^{+} + \pi^{+} + \pi^{0} \qquad (6.14)$$

Alternatively, mesons may be the single product of the collision between a proton and antiproton.

A possible reaction of this type is the following:

$$^{+1}_{1}p + ^{-1}_{1}p \longrightarrow \pi^{+} + \pi^{-} + \pi^{0} \qquad (6.15)$$

Evidently, there seems to be a hidden conservation law that prevents the first reaction eq 6.14 from happening yet allows the second eq. 6.15

Investigation of instances similar to that shown above have led particle physicists to formulate the law of conservation of baryon number.

Conservation of baryon number is obeyed in all interactions: strong, weak and electromagnetic.

Like the law of conservation of charges, the law of conservation of baryon number is a simple counting rule. Each baryon is assigned baryon number B = + 1 and each antibaryon is assigned baryon number B = −1. Leptons and mesons have baryon number zero. The law of conservation of baryons can be stated as:

In a particle interaction, the baryon number (B) must remain constant.

If we consider eq. 6.14, the proton-proton reaction, the initial baryon number is 2 while the product baryon number is zero. However, in the interaction between proton and antiproton, the baryon number is conserved.

Note: Every known particle has a corresponding antiparticle, a charge-opposite version of an ordinary particle. For example, Anti proton (p^-) is an anti-particle of proton (p^+).

Just as conservation of charge is responsible for the stability of the electron, conservation of baryon number is the root cause of stability of the proton.

A conservation law similar to that adhered to the baryons exists for leptons. Leptons are involved in weak interactions. It is observed that whenever leptons are created, they are created in pairs. This observation has led to a new conservation law called the conservation of leptons:

> In a particle interaction, lepton number (L) is conserved, with in each lepton family.

The six known leptons are divided into three families. The electron (e^-) and its neutrino (υ_e), the muon (μ^-) and its neutrino (υ_μ), the tau (τ^-) and its neutrino (υ_τ). They are all given a lepton number L = +1. The antiparticle of each e^+, $\bar{\upsilon}_e$, μ^+, $\bar{\upsilon}_\mu$, τ^+ and $\bar{\upsilon}_\tau$ are all given lepton number L = −1.

Conservation of Lepton number explains why the following reaction is not observed even though it doesn't appear to violate any conservation laws.

$$\mu^- \not\longrightarrow e^- + \gamma$$

muon electron photon

Muon is an elementary particle that is the same as the electron in all respects except for its rest mass. It happens to have rest mass of 207 times that of an electron. They were discovered in 1936 in cosmic rays. At sea level cosmic rays consist mainly of muons and electrons. They are in the ratio of 4 electrons to 1 muon. The reason why ordinary matter is not made up of muons as that of electrons is because muons can decay by weak interaction in to electrons with half - life of 1.5×10^{-6} seconds:

$$\mu^- \longrightarrow \upsilon_\mu + e^- + \bar{\upsilon}_e$$

Muon lepton No. $(+1) \rightarrow (+1) + (0) + (0)$

Electron lepton No. $0 \rightarrow 0 + (+1) + (-1)$

Charge $(-1) \rightarrow 0 + (-1) + (0)$

The above reaction satisfies all the relevant conservation laws.

Example

When high energy positive muons collide with electrons, they occasionally produce two neutrinos:

$\mu^+ + e^- \rightarrow 2\upsilon$

Name the kind of neutrinos produced.

Solution

We are looking for particles which completes the reaction in such a way that electric charge is conserved, muon lepton number and electron lepton number are conserved.

$$\mu^+ + e- \rightarrow 2\upsilon$$

Charge $(+1) + (-1) \rightarrow (0)$

Muon lepton No. $(-1) + (0) \rightarrow ?,\ \bar{\upsilon}_\mu$

Electron lepton No. $(0) + (+1) \rightarrow ?,\ \upsilon_e$

Because $\bar{\upsilon}_\mu$ is within the muon family with a lepton number (-1) and υ_e is within electron family with lepton number $(+1)$ which fits to the reaction with no violation to the conservation laws.

The Mesons

Just as the leptons are the particles of the weak interaction, the hardons are the particles of the strong interaction. Strong interactions take about 10^{-23}s to act while weak interactions takes about 10^{-10} s.

The two longest-lived mesons are the pion and the k-meson (kaon). Pions have spin zero and occur with negative, positive and neutral charge (π^+, π^-, π^o). As with photon, π^o is its own antiparticle. π^- is the antiparticle of pion particle, π^+.

Did you know?

The first man made pions were detected at Berkeley Synchrocyclotron.

Kaon has a mass of about one - half of a proton. They can be produced easily via strong interactions. A neutral kaon decays into two pions: $k^0 \rightarrow \pi^+ + \pi^-$ with half-life of 10^{-10}s instead of 10^{-23}s for strongly interacting particles (i.e a lifetime which is too long for strongly interacting particles).

Activity 12

Prepare a chart which shows classification of sub-atomic particles.

This property, where strongly interacting particles refuses to decay by strong interaction even though the decay products are also strongly interacting particles was strange for the physicists at the time of its discovery. Thus, they called it a strange particle and proposed the so called the law of conservation of strangeness. The kaons were assigned a new quantum number, s called strangeness which is conserved in strong and electromagnetic interaction but not in weak interaction. The law of conservation of strangeness can be written as:

> In strong and electromagnetic interactions, strangeness (s) is conserved; in weak interactions strangeness may change by ±1 unit. Strangeness is a partially conserved quantity.

Non-strange particles like nucleons, pions, have zero strangeness. k^+ and k^0 have S = +1 and their anti-particle k^- and k^0 have s = −1. In the decay, of neutral kaon, k^o,

$$k^o \rightarrow \pi^+ + \pi^-$$

Strangeness No. (+1) → (0) + (0)

Strangeness is not conserved. It has changed by −1 during the process. Thus, the decay mode must be via the weak interaction.

Did you know?

Murray Gell-Mann and Kazuhiko Nishijima independently proposed the quantum number, strangeness (s).

There were also some baryons heavier than proton discovered in 1950's, which are labeled as Lambdas ($\wedge^{o}$), Sigma's (Σ^{+}), Xi (Ξ^{-}) and Omega (Ω^{-}).

They are given strangeness of −1, −1, −2 and −3 respectively. The anti-particle of each will have the corresponding quantum number with opposite sign. They have half - lives of the order $\sim 10^{-10}$s.

In the decay of lambda $\wedge^{o}$ into a proton and pion with a half - life of 2×10^{-10}s:

$$\wedge^{o} \rightarrow p + \pi^{-}$$

Strangeness No. $-1 \rightarrow (0) + (0)$ weak interaction! (why?)

Lambda ($\wedge$) and neutral kaon (k^{o}) can be produced together by strong interaction in pion - proton collision:

$$\pi^{-} + p \rightarrow \wedge^{o} + k^{o}$$

Strangeness No. $(0) + (0) \rightarrow (-1) + (+1)$ conserved!

The 'twin' production in the context of conservation of strangeness explains how strange particles are produced by strong interaction, but, once produced, each strange particle can only decay by weak interaction.

Activity 13

Prepare a table which shows the link between quantities to be conserved in the conservation laws.

You have learned so far (Both classical and elementary particle physics) with interactions: strong, weak, and electromagnetic.

Although there are other conservation laws for the elementary particles, we have considered only three of them to show conservation laws of physics are in a sense stronger than other 'laws'.

Exercises

1. Complete the pion production given below
   (a) $p + p \rightarrow p + n + ?$
   (b) $p + n \rightarrow p + p + ?$

2. Which kind of neutrino $\left(\upsilon_e, \bar{\upsilon}_e, \upsilon_\mu, \bar{\upsilon}_\mu\right)$ is the correct choice for the reaction.
   $(?) + n \rightarrow p + \mu^-$
3. Consider a particle with a symbol π^-
   (a) Name the particle
   (b) Identify it as leptons or hadrons
   (c) Give its charge, baryon number and strangeness
   (d) Give its quark structure

The photoelectric effect cannot be explained by considering light as a wave. However, this phenomenon can be explained by the particle nature of light, in which light can be visualized as a stream of particles of electromagnetic energy. These 'particles' of light are called photons, which are massless and chargeless. After continuous research in this field, the explanation for the photoelectric effect was successfully explained by Albert Einstein. He concluded that this effect occurred as a result of light energy being carried in discrete quantized packets. For this excellent work, he was honored with the Nobel prize in 1921.

According to Einstein, the energy (E) of a photon is given by the equation.

$$E = hf$$

Where h = planks constant (6.626×10^{-34} J.s)

f = frequency of photon in Hz

Thus, it can be understood that different frequencies of light carry photons of varying energies. For example, the frequency of blue light is greater than that of red light. Therefore, the energy held by a photon of blue light will be greater than the energy held by a photon of red light.

Threshold energy or work function

For the photoelectric effect to occur, the photons that are incident on the surface of the metal must carry sufficient energy to overcome the attractive forces that bind the electrons to the nuclei of the metals. The minimum amount of energy required to remove an electron from the metal is called the threshold energy or work function (denoted by the symbol Φ).

KEY TERMS

- The minimum amount of energy required to remove an electron from the metal is called the threshold energy or work function.

For a photon to possess energy equal to the threshold energy, its frequency must be equal to the threshold frequency. It is the minimum frequency of light required for the photoelectric effect to occur. The threshold frequency is usually denoted by the symbol f_{th} and the associated wavelength (called the threshold wavelength) is denoted by the symbol λ_{th}. The relationship between the threshold energy (work function) and the threshold frequency can be expressed as follows.

$$\boldsymbol{\Phi = hf_{th}} = h\frac{c}{\lambda_{th}}$$

According to Einstein's explanation of the photoelectric effect:

The energy of photon	=	energy needed to remove an electron	+	kinetic energy of the emitted electron

i.e. $hf = W + E$

$$\mathbf{E_{photon} = \Phi + E_{electron}}$$

$$\Rightarrow \mathbf{hf = hf_{th} + \tfrac{1}{2}m_e v^2}$$

Where,

- E_{photon} denotes the energy of the incident photon, which is equal to hf
- Φ denotes the threshold energy of the metal surface, which is equal to hf_{th}
- $E_{electron}$ denotes the kinetic energy of the photoelectron, which is equal to $\frac{1}{2}m_e v^2$ (m_e = mass of electron = 9.1×10^{-31} kg)

If the energy of the photon is less than the threshold energy, there will be no emission of photoelectrons. Thus, the photoelectric effect will not occur if $f < f_{th}$. If the frequency of the photon is exactly equal to the threshold frequency ($f = f_{th}$), there will be an emission of photoelectrons, but their kinetic energy will be equal to zero.

Laws of photoelectric emission

- There is no time lag between the irradiation of the surface and the ejection of the electrons.
- At a particular fixed frequency of incident radiation the rate of the emission of photoelectrons (i.e. the photocurrent) increases with increase in the intensity of the incident light.
- Photoelectric effect does not occur at frequency less than threshold frequency.

- At the frequency above the threshold frequency, the kinetic energy of the ejected electrons depends only on the frequency of the exposed radiation and not on its intensity.

Applications of Photoelectric Effect

- Photoelectric effect is used to generate electricity in solar panels. These panels contain metal combinations that allow electricity generation from a wide range of wavelengths.
- Motion and position sensors: In this case, a photoelectric material is placed in front of a UV or IR LED. When an object is placed in between the Light-emitting diode (LED) and sensor, light is cut off and the electronic circuit registers a change in potential difference.
- Lighting sensors such as the ones used in smartphones enable automatic adjustment of screen brightness according to the lighting. This is because the amount of current generated via the photoelectric effect is dependent on the intensity of light hitting the sensor.
- Digital cameras can detect and record light because they have photoelectric sensors that respond to different colors of light.
- Photoelectric cells are used in burglar alarms.
- Photoelectric effect is used in photomultipliers to detect low levels of light.
- Photoelectric effect is used in video camera tubes in the early days of television.

Example

Monochromatic light of wavelength 400 nm strikes a plate of metal. This metal has a work function of 2.14 eV. Use Photoelectric Effect Formula to find the energy of the electrons that are ejected.

Solution

Planck's constant, $h = 6.62 \times 10^{-34}$Js

Wavelength, $\lambda = 400\text{ nm} = 400 \times 10^{-9}\text{ m}$

Thus frequency, $f = \dfrac{v}{\lambda} = \dfrac{3\times10^{8}\text{m/s}}{4\times10^{-7}\text{m}} = 7.5\times10^{14}\text{Hz}$.

Work function $= 2.14\text{eV} = 2.14\times1.6\times10^{-19}\text{J} = 3.424\times10^{-19}\text{J}$

Using the energy equation:

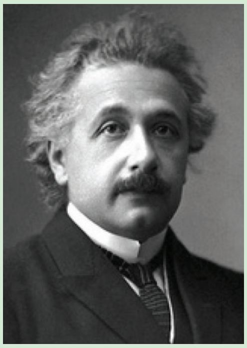
$$E = hf - \phi = 6.62\times10^{-34}\,Js\times7.5\times10^{14}\,Hz - 3.2424\times10^{-19}\,J$$

$$E = 4.965\times10^{-34}\,J - 3.424\times10^{-19}\,J$$

$$= 1.54\times10^{-19}\,J = 0.963eV$$

HISTORICAL NOTE

ALBERT EINSTEIN (born March 14, 1879, Germany died April 18, 1955, U.S.)



Albert Einstein was a German-born theoretical physicist, widely acknowledged to be one of the greatest and most influential physicists of all time.

Some of Einstein's inventions list, theories, and findings are:

1. **Quantum Theory of Light:** Einstein proposed his theory of light, stating that all light is composed of tiny packets of energy, called photons. He suggested these photons were particles but also had wave-like properties, a totally new idea at the time.
2. **Special Theory of Relativity:** This Einstein theory helped to explain that time and motion are relative to their observers, as long as the speed of light remains constant and natural laws are the same throughout the universe.
3. **Avogadro's Number:** On the existence of atoms, he could calculate the size of the atom and Avogadro's number.
4. The **photoelectric effect:** for which he had received the Nobel Prize for Physics. This theory later led to the invention of the **television**, which gave technologists a vision to come up with modern day screen devices *(smartphones, computers, laptops)*.
5. His famous equation: $E = mc^2$. He demonstrated the link between *mass* and energy that led to the **nuclear energy** today. This provided the first mechanism to explain the energy source of the Sun and other stars.
6. **Brownian Movement**: His observation of the zigzag movement of particles in suspension, helped to prove the existence of **atoms** and **molecules**. And we all know how fundamental this discovery is to almost every branch of science today.
7. **General Theory of Relativity:** Einstein proposed that gravity is a curved field in the space-time continuum created by the existence of mass.
8. **Manhattan Project:** Albert Einstein created the Manhattan Project, a research supported by the U.S., that led to the development of the atomic bomb in 1945.

9. **Einstein's refrigerator:** This may be one of the least known inventions that Einstein is famous for today. Einstein developed a refrigerator design that used ammonia, water, and butane, and required almost no energy to work.
10. **Sky is blue:** Though this seems to be a simple explanation, however, Einstein help put this argument to rest.

Note that these are very few contributions of Albert Einstein there are many more.

Throughout his life, Einstein published hundreds of books and articles. He published more than 300 scientific papers and 150 non-scientific ones. On 5 December 2014, universities and archives announced the release of Einstein's papers, comprising more than 30,000 unique documents. Einstein's intellectual achievements and originality have made the word "Einstein" synonymous with "genius".

Summary

- Fundamental laws of quantum mechanics are laws of probability rather than laws of certainty.
- Both particles and wave have dual nature.
- In Newtonian mechanics we consider a particle as a point.
- Particles are considered as a wave to understand the subatomic interactions.
- Many aspects of particle behavior is treated only in terms of probabilities.
- An atom can exist in a few sharply definite states of energy.
- A more recent and useful model of atomic structure is the electron cloud model.
- Cosmic rays are the natural sources of high energy particles produced by the sun and other stars.
- The devices used in elementary particle research are basically of two kinds, those which are used to create particles, the accelerators and those used to detect particles, detectors.
- There are four fundamental interactions in nature: weak, strong, electromagnetic and gravitational. Each interaction has mediator particles: Bosons, gluons, photons and gravitons respectively.
- The constituents of matter can be classified as leptons and hadrons. Hadrons can be further divided into mesons and baryons. Mesons and baryons consists of quarks.
- The laws of classical physics apply equally well to the interaction of elementary particles. They also led scientists to formulate new conservation laws.

- Weak, strong and electromagnetic interaction obey conservation of baryon number, lepton number in addition to the four classical conservation laws.
- Weak interaction violates conservation of strangeness.

Review Exercises

1. Is there a limitation on precision of our measurements in the macroscopic world? Microscopic world? If yes, what is this limitation?
2. What is the information we could obtain from quantum number of an atom?
3. According to an electron cloud model, where could an electron of an atom be found?
4. Name four particle accelerators and four particle detectors. Which accelerators are mostly used in hospitals?
5. Film badges are worn by people who are exposed to radiation in their jobs. What do you think the plastic badge contains?
6. Describe the four basic interactions.
7. Which of the four conservation laws in classical physics are obeyed by elementary particles?
8. Which of the four basic interaction violates the law of conservation of strangeness?
9. What is an anti-particle?
10. Compare muon with an electron.
11. How many quarks and anti quarks are there in the following particles a) a meson b) a baryon
12. A photon and an electron both have kinetic energies of 1 eV. What are the corresponding wavelengths?
13. In Bohr model of the hydrogen atom, what is the de Broglie wavelength for the electron when it is in $n = 4$ level
14. If a Geiger counter that is one meter away from the point source measures a count rate of 100 counts per minute, what will be its reading 2 meter from the source?
15. What is the principal quantum number corresponding to an energy level in hydrogen atom with energy 0.544 eV?
16. An atom in meta-stable state has a lifetime of 5.2 ms. What is its uncertainty in energy of its meta-stable state?
17. Identify the conservation laws that would be violated in each of the following reactions:

    (a) $\pi^{+} \rightarrow e^{+} + \gamma$ (b) $\pi^{-} + p \rightarrow k^{o} + p + \pi^{o}$

18. Figure out the missing particle

    (a) $p + \bar{p} \rightarrow n + (?)$ (b) $p + \bar{\upsilon}_{\mu} \rightarrow n + (?)$

19. Identify a reaction which involves weak interaction.

    (a) $\pi^- + p \rightarrow \wedge^o + k^o$ (b) $\wedge^o \rightarrow p + \pi^-$

20. Deuterons ($q = e$, $m = 3.3 \times 10^{-27}$kg) with KE of 24 KeV are injected near the center of a cyclotron with B = 1.7T and cyclotron diameter 1.1m.

    (a) What should be the angular frequency of the oscillator used to accelerate the deuterons $\left({}_{1}^{2}H\right)$?

    (b) Determine the final energy of the deuterons in electron volts.

21. A radiation of wavelength 300 nm is incident on a silver surface whose work function is 4.7 eV. Will photoelectrons be observed?

Sample Test

1. Suppose hydrogen atom is in its ground state. How much energy is needed to ionize it?
   (a) 3.4 eV (c) 13.6 eV
   (b) 1.51 eV (d) 1.89 eV
2. Which one of the following is the first antiparticle discovered?
   (a) Antineutrino (c) Lambda
   (b) Positron (d) Sigma
3. Which one of the following is a force carrying particle?
   (a) Muon (c) Photon
   (b) Kaon (d) neutrino
4. Which one of the following particles are grouped under baryon?
   (a) Neutrons (c) Muons
   (b) Photons (d) Electrons
5. Identify a particle with a lepton number, L= +1.
   (a) t^+ (c) e^-
   (b) m^+ (d) All of the above
6. In which one of the following interactions is conservation of baryon number obeyed?
   (a) Strong interaction (c) electromagnetic interaction
   (b) Weak interaction (d) All of the above
7. Which one of the following is a partially conserved quantity?
   (a) Lepton number (c) Baryon number
   (b) Strangeness (d) Spin
8. In which one of the following interactions is angular momentum conserved?
   (a) Weak interaction (c) Strong interaction
   (b) Electromagnetic interaction (d) All of the above
9. Which particle is missing in the reaction?
   $\bar{\nu}_{\mu} + p \rightarrow n + ?$
   (a) $\bar{P}$ (b) $\propto^{-}$ (c) $\propto^{+}$ (d) ν_{μ}
10. Which one of the following quantities is NOT conserved in interaction taking place by weak nuclear force?
    (a) Linear momentum (c) Strangeness
    (b) Electric charge (d) Mass- energy

11. Which one of the following particles is massless and chargeless?
    (a) Muon
    (b) neutrino
    (c) Kaon
    (d) All of the above
12. Which one of the following is a long-range interaction?
    (a) Strong interaction
    (b) Gravitational interaction
    (c) Weak interaction
    (d) Electromagnetic interaction
13. Which one of the following particles is believed to be stable?
    (a) Photon
    (b) Proton
    (c) electron
    (d) Neutron
14. Which conservation law is obeyed by subatomic particles?
    (a) Conservation of mass-energy
    (b) Conservation of linear momentum
    (c) Conservation of angular momentum
    (d) All of the above
15. A quark and an antiquark combine to form,
    (a) a photon
    (b) an electron
    (c) a méson
    (d) a baryon
16. Which set of interactions is in decreasing order?
    (i) Strong interaction
    (ii) Gravitational interaction
    (iii) Weak interaction
    (iv) Electromagnetic interaction
    (a) 1,2,3,4
    (b) 1,3,2,4
    (c) 1,3,4,2
    (d) 3,1,4,2
17. In which of the following detectors does measurement of radioactivity depend on tiny flashes emitted when ionizing radiation is incident on them?
    (a) GM tube
    (b) Cloud chamber
    (c) Scintillator
    (d) Bubble chamber
18. Which of the following particles participate in strong interaction?
    (a) Baryons
    (b) Leptons
    (c) Neutrinos
    (d) All the above

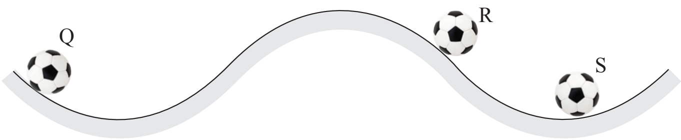
19. Which of the following quantities doesn't affect the cyclotron angular frequency?
    - (a) Mass
    - (b) Charge
    - (c) Magnetic field
    - (d) Speed
20. A cyclotron can accelerate,
    - (a) α-particle
    - (b) b^+ particle
    - (c) b^--particle
    - (d) All of the above
21. One can reduce exposure to radiation by:
    - (a) shielding
    - (b) keeping distance from the source
    - (c) reducing time of being around the source
    - (d) All of the above
22. What force is responsible for the radioactive decay of a nucleus?
    - (a) Weak nuclear force
    - (b) Electromagnetic force
    - (c) Strong nuclear force
    - (d) All of the above
23. Bohr's model of an atom considers:
    - (a) classical physics
    - (b) classical physics and quantum mechanics
    - (c) quantum mechanics
    - (d) none of the above
24. Bohr, In his model:
    - (a) assumed that electrons radiate energy when they are in a stable orbit
    - (b) explained why electrons always have certain stable orbits.
    - (c) gave an expression for radius of the atom and predicted energy level of hydrogen.
    - (d) all of the above.
25. What is the de-Broglie wavelength of an electron moving with a speed of 1.45×10^7m/s?
    - (a) 5×10^{-11} m
    - (b) 2×10^{-11} m
    - (c) 3.6×10^{-11} m
    - (d) 1.9×10^{-11} m
26. Which nature of electrons makes the electron microscope possible?
    - (a) Wave nature
    - (b) Particle nature
    - (c) Wave and particle nature
    - (d) None of the above

27. Which conservation law is violated in the reaction, $K^- + p \rightarrow n + \neq^0$
    (a) Law of conservation of baryon number
    (b) Law of conservation of charge
    (c) Law of conservation of strangeness
    (d) None of the above
28. Which particles give rise to heavy and short tracks in cloud chambers?
    (a) α-particle
    (b) b^+ particle
    (c) b^--particle
    (d) Neutrons
29. Among particle detectors, which one produces relatively very high energy particles?
    (a) Synchrotron
    (b) Cyclotron
    (c) Linear accelerator
    (d) All of the above
30. Which photon emission in excited hydrogen atom is in the visible range?
    (a) Transition to n = 3 from higher energy levels
    (b) Transition to ground state from high energy levels
    (c) Transition to n = 2 from higher energy levels
    (d) Transition to n = 4 from higher energy levels

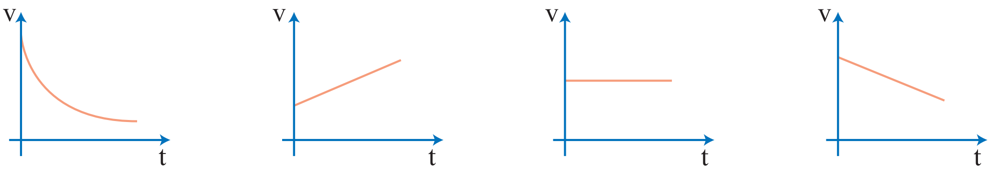
SAMPLE TEST-1

Now answer the following questions

1. The area under a force-time graph represents:
   (a) work done
   (b) change in internal energy.
   (c) change in kinetic energy
   (d) change in momentum
2. From the principle of flotation, a body sinks in a fluid until it displaces a quantity of fluid equal to its own:
   (a) weight
   (b) volume
   (c) density
   (d) mass
3. Suppose three identical steel balls Q, R and S are placed on an undulating ground as illustrated in the diagram above. Which of the balls is/are in neutral equilibrium?
   (a) R only
   (b) Q and S only
   (c) S only
   (d) Q only



4. The length of a simple pendulum is increased by a factor of four. By what factor is its period increased?
   (a) 2π (b) 4 (c) 0.5π (d) 2
5. The linear expansivity, α, and cubic expansivity, γ, of a material are related by the equation:
   (a) $\gamma = 2\alpha$
   (b) $\gamma = \frac{1}{3}\alpha$
   (c) $\gamma = \alpha$
   (d) $\gamma = 3\alpha$
6. Which of the illustrated graphs below represents a body moving with uniform retardation?
   (a) A (b) B (c) C (d) D



7. The **correct** relationship between the displacement, s, of a particle initially at rest in a linear motion and the time, t, is:
   (a) $s \propto t$ (b) $s \propto t^2$ (c) $s \propto t^{-2}$ (d) $s \propto t^{-1}$

8. Joule is equivalent to:
   - (a) kg m^2 s^{-2}
   - (c) kg m^2 s^{-1}
   - (b) kg ms^{-1}
   - (d) kg m^2 s^{-2}
9. Which of the following statements(s) is/are **correct** about a fixed mass of gas compressed in an in-expansible container?
   - (i) The average speed of the gas particles increases.
   - (ii) The temperature of the gas increases.
   - (iii) The molecules hit the walls of the container more often than before the compression.
   - (a) I and III only
   - (c) I only
   - (b) I, II and III
   - (d) I and II only
10. An example of a mechanical wave is:
    - (a) X-rays
    - (c) water waves
    - (b) light rays
    - (d) radio waves
11. Which of the following descriptions of the image formed by a plane mirror is **not correct**?
    - (a) erect and of the same size as the object.
    - (b) laterally inverted and of the same size as the object.
    - (c) erect and bigger than the object.
    - (d) virtual and of the same size as the object.
12. The efficiency of a wheel and axle is 100% and the ratio of their radii is 5:1. Calculate the effort required to lift a load of mass 20kg using this machine. {g = $10ms^{-2}$]
    - (a) 40N
    - (b) 100N
    - (c) 20N
    - (d) 25N
13. A freely suspended compass needle on the earth's surface settles in a plane called:
    - (a) magnetic declination
    - (c) geographic meridian
    - (b) isoganals
    - (d) magnetic meridian
14. An object is placed at different distance, u, from a converging lens of focal length 15.0 cm. For what value of u does the lens act as a simple microscope?
    - (a) $u < 15$ cm only
    - (c) $u = 15$ cm only
    - (b) $u > 30$ cm only
    - (d) $u = 30$ cm only
15. The periodic rise and fall in the intensity of sound produced when two notes of nearly equal frequencies are sounded together is called:
    - (a) doppler effect
    - (c) interference
    - (b) beat
    - (d) resonance

16. The power of a lens is 02.5 D. What is its radius of curvature?
    (a) 40.0 cm (c) 2.5 cm
    (b) 80.0 cm (d) 25.0 cm
17. The reason for having a large number of turns in the coil of a moving coil galvanometer is to:
    (a) make the deflection of the needle proportional to the current.
    (b) increase the sensitivity of the galvanometer.
    (c) decrease the magnetic flux produced by the magnet.
    (d) make the permanent magnet stronger.
18. Current is passed through two parallel conductors in the same direction. If the conductors are placed near each other, they will:
    (a) remain stationary (c) move in a circle
    (b) attract each other (d) repel each other
19. The period of a 10 kHz radio wave travelling at 3.0×10^8 ms^{-1} is:
    (a) 3.0×10^{-5}s. (c) 1.0×10^4s.
    (b) 3.0×10^4s. (d) 1.0×10^{-4}s.
20. The magnetic material produced from the chemical combination of metal oxides and has a very high resistance to electric current is called:
    (a) ferromagnetic substance.
    (b) diamagnetic substance.
    (c) ferrite substance.
    (d) paramagnetic substance.
21. A body of mass, **M** moving with velocity, V, has a wavelength, λ, associated with it. This phenomenon is called:
    (a) Compton effect
    (b) wave-particle paradox.
    (c) photoelectric effect.
    (d) Heisenberg's uncertainty principle
22. Which of the following statements about a straight current - carrying wire placed in a uniform magnetic field is correct? The wire experiences.
    (a) maximum motor force if the current reverses its direction.
    (b) no motor force if it is parallel to the field.
    (c) no motor force if it is perpendicular to the field.
    (d) a motor force with constant direction fi either the current or the magnetic field is reversed.

23. Three cells each of emf 1.1V and internal resistance 2Ω are connected in parallel across a 3Ω resistor. Determine the current in the resistor.
    (a) 0.30A (c) 0.90A
    (b) 0.01A (d) 0.39A
24. Which of the following statements about electric potential energy is not correct?
    (a) The electric potential energy of a positively charged particle increases when ti moves to a point of higher potential.
    (b) The electric potential energy of a negatively charged particle increases when it moves to a point of lower potential.
    (c) The electric potential energy of a positively charged particle decreases when it moves to a point of higher potential.
    (d) The work done is taking a charged particle around a closed path in an electric field is zero.
25. A galvanometer with a full scale deflection of 20 mA is converted to read 8 V by connecting a 395 Ω resistor in series with it. Determine the internal resistance of the galvanometer.
    (a) 8.0 Ω (b) 10.0 Ω (c) 2.5 Ω (d) 5.0 Ω
26. An inductor of inductance 10 H is connected across an a.c. circuit source of 50 V, 100 Hz. What is the current in the circuit? [$\pi = 3.14$]
    (a) 0.050A (c) 0.200A
    (b) 0.008A (d) 0.070A
27. The speed of fast moving neutrons in a nuclear reactor can be reduced by using:
    (a) iron rods (c) graphite rods
    (b) boron rods (d) concrete shield
28. In a series R-L-C circuit at resonance, impedance is:
    (a) capacitive (c) maximum
    (b) inductive (d) minimum
29. A lamp is rated 240 V, 60 W. Determine the resistance of the lamp when lit.
    (a) 540 Ω (b) 960 Ω (c) 120 Ω (d) 240 Ω
30. One major reason why electrical appliances in homes are normally earthed is that the
    (a) person touching the appliance is safe from electric shock.
    (b) appliances are maintained at a higher potential difference than the earth.
    (c) appliances are maintained at a lower potential difference than the earth.
    (d) appliances are maintained at the same potential difference with that of the earth.

31. In doping an intrinsic semiconductor to produce a p-type semiconductor,
    (a) an acceptor element is added.
    (b) the semiconductor is connected to a battery.
    (c) the semiconductor is heated up.
    (d) a donor element is added.
32. Arrange the following radiation in order of increasing ionization of air.
    (i) Alpha
    (ii) Gamma
    (iii) Beta
    (a) i < iiii < i
    (b) i < iii < iii
    (c) i < iii < ii
    (d) ii < i < iii
33. If the kinetic energy of an electron is 100 eV, what is the wavelength of the de-Broglie wave associated with it? [h = 6.6×10^{-34} J s, e = 1.6×10^{-19}C. me = 9.1×10^{-31} kg]
    (a) 3.90×10^{-10} m
    (b) 5.50×10^{-14} m
    (c) 1.22×10^{-10} m
    (d) 4.10×10^{-10} m
34. Gamma rays are produced when:
    (a) high velocity electrons are abruptly stopped in metals.
    (b) energy changes occur within the nuclei of atoms.
    (c) energy changes occur within the electronic structure of atoms.
    (d) electrons are deflected in very strong magnetic fields.
35. The half-life of a radioactive substance is 15 hours. If at some instance, the sample has a mass of 512 g, calculate the time it will take 7/8 of the sample to decay.
    (a) 45 hours
    (b) 60 hours
    (c) 15 hours
    (d) 30 hours
36. When the direction of vibration of the particles of a medium is perpendicular to the direction of propagation of a wave, the wave is said to be:
    (a) mechanical
    (b) a sound wave
    (c) longitudinal
    (d) transverse
37. A hunter fires a gun at a point 408m away from a cliff. If he hears an echo 2.4 s later, determine the speed of the sound wave.
    (a) 400 ms^{-1}
    (b) 680 ms^{-1}
    (c) 340 ms^{-1}
    (d) 380 ms^{-1}

38. The distance between the fixed points of a mercury - in - glass thermometer is 30 cm. Determine the temperature when the mercury level is 10.5 cm above the lower fixed point.

(a) 35.0°C
(b) 40.5°C
(c) 28.6°C
(d) 30.0°C

39. Which of the following statements about electromagnetic waves is not correct?

(a) The electric and magnetic fields are at right angles to each other.
(b) They travel with the speed of light.
(c) They carry energy as they travel through space.
(d) They are longitudinal

40. The diameter of a brass ring at 30°C is 50.0 cm. To what temperature must this ring be heated to increase its diameter to 50.29 cm?

[linear expansivity of brass = 1.9×10^{-5} K^{-1}]

(a) 306.1°C
(b) 335.3°C
(c) 152.6°C
(d) 182.6°C

41. Which of the following concepts is a method of heat transfer that does not require a material medium?

(a) Convection
(b) Radiation
(c) Conduction
(d) Diffusion

42. Which of the following statements about light travelling from one material medium to another is not correct?

(a) The refracted angle is less than the incident angle if the speed is higher in the first material.
(b) It bends away from the normal if the speed is lower in the first material.
(c) Its wavelength does not change.
(d) Its frequency changes.

43. The engine of a car provides a forward force of 1240 N and the total resistive force on the car is 800 n. If the mass of the car is 1220 kg, determine the car has to travel from rest before acquiring a speed of 4 ms^{-1}.

(a) 11.1 m
(b) 5.5 m
(c) 44.0 m
(d) 22.2 m

44. A quantity of water at 0°C heated to 30°C. For each degree rise in temperature, its density will:

(a) falls and then rises.
(b) rises and then falls.
(c) rise steadily
(d) fall steadily

45. A small object of mass 50 g is released from a point **A**. Determine the velocity of the object when it reaches a point **B**, a vertical distance of 30 m below **A**. [$g = 10\ ms^{-2}$]

(a) $17.3\ ms^{-1}$ (c) $1.5\ ms^{-1}$

(b) $24.5\ ms^{-1}$ (d) $6.0\ ms^{-1}$

46. Molecules move in random motion within a liquid. The total internal energy of the liquid depends on all of the following except its:

(a) specific heat capacity

(b) melting point

(c) temperature

(d) mass

47. The viscosity of a fluid depends on the following factors except the:

(a) surface area of the fluid in contact.

(b) normal reaction in the fluid

(c) relative motion between the layers of the fluid.

(d) nature of the material of the fluid

48. Using vernier calipers, which of the following readings gives the correct measurement for the length of a rod?

(a) 4.125 cm

(b) 4.1254 cm

(c) 4.1 cm

(d) 4.13 cm

49. The basic principle of operation of a beam balance is:

(a) Hooke's law

(b) principle of moments

(c) Archimedes principle.

(d) law of flotation

50. In which of the following situations is friction not useful?

(a) Application of brakes

(b) Moving piston in a sleeve

(c) Operation fo a grinding machine

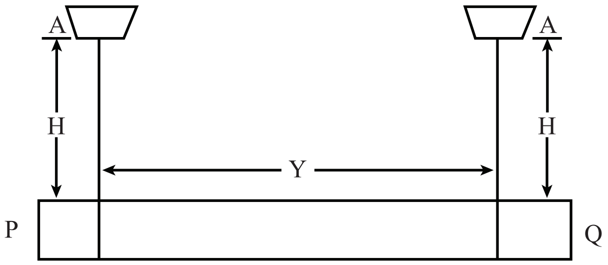
(d) Walking

SAMPLE TEST-2

Answer two questions Only

1.

(a)



A uniform meter rule is suspended symmetrically about its center by using two parallel strings each of fixed length, **H**, and spaced at a distance, **Y**, apart. The ends **P** and **Q** of the meter rule are displaced in opposite directions and released much that it oscillates about its mid - point.

The time, t, for 20 oscillations is measured and recorded. The procedure is repeated for four other values of **Y**.

Figure 1 (a) shows the raw values, y_1 while Figure 1 (b) shows the corresponding times t_1 where I = 1, 2, 3, 4, and 5.

(i) Measure and record the raw values y_i.

(ii) Convert y_i to values y_1 using the scale provided.

(iii) Read and record t_1.

(iv) Evaluate log **Y** and log t_i.

(v) Tabulate the results.

(vi) Plot a graph with log **Y** on the vertical axis and log t on the horizontal axis.

(vii) Determine the slope, s, of the graph.

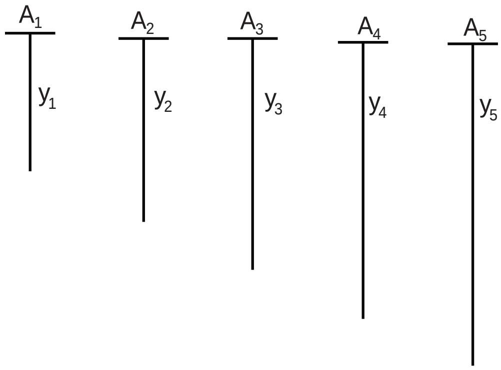
(viii) If **Y** and t are related by the equation log $Y = -\log t + \frac{1}{2}\log\frac{(2.63\times10^9)}{K}$ and the intercept on the vertical axis is 2.92, calculate **K**.

(ix) State two precautions necessary to ensure accurate results when performing this experiment in the laboratory.

(b)

(i) Distinguish between a couple and a torque.

(ii) Give two examples of torque in everyday life.



Scale 1 cm represents 10 cm

Figure 1 (a)

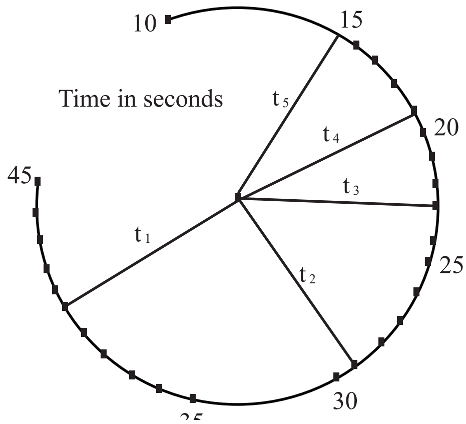
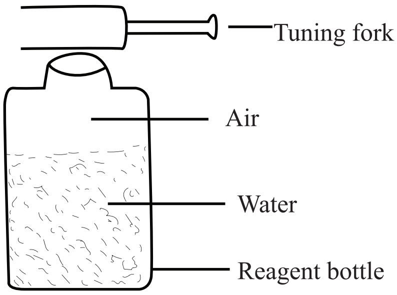


Figure 1 (b)



The diagram above is a set-up for a resonance experiment. A reagent bottle is filled with water up to the base of its neck. The water is poured into an empty measuring cylinder and its volume, V_o is read and recorded.

The water is then poured back into the reagent bottle. The air column is gradually increased by pouring out small volumes of the water and each time a vibrating tuning fork of frequency, f, is placed close to the mouth of the bottle, this process is continued until resonance occurs.

When resonance occurs, the water is poured into the measuring cylinder and its volume, V_o, is recorded. The bottle is refilled with water up to the base of its neck and the process is repeated for four other frequencies f.

Figure 2 (a) shows the volume, V_o and the volumes, V_i, at resonance, while Figure 2.(b) shows the corresponding frequencies, F_i, where i = 1, 2, 3, 4 and 5.

(i) Read and record V_o.

(ii) Read and record the volumes, V_i.

(iii) Evaluate $V_A = V_o - V_i$

(iv) Measure and record the values F_i.

(v) Convert F_i to real values f_i using the scale provided.

(vi) Evaluate $P = \frac{1}{f^2}$ in each case.

(vii) Tabulate the results

(viii) Plot a graph with VA on the vertical axis and P on the horizontal axis.

(ix) If VA and P are related by the equation: $V_A = \frac{K}{P} - C$ where K and C are constants, use the graph to determine the value of K.

(x) Use the graph to determine the volume VA which would resonate with a tuning fork of frequency 330.0Hz.

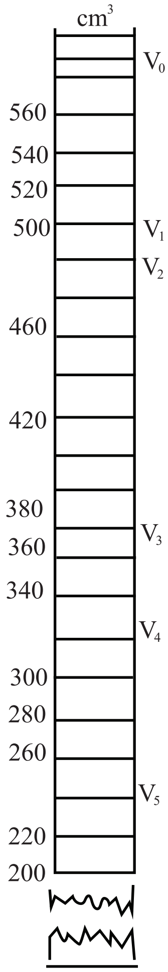
(xi) State two precautions necessary to ensure accurate results when performing this experiment.

(c)

(i) Explain the term resonance in sound.

(ii) A man can distinguish between a reflected sound from the original one if at least 0.10s elapses between them. Calculate the minimum distance between the listener and the reflector.

[Speed of sound in air = 343 ms^{-1}]



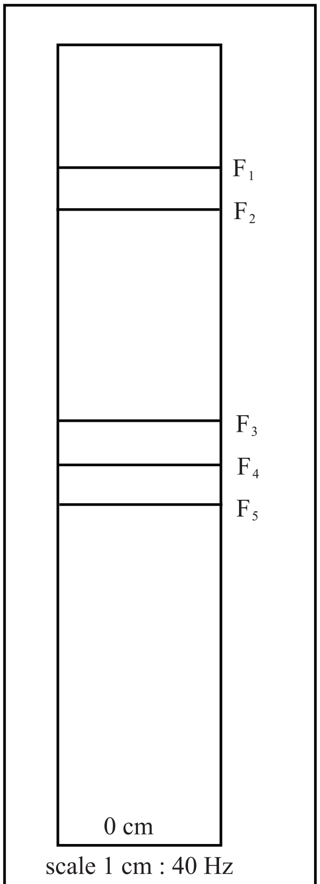
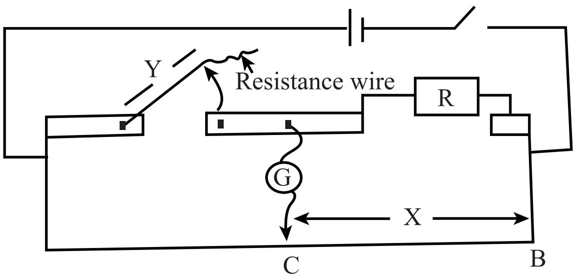


Fig. 2(b)

Figure 2 (a)



The diagram above shows a meter bridge circuit containing a resistance wire in the left hand gap and a fixed resistor, R, in the right side.

A crocodile clip is used to tap a length, **Y**, of the resistance wire in the circuit and the jockey is used to determine the balance point, **C**. The balance length **X** = **CB** is measured and recorded.

The procedure is repeated for four other lengths, **Y**, of the resistance wire.

Figure 3 a) shows the lengths, y_i, while Figure 3 b) shows the corresponding balance lengths, x_i, where i = 1, 2, 3, 4 and 5.

(i) Measure and record the values, y_i.

(ii) Use the scale provided to convert the values y_i to real values Y_i.

(iii) Measure and record the values, x_i.

(iv) Use the scale provided to convert the values x_i to real values X_i.

(v) Evaluate $z = (100 - x)$, $\frac{x}{z}$ and $T = \frac{10}{y}$ in each case.

(vi) Tabulate the results.

(vii) Plot a graph with $\left(\frac{x}{z}\right)$ on the vertical axis and T on the horizontal axis.

(viii) Determine the slope, s, of the graph.

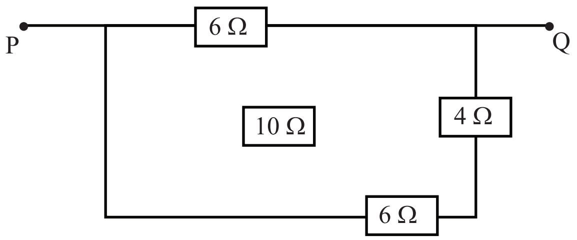
(ix) Given that $S = \frac{A}{P}$ where $A = 5.31 \times 10^{-4}$ cm^2, calculate P.

(x) State two precautions that are necessary to ensure accurate results when performing the experiment in the laboratory.

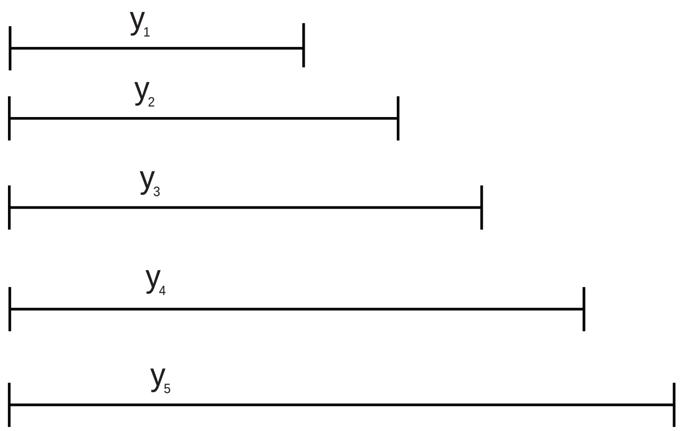
(d)

(i) Draw a graph to show the relationship between the resistance and temperature of a light dependent resistor.

(ii)

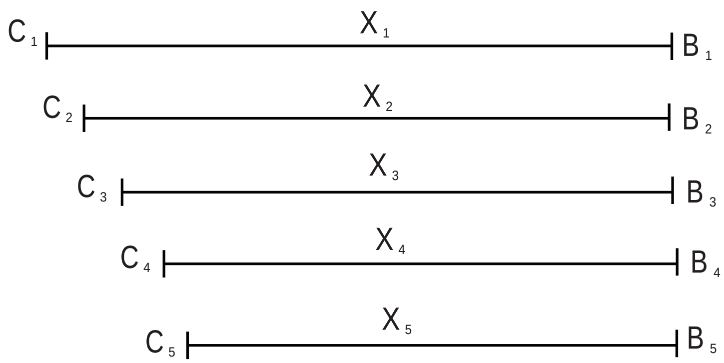


Calculate the equivalent resistance between the points P and Q in the circuit diagram above.



Scale 1 cm: 5 cm

Figure 3 (a)



Scale 1 cm: 5 cm

Figure 3 (6)

SAMPLE TEST-3

Answer two questions only from this section.

1. Express each of the following SI units in base units:
   (a) Pascal; (b) Joule; (c) Coulomb.
2.
   (a) Define force constant of an elastic material.
   (b) A graph of tensile stress is plotted against tensile strain. What does the slope of the graph represent?
3. A stone is projected with an initial speed, u, at an angle, θ, to the horizontal. Show that the maximum height, H_{max} attained is given by $H_{max} = \frac{u^2 Sin^2\theta}{2g}$, where is the acceleration due to gravity.
4.
   (a) Explain the term escape velocity of satellites.
   (b) A satellite is launched with a speed greater than the escape velocity. What would be the shape of the orbit?
5. State the:
   (a) method of reducing the depletion layer in a p-n junction.
   (b) reason why diodes are used as rectifiers.
6. State three uses of magnets.
7. An object is thrown horizontally with a speed of 25 ms^{-1} from the roof of a tall building. If the object takes 5s to hit the ground, calculate the height of the building. [g = 10 ms^{-2}]
8.
   (a) Define average velocity.
   (b) A body of mass 2.0 kg moving with a velocity of 10ms^{-1} undergoes elastic collision with a stationary body of mass 4.0 kg. Calculate the velocities of the bodies after collision.
   (c) A block of wood weighing 8N is placed on a horizontal table. It is pulled by a spring balance attached to one of its ends. The block just begins to move when the spring balance records 5N. Calculate the:
      (i) co-efficient of static friction;
      (ii) acceleration of the block when the spring balance records 12 N. [g = 10 ms^{-2}]
   (d) When a bicycle tyre is inflated, its pressure increases, explain the reason for this observation.

9.

(a) State three differences between evaporation and melting.

(b) Define each of the following terms:

(i) absolute zero temperature;

(ii) heat energy.

(c) A thermometer which was wrongly calibrated indicates -5^oC at the lower fixed point and 106^oC at the upper fixed point. Calculate the reading on the thermometer when the actual temperature is 60^oC.

(d) State two physical properties that are used in the measurement of temperature.

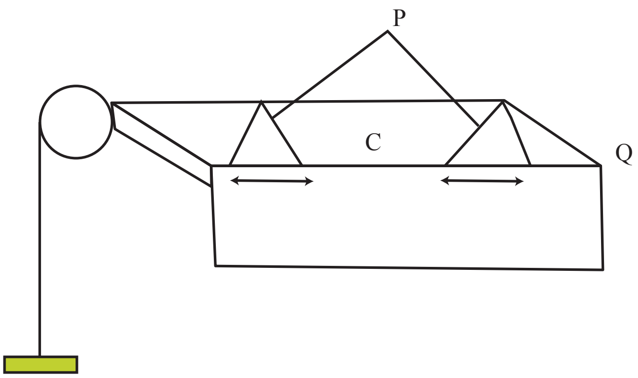
(e) Explain briefly how conduction, convection and radiation effects are minimized in a thermos flask.

10.

(a) State one law of vibration of stretched strings.

(b)

(i) The diagram below illustrates a sonometer set-up. Identify the parts labeled P and Q.



(ii) State two functions of a sonometer.

(iii) A sonometer wire of length, l, vibrates at a frequency of 350 Hz. If the length of the wire is increased by 15 cm, the wire vibrates at 280 Hz under constant tension. Calculate the value of l.

(c) Define each of the following terms:

(i) Fundamental note:

(ii) Overtone;

(iii) Harmonics.

11.

(a) State the laws of electromagnetic induction.

(b) Explain why a positively charged conductor placed near a candle flame causes the flame to spread out.

(c) An alternating current 20 A flows through a pure inductor of inductance 30 mH. Calculate the energy stored in the inductor.

(d)

(i) Define each of the following terms:

(ii) Electron volt;

(iii) Resonant frequency of an a.c. series circuit.

(iv) Show that the resonant frequency, f_o of an L-C-R series a.c. circuit is given by $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

12.

(a) Light of wavelength 7.0×10^{-7} m is incident on a metal of work function 1.20 eV. Calculate the:

(i) energy of the photon;

(ii) energy of the most energetic photo electrons.

[Planck's constant, $h = 6.6 \times 10^{-34}$ J s; $c = 3.0 \times 10^{8}$ m s^{-1}]

(b) Explain each of the following terms:

(i) Nuclear fusion;

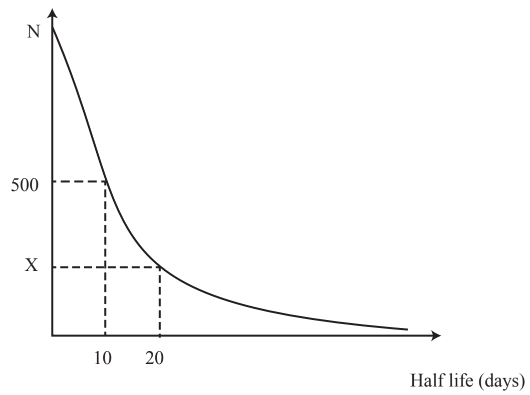
(ii) Nuclear fission.

(c) The diagram below illustrates a decay curve for a radioactive sample of orginal number, N:

Using the graph, calculate the

(i) values of X and N:

(ii) number of sample disintegrated after 40 days.



(d) State two properties of alpha particles.

What is Bullying?

Any unwanted written, verbal, graphic, or physical act by an individual or group toward another person(s) that causes harm or distress.

Types of Bullying

- Physical
- Verbal
- Social
- Emotional
- Cyber

Signs of Bullying

- Headaches
- Depression
- Loss of friends
- School absenteeism
- Academic problems

What You Can Do

PREVENT

- Be a role model for positive communication, healthy relationships, and self-care.
- Reinforce acts of kindness, respect, and inclusion.
- Set policies and rules about bullying.

RECOGNIZE

- Know the definition of bullying and its many forms.
- Talk with and actively listen to the youth who confide in you.
- Watch for warning signs of bullying.

INTERVENE

- If you witness bullying behavior
- Respond quickly and consistently to send the message that it is not acceptable.
- Separate the students involved.
- Meet any immediate medical or mental health needs.
- Stay calm and model respectful behavior.

Source: Teacher's Diary on *Cyber-Crime Awareness* by UNODC, Cybercrime and MoE, Republic of Liberia

REPUBLIC OF LIBERIA
MINISTRY OF EDUCATION

TEXTBOOK MANAGEMENT
Student Textbook Assignment Form

School : ____

District : ______________________Country: ________________

Teacher : _________________________Subject: _______________

Grade : ____

No.	Name of the Student	Name of the Textbook	Barcode Number	Condition of the TB when received	Date Received	Date Returned	Condition of TB when returned	Signature of Student
1								
2								
3								
4								
5								
6								
7								
8								
9								
10								
11								
12								
13								
14								
15								